EE354 Study Guide for Fourier Exam

Provided Reference Sheets

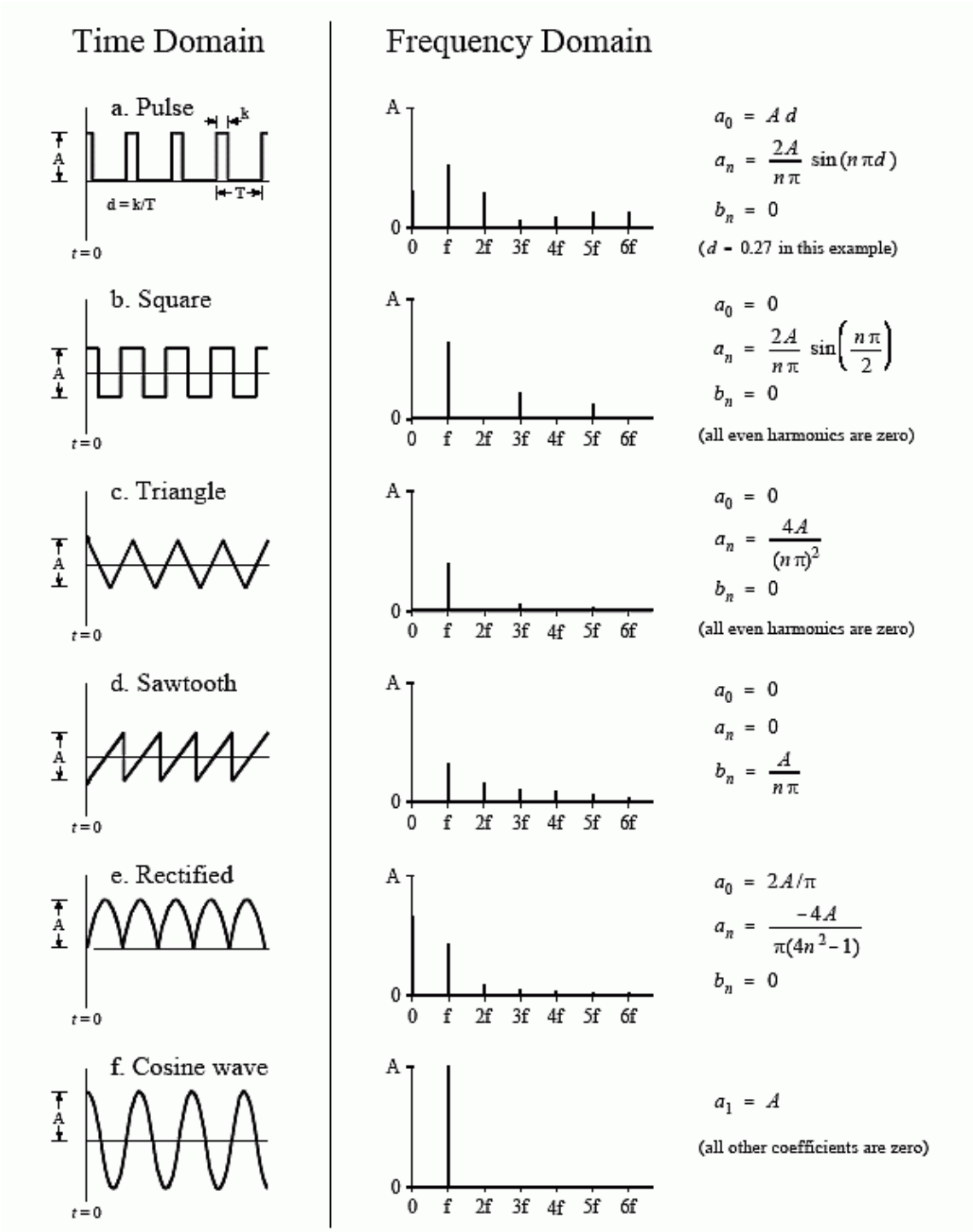

FIGURE 13-10
Examples of the Fourier series. Six common time domain waveforms are shown, along with the equations to calculate their "a" and "b" coefficients.

Trigonometric Fourier Series

$$s(t) = a_0 + \sum_{n=1}^{\infty}\left[a_n \cos(2\pi n f_0 t) + b_n \sin(2\pi n f_0 t)\right] \qquad \text{Eqn. 2.73}$$

Where:

$$a_0 = \frac{1}{T}\int_{t_0}^{t_0+T} s(t)\,dt \qquad \text{Eqn. 2.76a}$$

$$a_n = \frac{2}{T}\int_{t_0}^{t_0+T} s(t)\cos(2\pi n f_0 t)\,dt \qquad \text{Eqn. 2.76b}$$

$$b_n = \frac{2}{T}\int_{t_0}^{t_0+T} s(t)\sin(2\pi n f_0 t)\,dt \qquad \text{Eqn. 2.76c}$$

Complex Exponential Fourier Series

$$s(t) = \sum_{n=-\infty}^{\infty} D_n e^{j2\pi n f_0 t} \qquad \text{Eqn. 2.90}$$

Where:

$$D_n = \frac{1}{T}\int_{t_0}^{t_0+T} s(t) e^{-j2\pi n f_0 t}\,dt \qquad \text{Eqn. 2.91}$$

Fourier Transform and Inverse Fourier Transform

$$S(f) = \Im\left[s(t)\right] = \int_{-\infty}^{\infty} s(t) e^{-j2\pi f t}\,dt \qquad s(t) = \Im^{-1}\left[S(f)\right] = \int_{-\infty}^{\infty} S(f) e^{j2\pi f t}\,df$$

Convolution

$$g(t) * w(t) = \int_{-\infty}^{\infty} g(\tau) w(t-\tau)\,d\tau = \int_{-\infty}^{\infty} w(\tau) g(t-\tau)\,d\tau \qquad \S 3.3.6$$

$$g(t) * w(t) = G(f) W(f)$$
$$G(f) * W(f) = g(t) w(t)$$

Parseval's Theorem

The energy of a time domain function is equal to the energy of its Fourier Transform.

$$\int_{-\infty}^{\infty} \left|r^2(t)\right| dt = \int_{-\infty}^{\infty} \left|R^2(f)\right| df$$

Fourier Series

- Given an *arbitrary* time-domain signal, compute the trigonometric and exponential Fourier Series Coefficients, write the Fourier Series approximation of the signal, and draw the Fourier Spectrum.
- Given an *arbitrary* set of Fourier Coefficients, draw the Fourier Spectrum and write the time domain trigonometric or exponential Fourier Series.
- Given a Fourier Series or Fourier Spectrum of an arbitrary signal, compute the Fourier Series or Fourier Spectrum after passing the signal through a high-pass, low-pass, or bandpass filter.

Fourier Transforms

- Know (memorize) the following Fourier Transform Pairs:
    a. Rect / Sinc
    b. Delta / Constant
    c. Cosine / Even Delta
    d. Sine / Odd Delta
    e. Triangle / Sinc^2
    f. Impulse Train / Impulse Train
- Given an *arbitrary* signal in the time-domain, compute the Fourier Transform and draw the signal spectrum.
- Given an *arbitrary* signal spectrum, compute the Inverse Fourier Transform and write the signal in the time-domain.
- Numerically evaluate the Fourier Transform of a signal for a specific frequency value.
- Given an *arbitrary* signal, apply the frequency domain version of convolution to determine the output (time-domain or frequency domain) of a high-pass, low-pass, or bandpass filter.
- Understand the difference between continuous-time and discrete-time Fourier Transforms.

Convolution

- Given two *arbitrary* (but simple) signals, $a(t)$ and $b(t)$, use the convolution integral and compute the result of $a(t)*b(t)$.
- Understand and be able to apply the following convolution properties:
    a. Identity
    b. Commutative
    c. Associative
    d. Distributive
- Apply the properties of convolution with special functions (e.g., delta-functions or constants) to simplify and solve convolution of an arbitrary signal with the special function.
- Given an *arbitrary* signal, apply the convolution theorem and properties to determine the output (time-domain or frequency domain) of a high-pass, low-pass, or bandpass filter.