EE334
Fourier and Filters Worksheet

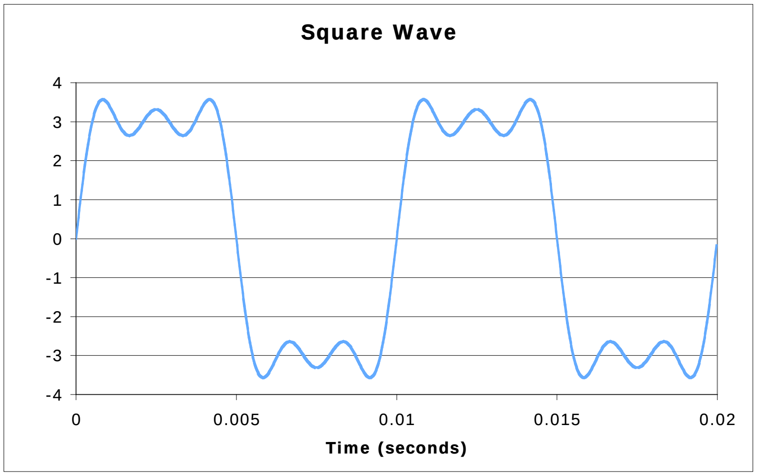

Figure 1. An approximated square wave formed from adding three sine waves.

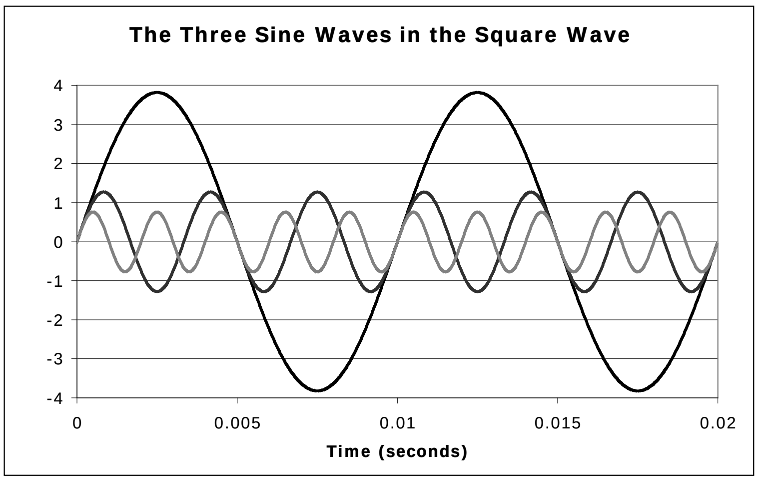

Figure 2. Three sinusoidal components that make the square wave approximation in Figure 1.

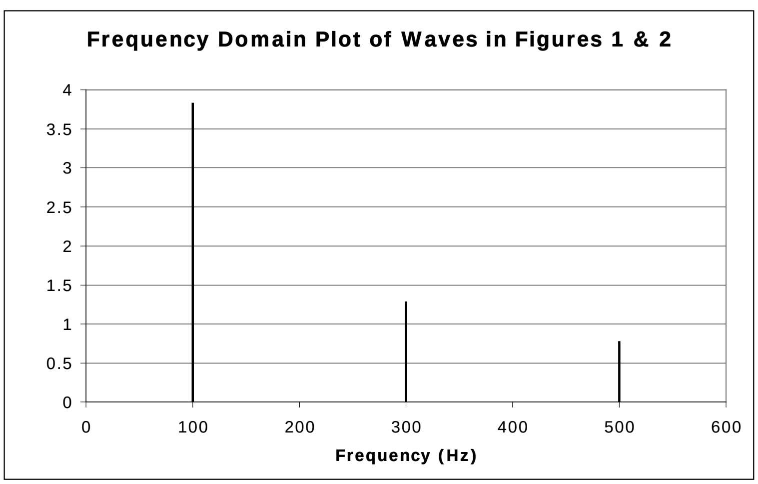

Figure 3. A frequency domain plot of the waves in Figures 1 and 2.

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Figure 1 shows a square wave generated by combining three sine waves each of a different frequency and amplitude. Figure 2 shows the waves in Figure 1 before they were combined into the compound wave of Figure 1. Notice that the waves in Figures 1 and 2 are plotted in terms of volts and time. Figure 3 is a different view of the same waves in Figures 1 and 2. The difference is that we're not looking at the waves in time, but rather by their frequencies. Each of the three sine waves is plotted according to its frequency and peak voltage (amplitude).

FOURIER THEORY (simplified): All waveforms (everything you hear for example) can be reproduced by combining sine waves of the appropriate frequencies and amplitudes. Moreover, even waveforms that weren't created by the combination of sine waves behave as if they were. This means that complex waveforms can be broken down into their sine wave components, i.e., we can separate (filter) out certain frequencies from the mish-mash of frequencies.

Figure 4 shows the frequency response curve for an <u>ideal low-pass filter</u>. The filter is said to be low-pass because it only lets frequencies below 200 Hz pass through it. This is reflected in Figure 4 by the vertical axis that shows the ratio of voltage into the filter to voltage out of the filter: v_{out}/v_{in}. Frequencies higher than 200 Hz are completely blocked (attenuated) – the in/out ratio is zero past 200 Hz.

1. Apply the frequency components of the approximated square wave of Figures 3 to the filter in Figure 4 (draw them on the same Figure 4 plot in the correct frequencies). Plot the resulting waveform in the time domain to the right side

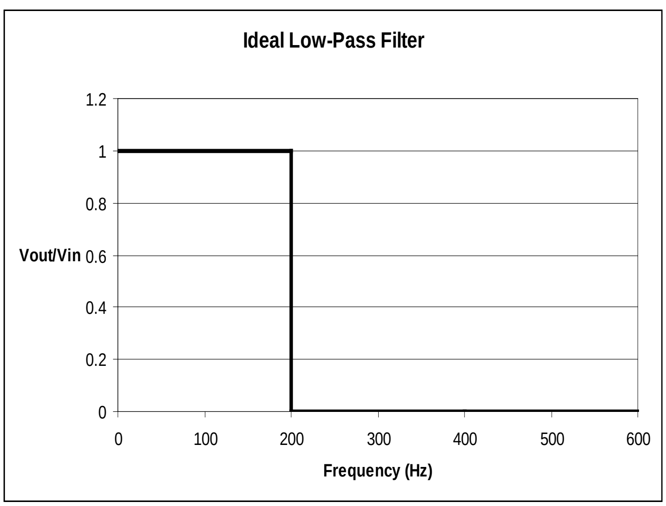

Figure 4. An ideal low-pass filter.

2. Figure 5 shows an ideal high-pass filter. Apply the frequency components of the approximated square wave of Figures 3 to the filter in Figure 5 (draw them on the same Figure 5 plot in the correct frequencies). Plot the resulting waveform in the time domain.

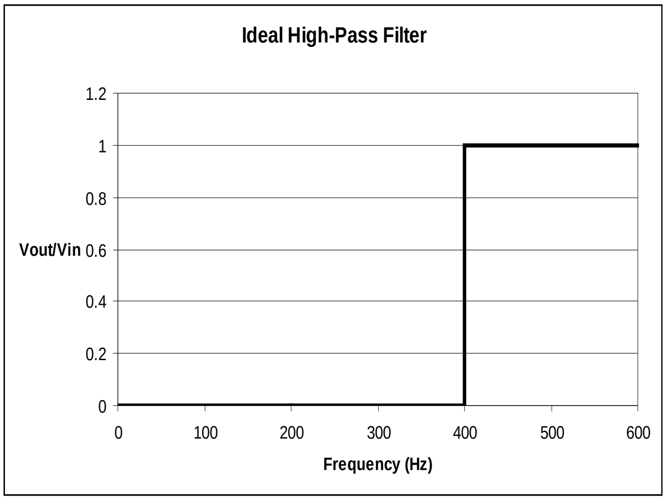

Figure 5. An ideal high-pass filter.

3. Figure 6 shows an ideal band-pass filter. The band-pass filter has two cut-off frequencies: a low cut-off f_{LOW} = 200 Hz and a high cut-off f_{HIGH} at 400 Hz. Signals with frequencies inside the band of f_{LOW} to f_{HIGH} pass through the filter ($v_{out}/v_{in} = 1$) while anything outside the band is completely filtered out ($v_{out}/v_{in} = 0$). Apply the band-pass filter to the compound wave of Figures 1, 2, and 3. Plot the resulting waveform in the time domain.

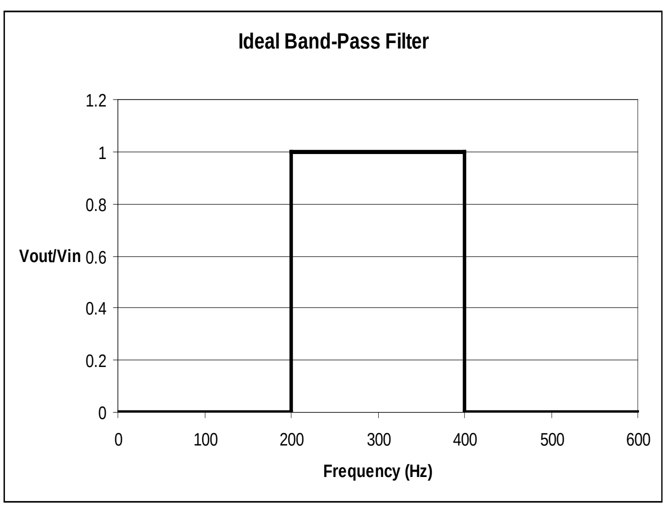

Figure 6. An ideal band-pass filter.

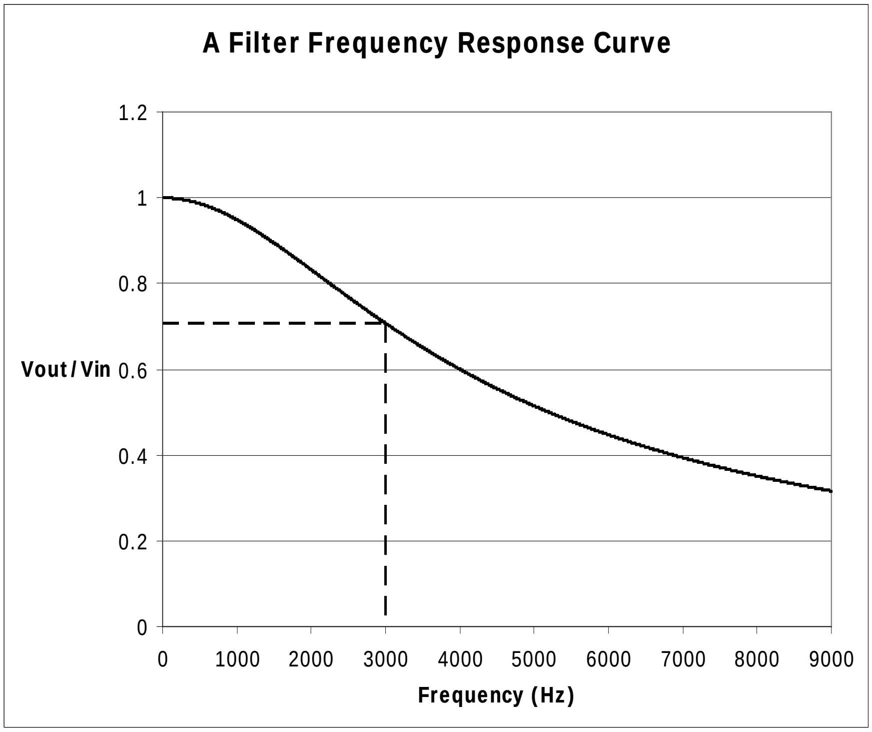

Figure 7. A frequency response curve for some filter.

4. Describe what is going on in Figure 7. Is this a very good filter? Why or why not? Use 2-3 sentences.

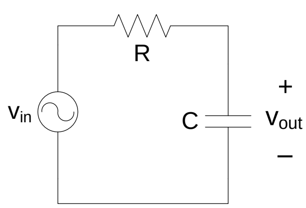

Figure 8. A filter made from a resistor and a capacitor.

The plot in Figure 7 can be made by several simple electronic circuits. The circuit in Figure 8, which consists of just a resistor and a capacitor, is one such circuit. To understand how the circuit in Figure 8 works, we need to know some facts about capacitors.

Capacitor Facts

- The impedance of a capacitor is dependent upon frequency (as opposed to a resistor, which theoretically behaves the same way regardless of input frequency). We remember that:

$$Z_C = \frac{1}{j\omega C} = \frac{1}{j2\pi f C}$$

- The cut-off, by definition, is the half power point (-3 dB), which equates to voltage and power ratios of:

$$\frac{v_{out}}{v_{in}} = \frac{1}{\sqrt{2}} = 0.707 \qquad \frac{P_{out}}{P_{in}} = \frac{1}{2} = 0.5$$

- Therefore, some quick math using voltage division and Figure 8 tells us that the cut-off frequency for an RC filter (both low-pass and high-pass) is given by:

$$f_{co} = \frac{1}{2\pi RC}$$

5. Express v_{out} (write an equation) as a function of v_{in}, R, and Z_C using voltage division.

6. Pretend the voltage supply in Figure 8 is at 0 Hz (a DC signal). Without doing any equations, how much of the voltage produced by the voltage supply will be across the capacitor? How much across the resistor?

7. Now suppose the power supply is operating at ∞ Hz. How much of the power supply's voltage, v_{in}, will be across the capacitor? How much across the resistor?

8. What kind of filter is this?

9. Modify the previous equation to be of the form v_{out}/v_{in}. This is called the <u>transfer function</u> or H(f) for short.

$$H(f) = \frac{v_{out}}{v_{in}}(f) =$$

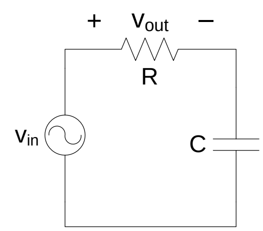

Figure 9. Another RC filter.

10. Figure 9 is the same as Figure 8, but v_{out} has been moved to be across the resistor. Derive the transfer function for the circuit in Figure 9.

11. What is the ratio of v_{out} to v_{in} when the power supply's frequency is 0 Hz (f = 0 Hz)? What is the ratio when $f = \infty$?

12. What kind of filter is this?

13. Draw a frequency response curve (in the fashion of Figure 7) for the circuit in Figure 9. A sketch is OK.

Figure 10. Generalized frequency response curve for the circuit in Fig. 9.

14. The facts section gives an equation for the cut-off frequency for RC filters. Substitute $f = f_{co}$ into the transfer functions you derived for parts #13 and #14. Any surprises? Look at Figure 7. Notice anything about the dashed lines on the diagram and the transfer function when $f = f_{co}$? Add similar dashed lines to your curve in Fig 10.

Inductor Facts

- Recall:

$$Z_L = j\omega L = j2\pi fL$$

- At the cut-off frequency the following are true (same as for an RC filter):

$$\frac{v_{out}}{v_{in}} = \frac{1}{\sqrt{2}} = 0.707 \qquad \frac{P_{out}}{P_{in}} = \frac{1}{2} = 0.5$$

- The cut-off frequency for an RL filter is given by:

$$f_{co} = \frac{R}{2\pi L}$$

15. At what frequency is an inductor's impedance the lowest? When is it the highest? How do these answers compare to a capacitor's extremes?

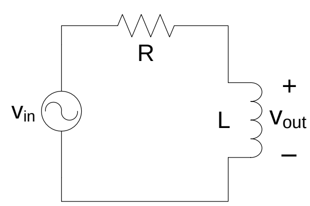

Figure 11. An RL filter.

16. Derive the transfer function, H(f), for the circuit in Figure 11.

17. Draw a frequency response graph for the circuit of Figure 11. Make sure you label axes and indicate f_{co}.

18. Is this a low-pass or high-pass filter? Why?

19. Based on what you saw with RC filters, what kind of filter do you think would result if we moved v_{out} from the inductor to the resistor? Why do you think this? No need to do math.

20. Redraw the circuit of Figure 11 below as Figure 12 but move v_{out} to the resistor.

Figure 12. Another RL filter.

21. Derive H(f) for the circuit in Figure 12.

22. Draw a frequency response curve for the transfer function. Label axes.

23. Substitute $f = f_{co} = \frac{R}{2\pi L}$ into the transfer functions you derived. How do these results compare to the transfer functions for the RC filters when $f = f_{co}$?

24. You are presented with an RC filter and told that $R = 2.5k\ \Omega$ and that $C = 4.7n$ F. What is the cut-off frequency for this filter? Is this filter high-pass or low-pass?

25. You are told to create a low-pass filter using an inductor and a resistor. The resistor has a value of $5.6k\ \Omega$. The cut-off frequency needs to be 500k Hz. What size of inductor should you use? What size inductor would you use to create a high-pass filter using the same sized resistor?

26. Examine the circuit in Figure 13. It's a kind of filter. Make and educated guess about what frequencies produced by the power supply at v_{in} will make it to v_{out} without being attenuated too much. **Hint:** Look at the low-pass and high-pass RC and RL filters where v_{out} was measured across the resistor. Think of superimposing those filters on top of each other to produce the one in Figure 13.

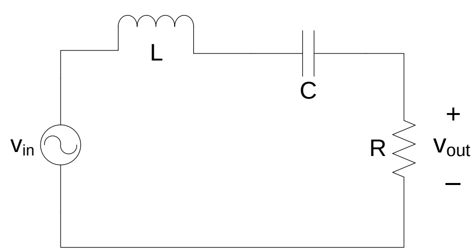

Figure 13. A new type of filter.

27. Derive the transfer function for the circuit in Figure 13.

28. Recall that the transfer function tells us how much of v_{in} makes it (passes through) to v_{out}. What occurs at frequencies of 0 Hz and ∞ Hz? Does this make sense?

29. Solve for the frequency that produces this maximum value from this transfer function.

30. Sketch the frequency response curve for the circuit in Figure 13.

31. What kind of filter is this? Hint: You saw an ideal version earlier in this packet.

32. The frequency where the ratio v_{out}/v_{in} is highest is called the center frequency, f_c. Label f_c on the graph in #38.

33. There are two cut-off frequencies for this filter. One is on the low frequency side of fc and is usually called f_{LOW}. The other is on the high side and is called f_{HIGH}. Label f_{LOW} and f_{HIGH} on the graph in #38. Make sure you indicate the value of v_{out}/v_{in} at the cut-off frequencies.