

EE 334  
Digital Modulation and Channel Capacity

We've talked often in this course about bandwidth – the frequency range over which a device operates. For regular analog transmissions typical bandwidths include:

- 3 kHz for telephone quality voice
- 15-20 kHz for high-fidelity music
- 6 MHz for television

Bandwidth is related to the concept of **channel capacity**,  $C$ , which is the binary data transmission rate of a transmission medium in bits per second.

It is rare that binary data can be transmitted directly on any medium (short distances over copper and fiber optic cables are the notable exceptions). Instead, binary data is usually converted to some form of analog signal as in the ASK, FSK, and PSK which we'll get to in a few moments.

### **Binary Digital Encoding**

First of all, we realize that a binary signal (0s and 1s) is really just voltages. For example, binary 0 might be 0V and binary 1 might be 5V.

The bit rate of a binary stream is the sampling frequency times the number of bits generated for each sample:

$$\text{bit rate} = (\text{sample rate}) \cdot (\text{bits per sample})$$

1. A signal is sampled at 8kHz using an 8-bit ADC. What is the bit rate?

The raw binary data resulting from ADC (or from any source generating binary data) is sometimes called the **baseband signal**.

The **channel** (a medium such as free-space, copper wire, fiber optic cable, etc.) used for transmission must have sufficient bandwidth.

### Transmitting Binary Data

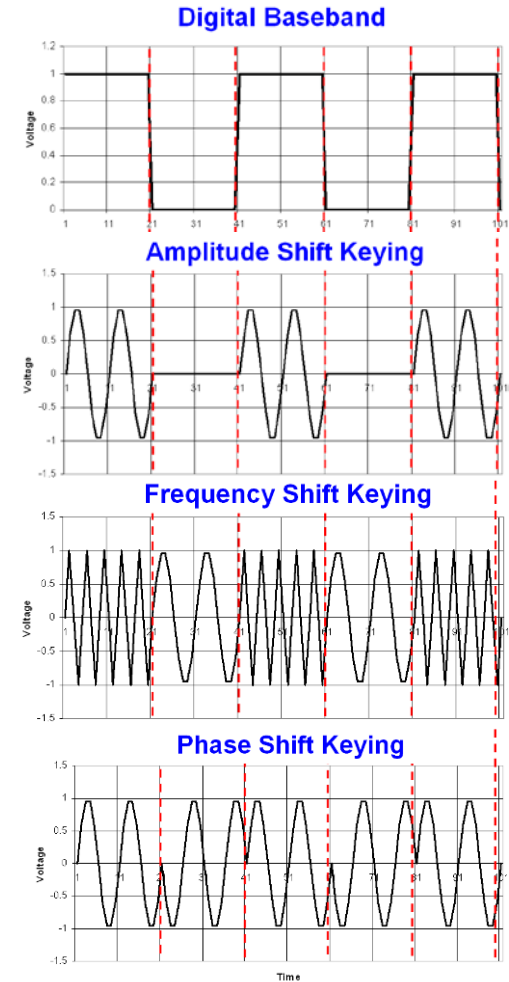
In some cases, such as fiber optic cables and short distances over copper wires, binary signals can be transmitted as true binary signals. That is, we can transmit binary signals by alternating voltages (0V and 5V) on a copper wire or by turning a light off (binary 0) and on (binary 1) very quickly for a fiber optic cable.

However, for long distances over copper and over-the-air transmission (aka radio), we cannot send binary signal directly. Instead, the baseband signal must be modulated onto an RF carrier using some modulation scheme:

$$v_c = V_c \cos(2\pi f_c t + \theta)$$

- Amplitude Shift Keying (ASK) – aka Amplitude Modulation (AM). Data relayed by modifying the carrier's voltage,  $V_c$ .
- Frequency Shift Keying (FSK) – aka Frequency Modulation (FM). Data relayed by modifying the carrier's frequency,  $f_c$ .
- Phase Shift Keying (PSK) – Data relayed by modifying the carrier's phase,  $\theta$ .

The graphs to the right show the simplest instances of each modulation scheme. Each scheme is transmitting the binary data shown in the top graph (Digital Baseband).



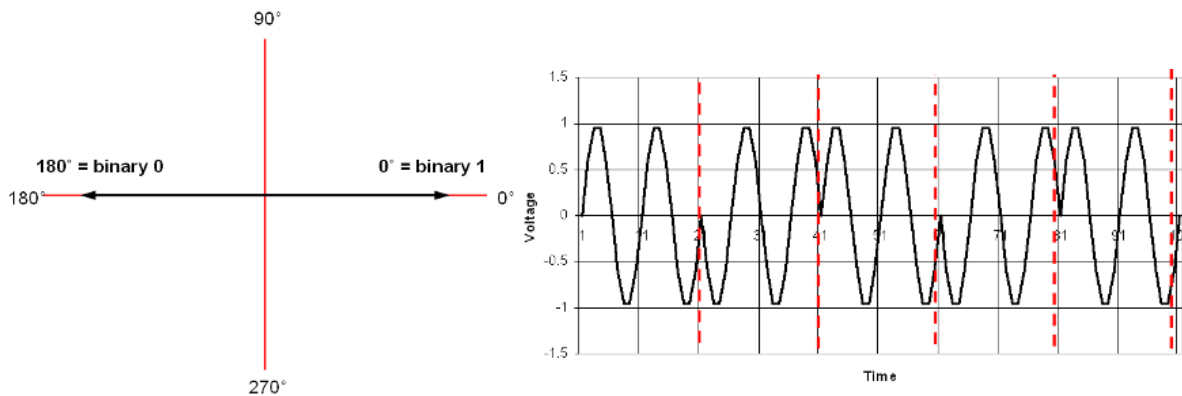
Combinations of the three modulation techniques are also possible. For example, ASK and PSK are often combined in what is called Quadrature Amplitude Modulation (QAM). We'll see more about QAM in a bit.

2. FSK and PSK could also be combined, but this is not commonly done (if ever). Why?
3. An FSK system uses 16 different frequencies to encode bits. How many bits are represented by each frequency?
4. If a modulation scheme uses 3 bits per symbol (a symbol is an amplitude, frequency, or phase), how many symbols are being used?

One of the most common digital modulation techniques is a combination of ASK and PSK called Quadrature Amplitude Modulation (QAM). To understand how QAM encodes bits, we need to study high-level PSK first.

### Binary PSK (BPSK)

BPSK is the simplest PSK system. It uses two phases to encode bit values. A phase diagram for BPSK and a time domain diagram are shown below.



The phase diagram describes the two symbols and their meaning: a phase shift of 0 = binary 1 and a shift of 180 = binary 0.

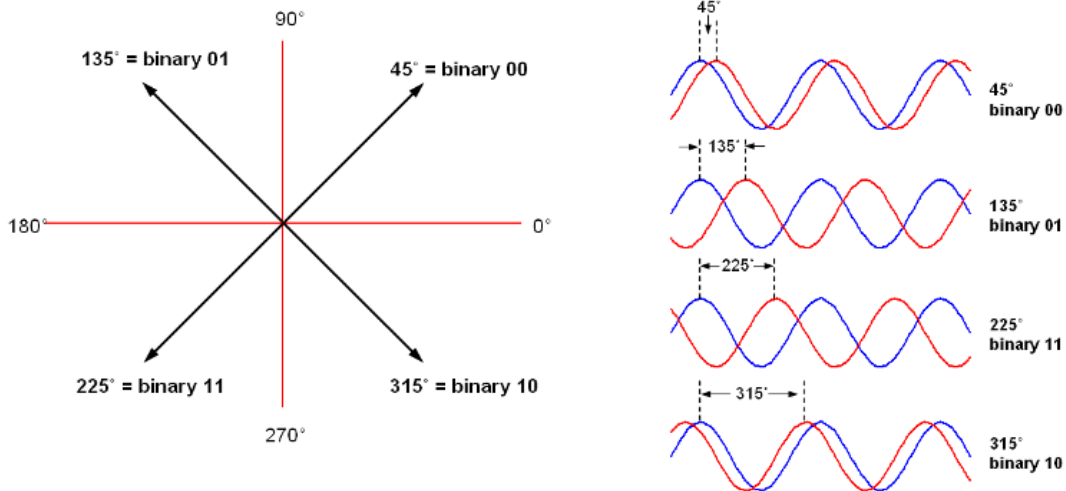
5. Write down the bits transmitted by the signal in the BPSK time domain diagram above.
6. How many bits per symbol are transmitted under BPSK?

### ANSWER 7-10 AFTER COVERING HARTLEY EQUATIONS

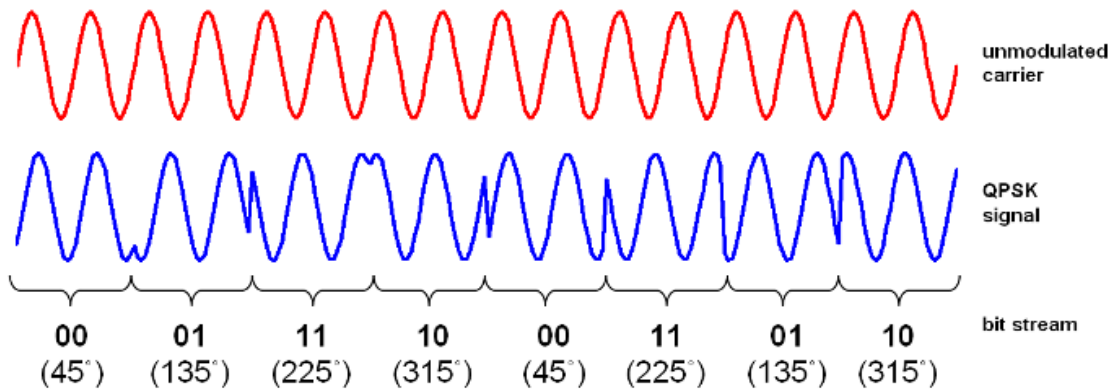
7. In the time domain BPSK diagram we see that the carrier wave maintains a phase shift for two cycles. How many bits per cycle are transmitted by this BPSK signal?  
 $k =$
8. If the carrier in the BPSK diagram above has a bandwidth of 500MHz what is the bit rate of the BPSK signal?
9. What does Hartley's Theorem predict the bit rate should be for the system described in the previous problem?
10. Why the discrepancies between the bit rates of the two previous problems?

**Quad PSK (QPSK)**

QPSK uses four phase shifts to encode bits. A phasor diagram and example of the four possible phase shifts in a QPSK system are shown in the following illustrations.



QPSK phasor diagram (above left) and example phase shifts (above right).

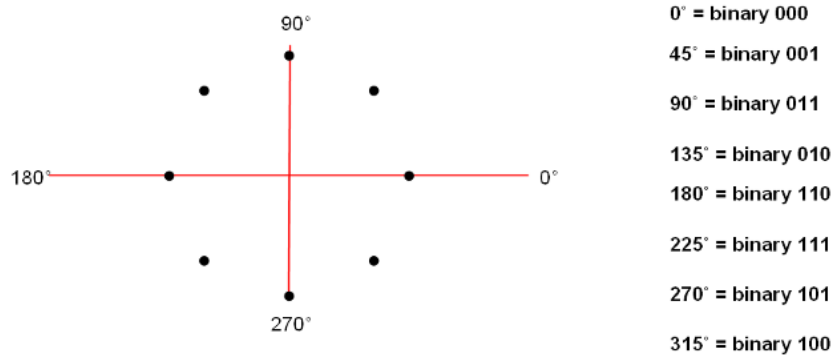


A QPSK system transmitting a bit stream. PSK systems are difficult for humans to interpret just by looking. The unmodulated carrier provides a reference that helps some.

**M-ary PSK**

In order to increase the data rate without increasing bandwidth, we can further increase the number of bits per symbol. For example, in the 8-PSK constellation below, 8 possible phase shifts allow 3 bits to be transmitted by each symbol.

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11. Consider BPSK, QPSK, and 8-PSK signals. Put them in order from most noise resistant to most vulnerable to noise.

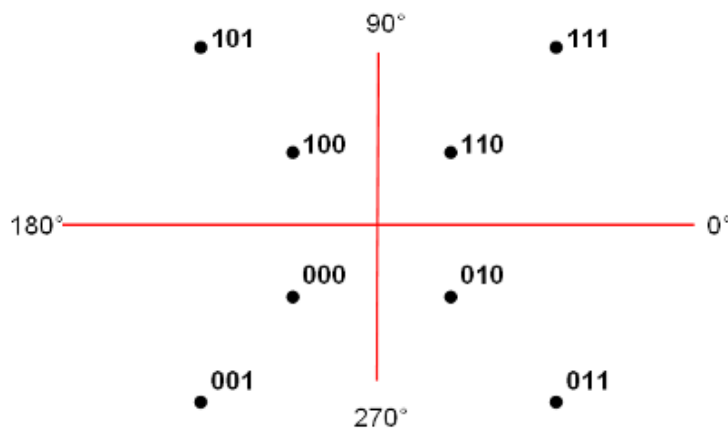
Most Noise Resistant

Most Vulnerable to Noise

12. Why did you answer the previous problem as you did?

**Quadrature Amplitude Modulation (QAM)**

In order to increase the distance between points in the signal constellation, another option is to modulate both the amplitude and the phase. This is called **quadrature amplitude modulation (QAM)**. An example constellation for QAM is shown below.



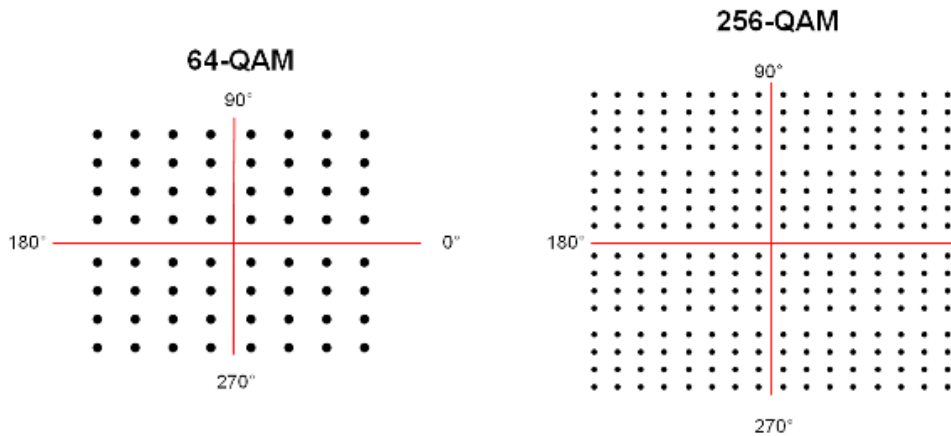
13. Draw concentric circles to show the voltage levels in the QAM system shown above.

14. How many symbols are in the QAM system shown above?

15. Enumerate the symbols for the QAM system show above (amplitude, phase)
16. Construct a constellation for a 16-QAM system. Note that there are at least two possibilities.

**Higher level QAM signals**

QAM signals can be extended to much higher bit rates as depicted. 64-QAM and 256-QAM are common in cable modems, satellites, and high-speed broadband wireless.



17. Put QAM, 16-QAM, 64-QAM, and 256-QAM in order from most noise resistant to most vulnerable to noise.

Most Noise Resistant

Most Vulnerable to Noise

## Bit Rates and Baud

One of the benefits of using digital modulation techniques like 16-PSK and multi-level QAM is that we can transmit lots of bits even though the transmission rate is lower. Sounds weird, but it makes sense once you think about it. Let's investigate beginning with a few definitions.

- **Bit rate** is the effective number of bits that transmitted per second.
- The inverse of the bit rate is the **bit time**. It's the effective time between consecutive bits. It's also the inverse of the bit rate.
- **Baud** is the number of symbols transmitted per second. You might hear the term "baud rate" sometimes, but saying "baud rate" is redundant.
- **Symbol time** is the inverse of baud; it's the time between consecutive symbols. Symbol time is sometimes called the **symbol period** since in many systems the symbol has to be held steady for some period of time to allow the receiver to see the symbol for long enough to make sense of it.

Bit rates can be higher than the baud because each symbol transmitted can represent several bits. Reason about it by doing the following problem concerning a 64-QAM system.

18. A 64-QAM system uses (obviously) 64 combinations of phase and amplitude to transmit binary data. There are 1000 phase/amplitude changes per second.
  - a. How many bits are needed to represent 64 things (things being frequencies in this case)? IOW, how many bits are represented by each combination of phase and amplitude?
  - b. Thinking of each phase/amplitude combo as a symbol, how what is the baud of the system?
  - c. What is the symbol time?
  - d. What is the bit rate?
  - e. What is the bit time?
  - f. How many times faster is the bit rate than the baud?

### Hartley's Law

The relationship between channel capacity,  $C$ , and the bandwidth of the channel,  $B$ , is captured in the following relationship:

$$C \propto kB$$

where  $k = \text{bits/cycle}$ . "Cycle" can be loosely interpreted as one cycle of the carrier wave. However, in many transmission systems the carrier goes through several cycles. For example, in PSK the carrier will go through several cycles (will stay phase shifted for several cycles) in order to relay one symbol. A symbol may represent one or more bits.

19. In your own words explain what  $C \propto kB$  means.

Some years ago an engineer named Hartley derived an expression for the maximum possible channel capacity. This is known as **Hartley's Law**:

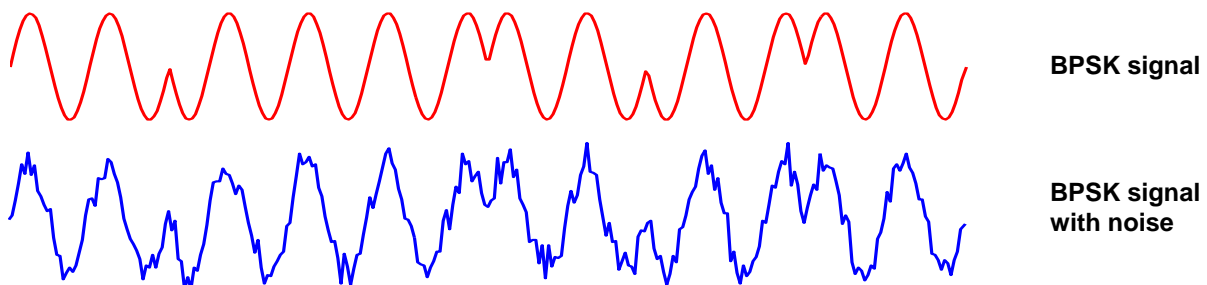
$$C = 2B \log_2 N = 2Bn$$

where  $n$  is the number of bits represented by each symbol transmitted (bits/symbol) and  $N$  is the number of symbols used (e.g., 2, 4, 8, 16, ...). Note that  $n = \log_2 N$ .

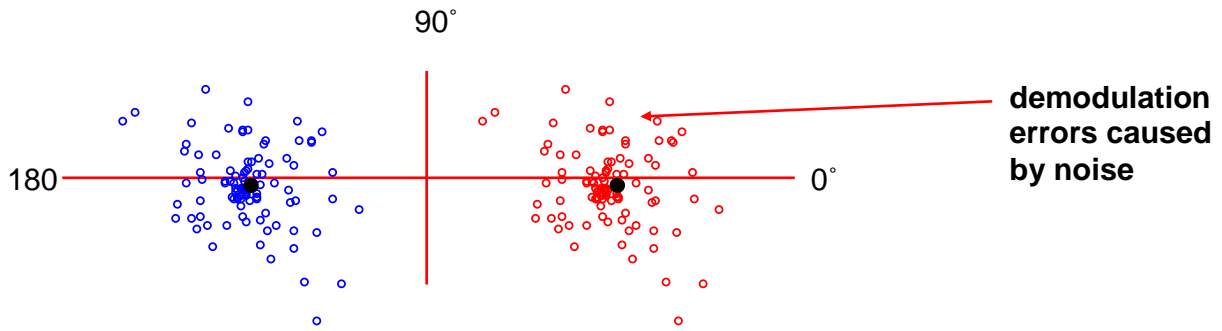
20. Hartley's Law is a best case law. That is, it gives the upper limit on channel capacity. That upper limit is only possible when conditions are perfect. What are "perfect conditions"?

### Noise Effects

Look at the BPSK signal below. The first plot is the signal as transmitted. The second one is how the signal looked when received. The distortions in the received signal were caused by noise (random voltage fluctuations).

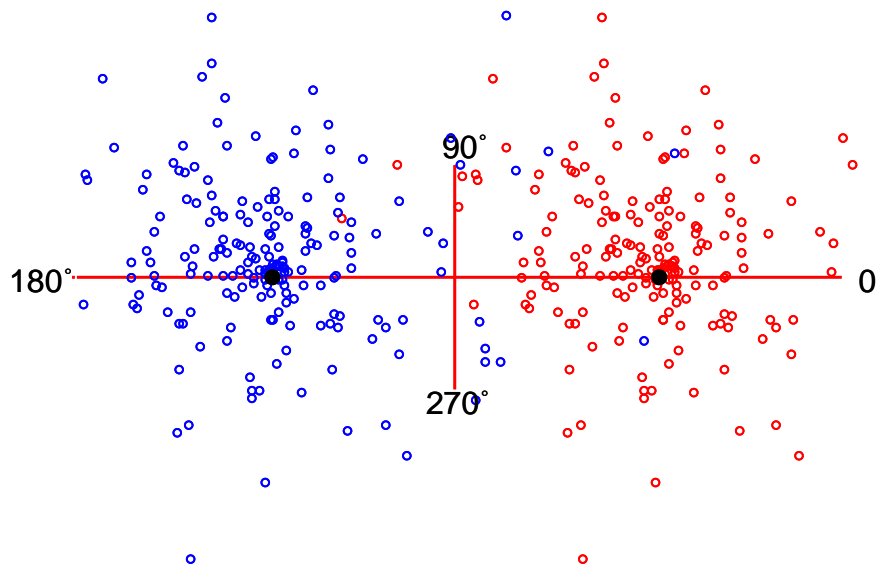


Here's how received BPSK signal looks as a phasor diagram. The SNR is 10dB.



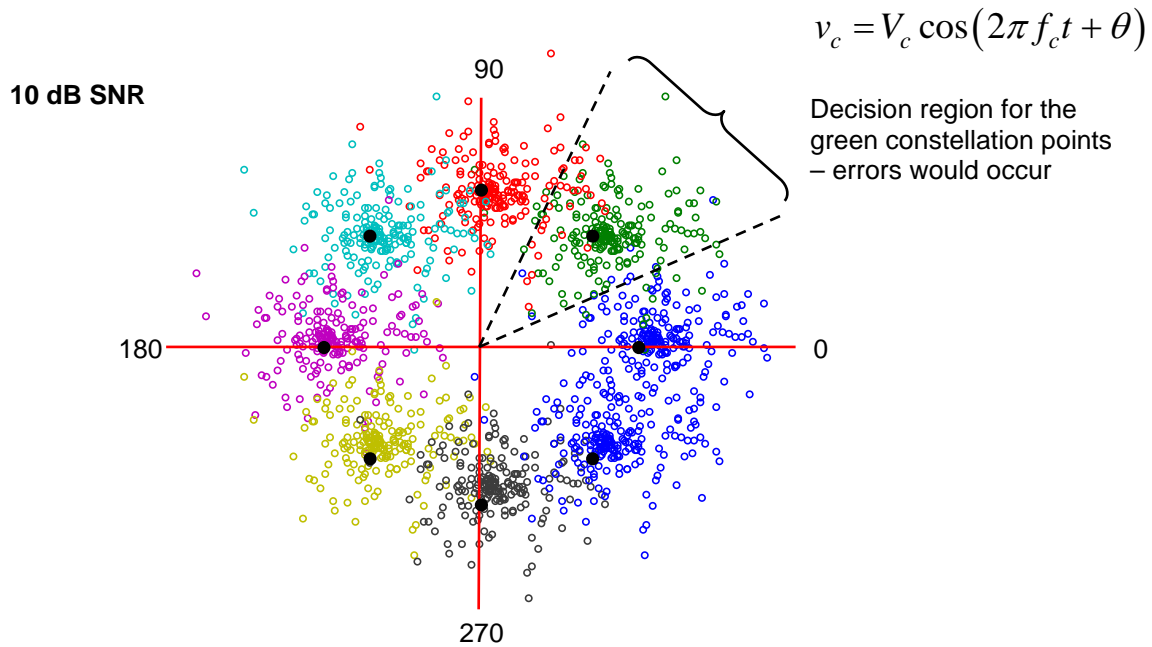
21. Looking at the diagram above, will the received signal be interpreted correctly? Why or why not?

22. Examine the BPSK phasor diagram show below. It has an SNR of 2dB; it's noisier than the BPSK signal above. Will the receiver misinterpret some of the bits now?



So, at 10dB a BPSK signal is received correctly a vast majority of the time (near 100% error-free). But at 2dB errors are fairly common.

23. Will the error rate for 8-PSK be higher than BPSK if both signals have an SNR of 10dB? Hint: 8-PSK uses eight phase angles instead of BPSK's two angles.



24. Based on what you've seen. What will happen to the error rates for 16-PSK, 32-PSK. Etc., if the SNR stays at 10dB?

25. Which transmission scheme would you use with an SNR of 10dB?

26. What's the advantage of 8-PSK over BPSK?

27. When is it safe to use 8-PSK?



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29. (Homework Problem) A voice-grade telephone line has a bandwidth of 3100 Hz and an S/N ratio of 30 dB.

- a. Calculate the maximum channel capacity
  
  
  
  
  
  
  
  
  
  
- b. How many symbols will a modulation scheme have to support in order to transmit this bit stream on the telephone line?
  
  
  
  
  
  
  
  
  
  
- c. What is the baud?

30. (Homework Problem) A cable television company uses a TV channel's worth of bandwidth (6 MHz) for delivering Internet data to a neighborhood. To keep customers happy, the cable company needs to support a peak data rate of  $60 \times 10^6$  bits/sec on the channel. What is the minimum SNR that allows this data rate?