

## EE334 DAC

A digital recording is fine as far as it goes. It can be stored, manipulated easily (edited), transmitted, etc. However, a digital recording can't be understood directly by humans. To make sense of a digital recording, the recording must be converted back to analog. Figure 1 shows, at a high level, the **digital-to-analog conversion (DAC)** process for audio.

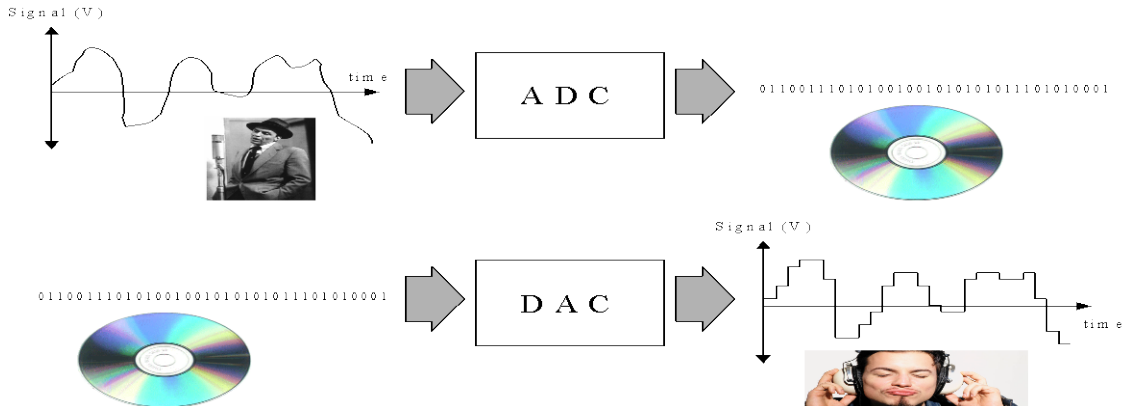


Figure 1. The digital-to-analog conversion process for audio.

In this lesson, you'll learn how to do DAC, compare the reconstructed signal to the original, and experience how errors in the ADC encoding affect the reconstructed analog signal.

1. Here's a digital signal: 101110111101011010011000. Can you convert it back to analog? Yes? No? Do you need more information?
2. You are told that the signal above was encoded using a 3-bit ADC. How many quantization levels did the ADC have?
3. How many quantization levels should be present in the reconstructed analog signal?
4. You are now told that the reconstructed analog signal needs to span a range from 0V to 1.4V. What is the resolution of the analog signal? Hint: Starting at 0V there should be even jumps all the way to 1.4V. Also, 000 corresponds to 0V and 111 to 1.4V. **Be careful!** The bottom and top of the range are set. This isn't as simple as dividing the voltage range into 8 parts.

EE 334  
DAC

Here's the generic equation for determining the resolution of a DAC device.  $N$  = number of bits per sample.

$$resolution_{DAC} = \frac{V_{max} - V_{min}}{number\_levels - 1} = \frac{V_{max} - V_{min}}{2^N - 1}$$

5. Label the vertical axis of the graph in Figure 2 below for the voltage levels of the reconstructed signal. You may wish to include both the binary number and voltages (one represents the other).
6. Label the horizontal axis of the graph below for reconstructing a signal that was sampled at 1 msec increments.
7. Partially reconstruct the signal by placing dots at where the sample time and voltage level of each sample meet.
8. This is almost a completely reconstructed signal. What's missing or wrong?
9. Complete the reconstructed signal by connecting the reconstructed sample points as directed by the instructor.

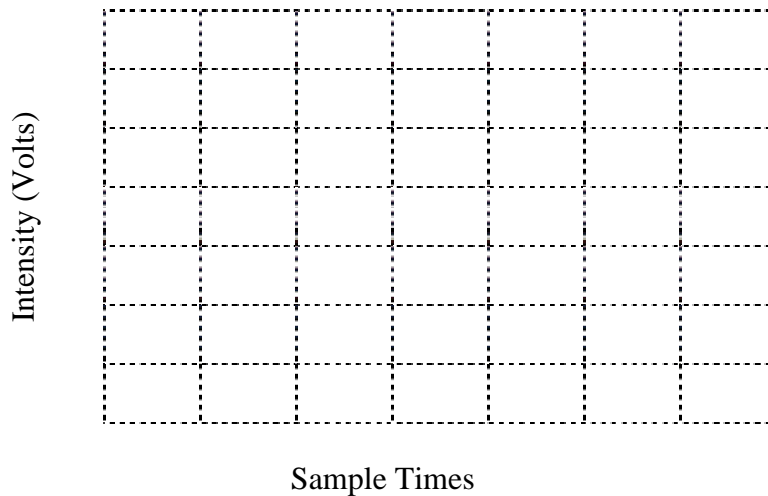


Figure 2. A chart for doing DAC.

10. Compare this reconstructed signal to the original analog signal in Figure 4 of the ADC note set. Make observations about the following things.
- Is there a DC bias in the reconstructed signal? If so, how do you make it disappear?
  - The end voltages of the original analog signal aren't the same as the reconstructed signal. However, they do have about the same overall range ( $\sim 2$  V). If the original signal and the reconstructed signal were sent to a speaker, would the original be louder, quieter, or about the same as the reconstructed signal?
  - List at least two ways we could adjust the loudness of the reconstructed signal.
  - Do the time scales match?
  - What would happen if the digital signal was played back at 0.5 msec per sample?

### A Simple Circuit for DAC

As is the case with ADC, there are several circuits that do DAC. The simplest and probably most commonly used circuit is the summing opamp. Figure 3 shows a summing opamp set up to do 3-bit DAC for the signal we just converted. The circuit uses simple resistor values so that you can more easily see the relationships between the input resistors and the feedback resistor. (Instructor reminder:  $R_f = 4$  ohms,  $R_0 = 100$  ohms)

Figure 3. A summing OpAmp circuit for 3-bit DAC.

EE 334  
DAC

In the circuit, the three input voltages behave as follows:

- If the bit corresponding to the input is 0, the input voltage is 0V. E.g. if  $b_0 = 0$ , then  $v_0 = 0V$ .
- If the bit corresponding to the input is 1, the input voltage is 5V. E.g., if  $b_0 = 1$ , then  $v_0 = 5V$ .

**Note:** In this example, the voltage corresponding to a bit-value of 1 is 5V. Generally, the voltage for a HIGH, can be any voltage that's greater than zero.

Recall that the summing amplifier's output is the sum of following equation for each input resistor and feedback resistor combination.

$$v_{out} = -\frac{R_f}{R_{in}} v_{in}$$

More specifically, the voltage out of an summing opamp DAC is:

$$v_{out} = \sum_{b=0}^{N-1} -\frac{R_f}{R_b} v_b = -R_f \sum_{b=0}^{N-1} \frac{1}{R_b} v_b$$

11. The bit pattern 001 ( $b_2b_1b_0$ ) is fed to the DAC circuit. What is the output of the opamp? Does value agree with your earlier DAC results?

12. How does this number compare to the resolution of the DAC from earlier? In other words, what's special about binary 001 and the resolution? Hint: What is the value of 001 in decimal?

13. Without solving an equation, predict the output voltage of the DAC circuit for the bit pattern 010. Hint: What is the value of 010 in decimal?

14. Now solve an equation for the output given bit pattern 010.

EE 334  
DAC

15. Do you see a pattern or a shortcut to solving the summing amplifier equation? Describe the pattern or shortcut.
16. Quick! What's the DAC output for 110?
17. What's the DAC voltage output for 100?
18. What's the relationship between the input resistors  $R_2$ ,  $R_1$  and  $R_0$ ? Surprised?
19. What's the relationship between ratios  $R_f/R_2$ ,  $R_f/R_1$ , and  $R_f/R_0$  and the relative values of  $b_2$ ,  $b_1$ , and  $b_0$ ?

The summing amplifier DAC can be extended to any number of bits. The relationships described in problems 18 and 19 will, of course, need to hold no matter how many bits are used.

## Aliasing

Aliasing occurs when the sample rate is too low. Figure 4 shows an example of aliasing for an audio signal.

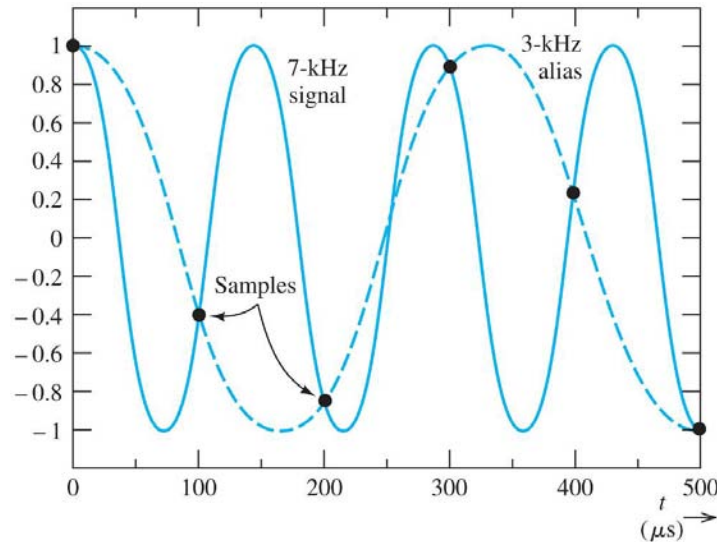


Figure 4. An example of aliasing caused by sampling at too low a rate.

20. What is the sample rate in Figure 4?

When reconstructed, the sampled signal is reproduced as 3 kHz signal (maybe not a sine wave as the figure shows, however). So even though the original signal is a 7 kHz sine wave, a 3 kHz **alias** results.

21. Did the ADC-to-DAC process in Figure 4 lose information or introduce information that wasn't there?

22. Describe some other examples of aliasing. Hint: you might have seen visual aliasing while watching a movie or TV show.

## The Nyquist Rate

Another one of our famous, but dead scientists is Nyquist. Nyquist is most famously remembered for answering the question (at least for audio) of “How fast do I have to sample?” It turns out that you must sample at least twice as fast as the highest frequency present in the input signal. If you don’t sample at least this fast, aliasing will result. This sample rate is called the Nyquist frequency. Here’s the equation

$$f_{\text{Nyquist}} \geq 2f_{\text{max}},$$

where  $f_{\text{max}}$  is the highest frequency present in the signal being sampled.

**Note that sampling at the minimum Nyquist rate ( $2f_{\text{max}}$ ) only guarantees freedom from aliasing.** It doesn’t mean that you necessarily have a high-quality ADC product. **Sampling audio at 2.5 to 3 times  $f_{\text{max}}$  is usually enough to fool human ears.**

## Quantization Error

The other way to get poor ADC is to have too few quantization levels. Consider the image which in Figure 5 which shows an analog wave and its reproduced digital version.

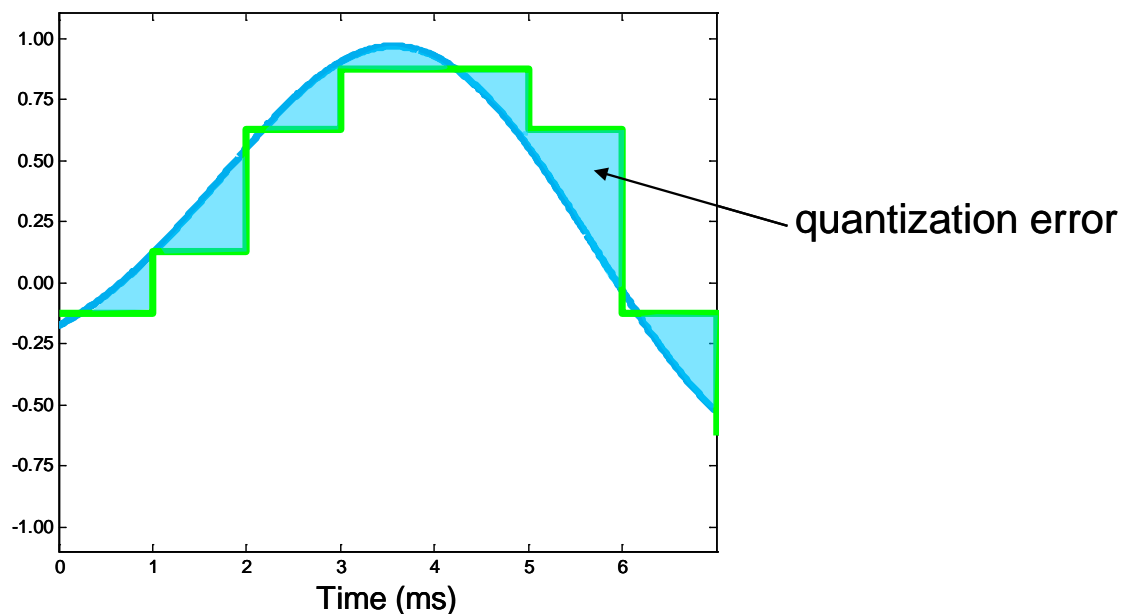


Figure 5. An example of quantization error.

The shaded regions show the error between the original analog signal and the reconstructed signal resulting from DAC.

23. Review: What are the two things we can do to reduce the quantization error?

Increasing the sampling rate in this example will help some, especially if the minimum Nyquist rate is not being satisfied. Unfortunately, increasing the sampling rate will not reduced the voltage level (intensity levels) errors; instead it will just make more errors.

Why do we care about the voltage level errors? The simple answer is this: When played back, the voltage errors are heard as noise.

24. Review: What is noise?

25. Do we have unwanted voltages in the recovered signal?

26. Thus, do we have noise?

The following equation gives the rms noise voltage in any quantization process.

$$v_{\text{noise}} = \frac{\text{resolution}}{2\sqrt{3}} = \frac{q}{3.46}$$

Figure 6 shows the effects of increasing sample rates and the number of quantization levels on a signal.

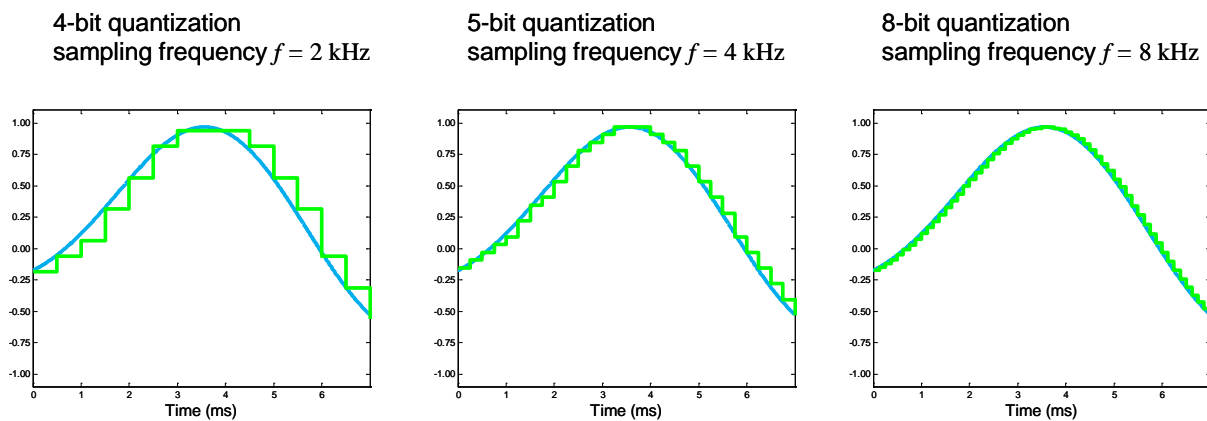


Figure 6. Effects of increasing sample rate and the number of quantization levels.

Setting Resolution Levels

It can be tricky to find just the right number of quantization levels to use. The best guide is probably subjective evaluation (let your ears or eyes be your guide). Sometimes the specifications will be provided for you. Here are two common cases.

First, someone might say that they want the resolution to be “ $x$  percent or better” of the range of the analog signal. In other words, you need to have enough quantization levels such that the resolution of each level is less than  $x$  percent of the total range. Let’s work a quick example.

27. The voltage range of an audio signal is 2V. We want a resolution that’s at least 1% of the range. Task: What is 1% of 2V?

28. Find the minimum number of sections needed to carve up the 2V range so that each section has a 1% resolution or better. The number of sections must be a power of 2! IOW, find the smallest  $N$  that satisfies the following equation

$$\frac{\text{signal\_volt\_range}}{2^N} \leq \frac{x\%}{100} \cdot \text{signal\_volt\_range} \text{ OR } \frac{100}{2^N} \leq x\%$$

29. For your answer to problem 28, how many bits will the ADC need? IOW, how many bits are needed for each sample?