TABLE 2–1 SOME FOURIER TRANSFORM THEOREMS[a]

Operation	Function	Fourier Transform
Linearity	$a_1 w_1(t) + a_2 w_2(t)$	$a_1 W_1(f) + a_2 W_2(f)$
Time delay	$w(t - T_d)$	$W(f)\, e^{-j\omega T_d}$
Scale change	$w(at)$	$\frac{1}{\lvert a \rvert} W\left(\frac{f}{a}\right)$
Conjugation	$w^*(t)$	$W^*(-f)$
Duality	$W(t)$	$w(-f)$
Real signal frequency translation [$w(t)$ is real]	$w(t) \cos(w_c t + \theta)$	$\frac{1}{2}[e^{j\theta} W(f - f_c) + e^{-j\theta} W(f + f_c)]$
Complex signal frequency translation	$w(t)\, e^{j\omega_c t}$	$W(f - f_c)$
Bandpass signal	$\mathrm{Re}\{g(t)\, e^{j\omega_c t}\}$	$\frac{1}{2}[G(f - f_c) + G^*(-f - f_c)]$
Differentiation	$\frac{d^n w(t)}{dt^n}$	$(j2\pi f)^n W(f)$
Integration	$\int_{-\infty}^{t} w(\lambda) d\lambda$	$(j2\pi f)^{-1} W(f) + \frac{1}{2} W(0)\, \delta(f)$
Convolution	$w_1(t) * w_2(t) = \int_{-\infty}^{\infty} w_1(\lambda) \cdot w_2(t - \lambda)\, d\lambda$	$W_1(f) W_2(f)$
Multiplication[b]	$w_1(t) w_2(t)$	$W_1(f) * W_2(f) = \int_{-\infty}^{\infty} W_1(\lambda)\, W_2(f - \lambda)\, d\lambda$
Multiplication by t^n	$t^n w(t)$	$(-j2\pi)^{-n} \frac{d^n W(f)}{df^n}$

[a] $\omega_c = 2\pi f_c$.

[b] $*$ denotes convolution as described in detail by Eq. (2–62).

TABLE 2–2 SOME FOURIER TRANSFORM PAIRS

Function	Time Waveform $w(t)$	Spectrum $W(f)$
Rectangular	$\Pi\left(\frac{t}{T}\right)$	$T[\mathrm{Sa}(\pi f T)]$
Triangular	$\Lambda\left(\frac{t}{T}\right)$	$T[\mathrm{Sa}(\pi f T)]^2$
Unit step	$u(t) \triangleq \begin{cases} +1, & t > 0 \\ 0, & t < 0 \end{cases}$	$\frac{1}{2}\delta(f) + \frac{1}{j2\pi f}$
Signum	$\mathrm{sgn}(t) \triangleq \begin{cases} +1, & t > 0 \\ -1, & t < 0 \end{cases}$	$\frac{1}{j\pi f}$
Constant	1	$\delta(f)$
Impulse at $t = t_0$	$\delta(t - t_0)$	$e^{-j2\pi f t_0}$
Sinc	$\mathrm{Sa}(2\pi W t)$	$\frac{1}{2W}\,\Pi\left(\frac{f}{2W}\right)$
Phasor	$e^{j(\omega_0 t + \varphi)}$	$e^{j\varphi}\,\delta(f - f_0)$
Sinusoid	$\cos(\omega_c t + \varphi)$	$\frac{1}{2}e^{j\varphi}\,\delta(f - f_c) + \frac{1}{2}e^{-j\varphi}\,\delta(f + f_c)$
Gaussian	$e^{-\pi(t/t_0)^2}$	$t_0 e^{-\pi(f t_0)^2}$
Exponential, one-sided	$\begin{cases} e^{-t/T}, & t > 0 \\ 0, & t < 0 \end{cases}$	$\frac{T}{1 + j2\pi f T}$
Exponential, two-sided	$e^{-\lvert t\rvert/T}$	$\frac{2T}{1 + (2\pi f T)^2}$
Impulse train	$\sum_{k=-\infty}^{k=\infty} \delta(t - kT)$	$f_0 \sum_{n=-\infty}^{n=\infty} \delta(f - nf_0)$, where $f_0 = 1/T$

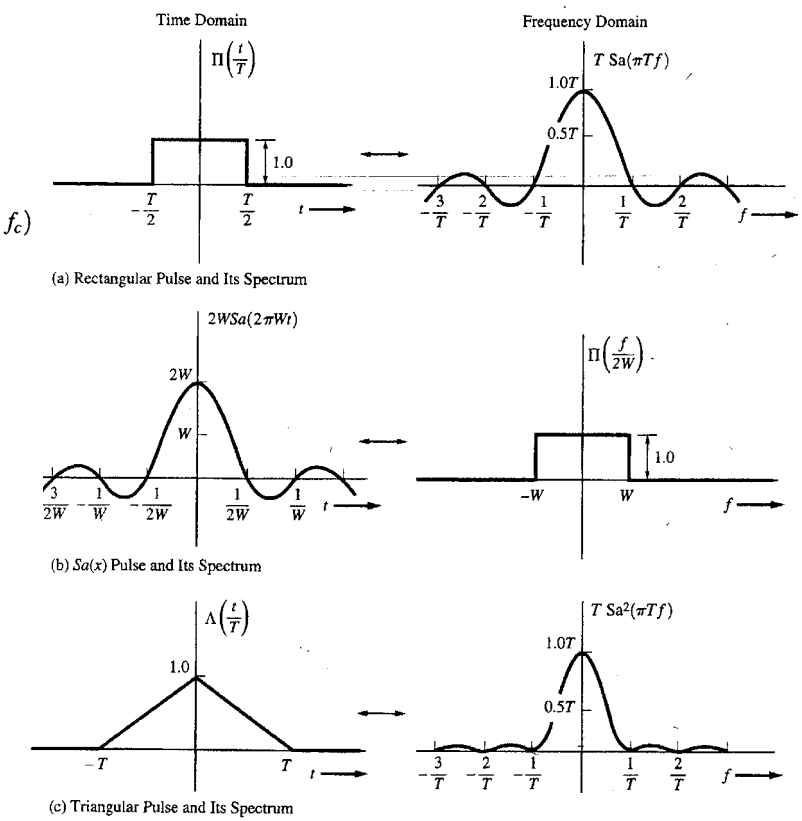

Figure 2–6 Spectra of rectangular, $(\sin x)/x$, and triangular pulses.