Name: ____________________
Section: ____________________

EE334 Homework PS1- Fourier and Transfer Functions

Problems from Hambley:

- P6.1, P6.7, P6.8, P6.16

Additional Problems (Instructor Option):

- Any as assigned by instructor

P6.1 The fundamental concept of Fourier theory is that all signals are sums of sinewaves of various amplitudes, frequencies, and phases.

P6.7 The transfer function shows how a filter affects the amplitude and phase of input components as a function of frequency. It is defined as the output phasor divided by the input phasor as a function of frequency.

To determine the complex value of the transfer function for a given frequency, we apply a sinusoidal input of that frequency, wait for the output to achieve steady state, and then measure the amplitude and phase of both the input and the output using voltmeters, oscilloscopes or other instruments. Then, the value of the transfer function is computed as the ratio of the output phasor divided by the input phasor. We change the frequency and repeat to determine the transfer function for other frequencies.

P6.8* The given input signal is

$$v_{in}(t) = 5 + 2\cos(2\pi 2500t + 30^\circ) + 2\cos(2\pi 7500t)$$

This signal has a component $v_{in1}(t) = 5$ with $f = 0$,
a second component $v_{in2}(t) = 2\cos(2\pi 2500t + 30^\circ)$ with $f = 2500$, and a third component $v_{in3}(t) = 2\cos(2\pi 7500t)$ with $f = 7500$. From Figure P6.8, we find the transfer function values for these frequencies:

$$H(0) = 1, \quad H(2500) = 1.25\angle -22.5^\circ, \text{ and } H(7500) = 1.75\angle -67.5^\circ$$

The dc output is $v_{out1} = H(0)v_{in1} = 5$

The phasors for the sinusoidal input components are

$$\mathbf{V}_{in2} = 2\angle 30^\circ \quad \text{and} \quad \mathbf{V}_{in3} = 2\angle 0^\circ$$

Multiplying the input phasors by the transfer function values , results in:

$$\mathbf{V}_{out2} = \mathbf{V}_{in2} \times H(2500) \qquad \mathbf{V}_{out3} = \mathbf{V}_{in3} \times H(7500)$$

$$= 2.5\angle 7.5^\circ \qquad = 3.5\angle -67.5^\circ$$

The corresponding output components are:

$$v_{out2}(t) = 2.5\cos(2\pi 2500t + 7.5^\circ)$$

$$v_{out3}(t) = 3.5\cos(2\pi 7500t - 67.5^\circ)$$

Thus, the output signal is

$$v_{out}(t) = 5 + 2.5\cos(2\pi 2500t + 7.5^\circ) + 3.5\cos(2\pi 7500t - 67.5^\circ)$$

P6.16 The input signal is

$$v_{in}(t) = 1 + 2\cos(2000\pi t) + 3\sin(3000\pi t) + 4\cos(4000\pi t)$$

which has components with frequencies of 0, 1000, 1500, and 2000 Hz. We can determine the transfer function at these frequencies by dividing the corresponding output by the input. Thus, we have

$$H(0) = 3/1 = 3 \qquad H(1000) = \frac{4\angle 30^\circ}{2\angle 0^\circ} = 2\angle 30^\circ$$

$$H(1500) = \frac{3\angle 0^\circ}{3\angle -90^\circ} = 1\angle 90^\circ \qquad H(2000) = \frac{0}{4\angle 0^\circ} = 0$$

$$V_{out}(t) = 3 + 4\cos(2000\pi t + 30^\circ)$$
$$+ 3\cos(3000\pi t)$$