Name: Key

Section: 5511

EE432 Fall 2009 Exam 1

Problem	Possible Points	Score
1	25	
2	25	
3	25	
4	25	
Total	100	

- Turn this in by Friday, Oct. 2, 2009 at the beginning of class.
- This is an open book, open notes and open computer exam. USE ONLY YOUR COURSE TEXTBOOK ("A Digital Signal Processing Primer"), and your notes from this course. You can also use the "Introduction to MATLAB 7" book by Dr. Etter, and MATLAB Help.
- You ***must show your work*** to get full credit for problems. Don't just write answers. Use additional sheets as necessary.
- Label your plots carefully, and turn in all of your code when using MATLAB.

- If you are stuck on a problem, you may ask for guidance...but it might cost you in points. You ask your question, and I will let you know how much it will cost. Then you can agree to obtain the guidance for the specified number of points off of your final score, if you wish.

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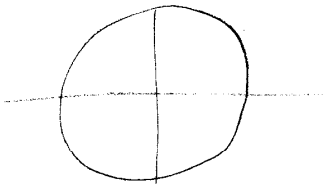
1. (25 pts) Sampling & Quantization. An analog signal contains frequencies of 200 Hz, 600 Hz, 900 Hz and 2300 Hz.

   a. When converting this signal to digital, what is the minimum sample frequency to prevent aliasing ?

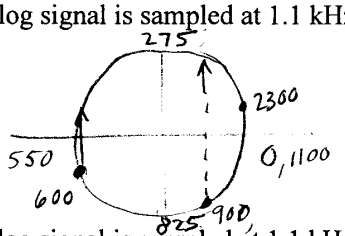
   $2 \times f_{max} = 2 \cdot 2300\ Hz = \boxed{4.6\ kHz}$

   b. If the analog signal is sampled at 5 kHz, what alias frequencies will appear out of the A/D?



   $f_s > 2 f_{max} \Rightarrow$ no aliasing

   c. If the analog signal is sampled at 1.1 kHz, what alias frequencies will appear out of the A/D?



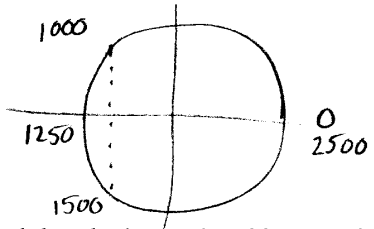
   600 Hz aliases to 500 Hz
   900 Hz aliases to 200 Hz
   2300 Hz aliases to 100 Hz

   d. If the analog signal is sampled at 1.1 kHz, what is the highest frequency that can appear out of the A/D?

   $\frac{f_s}{2} = \boxed{550\ Hz}$

   e. Unrelated question: If the sample rate in an A/D system is 2.5 kHz, name 3 frequencies in an input analog signal that could show up as aliased to 1 kHz frequency in the digitized signal?

Some freqs
1500 Hz
3500 Hz
4000 Hz
⋮



- 1500 aliases to 1000 Hz, so $1500 + k \cdot 2500$ will also alias to 1000 $k = 1, 2, 3, \ldots$
- $1000 \pm k \cdot 2500$ also aliases to 1000 Hz

   f. If it is desired that the input signal be quantized with 11 bits, and the input signal varies from +3 Volts to -3 Volts, what is the maximum expected quantization error?

$res = \frac{3-(-3)}{2^{11}} = \frac{6}{2048} = \frac{3}{1024}$

Max quant error $= \frac{res}{2} = \frac{3}{2048}$

$=$

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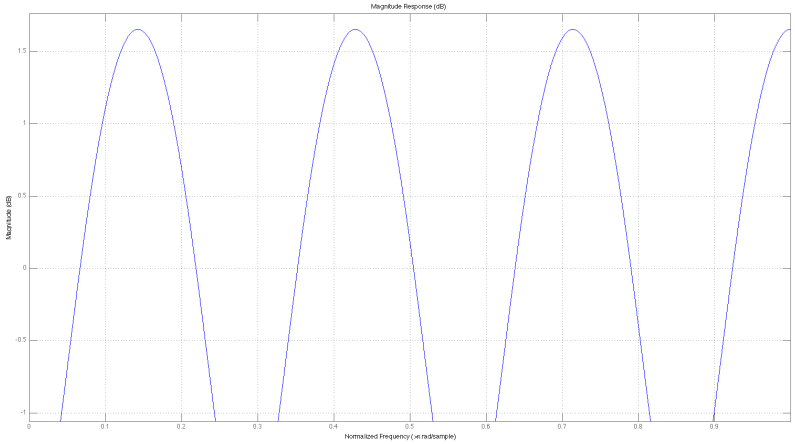
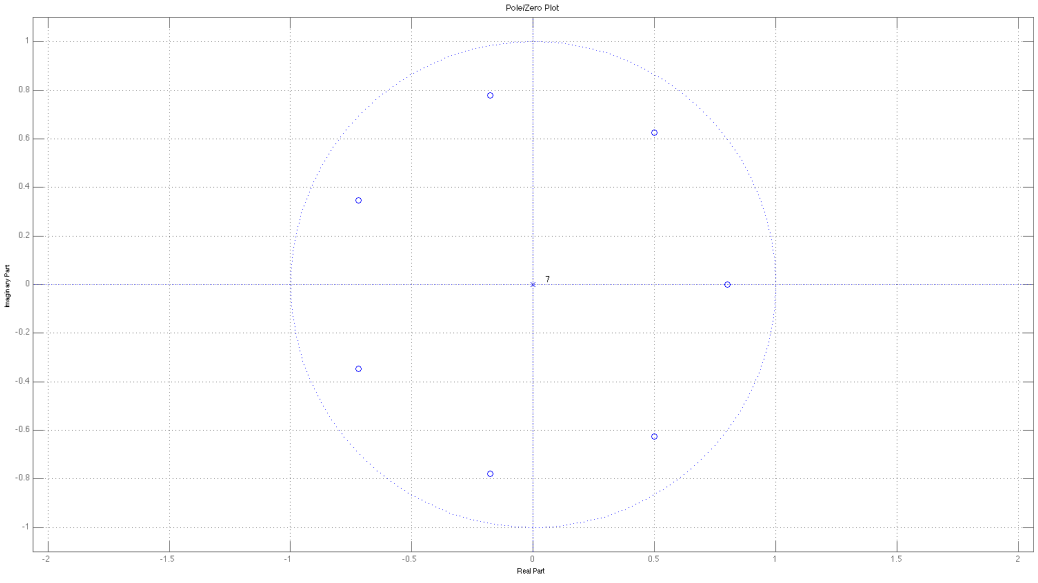
```
% ee432 exam 1
% prob 2
fvtool([1 0 0 0 0 0 0 -(0.8)^7],1)
roots([1 0 0 0 0 0 0 -(0.8)^7])

% ans =
%
%   -0.7208 + 0.3471i
%   -0.7208 - 0.3471i
%   -0.1780 + 0.7799i
%   -0.1780 - 0.7799i
%    0.8000
%    0.4988 + 0.6255i
%    0.4988 - 0.6255i
```





3. (25 pts) Feedback Filters.

a. Find the transfer function (in positive powers of z) for a system that has input/output equation $y_t = 0.5x_t - 0.3x_{t-1} + 0.6x_{t-3} + 0.2y_{t-1} - 0.5y_{t-2} + 0.25y_{t-3}$.

(5)

$$Y = 0.5X - 0.3z^{-1}X + 0.6z^{-3}X + 0.2Yz^{-1} - 0.5Yz^{-2} + 0.25Yz^{-3}$$

$$Y(1 - 0.2z^{-1} + 0.5z^{-2} - 0.25z^{-3}) = X(0.5 - 0.3z^{-1} + 0.6z^{-3})$$

What are its poles and zeros?

$$H(z) = \frac{Y}{X} = \frac{0.5 - 0.3z^{-1} + 0.6z^{-3}}{1 - 0.2z^{-1} + 0.5z^{-2} - 0.25z^{-3}} \left(\frac{z^3}{z^3}\right)$$

(3) Zeros:

$$z = \begin{cases} 0.7479 \pm 0.8834j \\ -0.8957 \end{cases}$$

(3) Poles:

$$z = \begin{cases} -0.1107 \pm 0.7623j \\ 0.4214 \end{cases}$$

$$H(z) = \frac{0.5z^3 - 0.3z^2 + 0.6}{z^3 - 0.2z^2 + 0.5z - 0.25}$$

(5) →Use MATLAB to generate a plot of the magnitude response, and a pole-zero plot. Give the plots a title that includes your name and the problem #. Turn in these plots with your exam.

→If a digital signal that has been sampled at 48 kHz is input to the filter, what frequency of the digital signal is the most amplified? What frequency or frequencies are most attenuated?

(3) Most amplified:

$$f_N = \frac{f_s}{2} = 24\text{ kHz}$$

$$0.55\, f_N = 0.55(24\text{ kHz}) = \boxed{13.2\text{ kHz}}$$

(3) Most attenuated:

$$\sim 0.27\, f_N = 0.27(24\text{ kHz}) = \boxed{6.48\text{ kHz}} \text{ AND}$$

$$\sim 0.95\, f_N = \boxed{22.8\text{ kHz}}$$

(3) →Is this a stable filter? Why or why not?

Yes – all poles inside unit circle.

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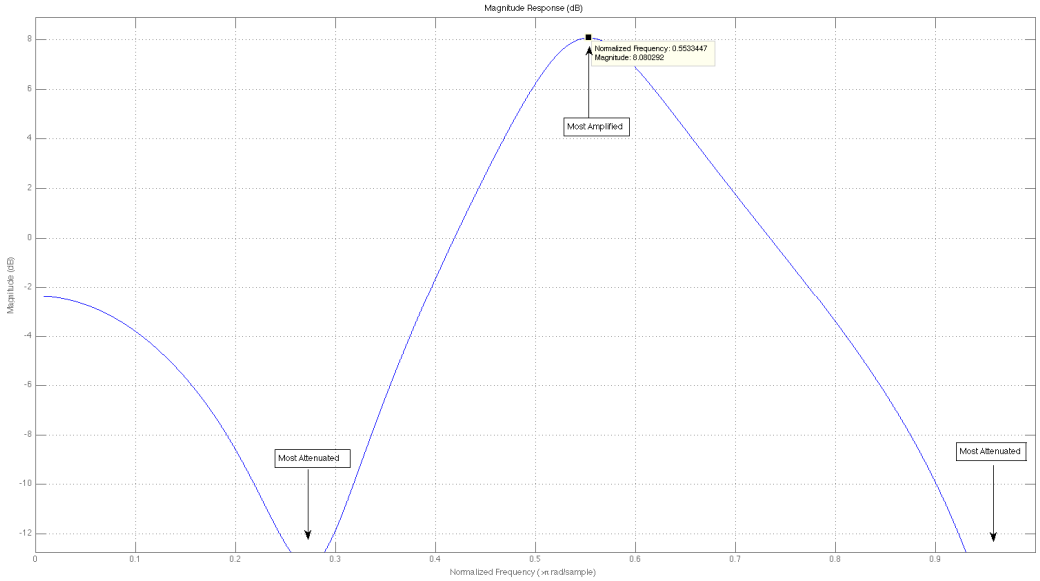
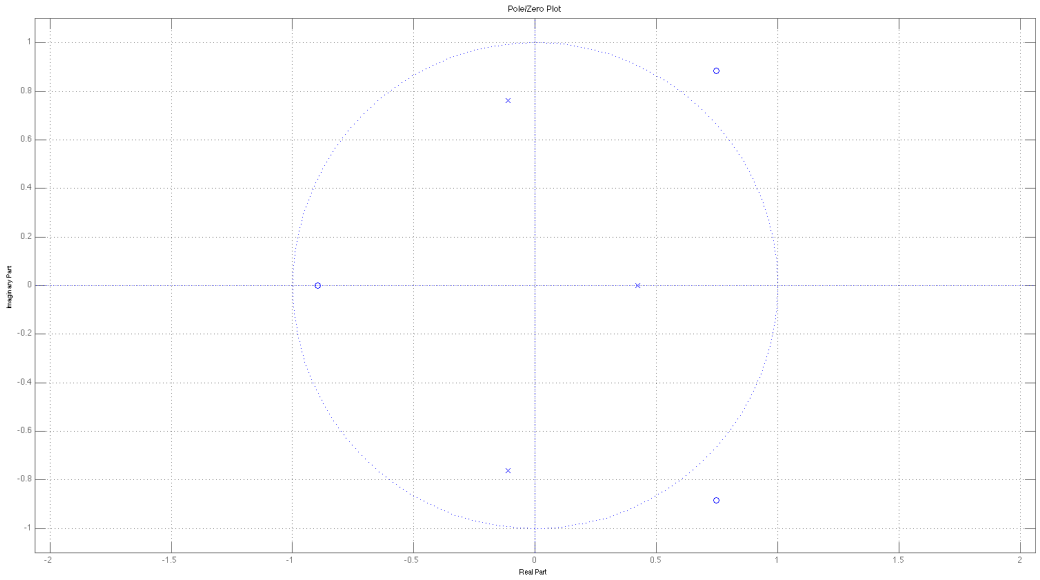
```
% prob 3
denom=[1 -0.2 +0.5 -0.25];
num=[0.5 -0.3 0 0.6];
fvtool(num,denom)
```





4. (25 pts) More Feedback filters. A feedback filter has the following 6 poles and 6 zeros:

Zeros:	Poles:
$z = 0.25 \pm j0.9$	$z = 0,0$ (2 poles)
$z = 1.13 \pm j0.8$	$z = 0.75e^{\pm j\frac{\pi}{3}}$
$z = e^{\pm j\frac{5\pi}{8}}$	$z = 0.2 \pm j0.8$

a. Determine the system transfer function in positive powers of z. Your answer should have its numerator terms multiplied out, and its denominator terms multiplied out.

$$H(z) = \frac{z^6 - 1.9946z^5 + 2.8070z^4 - 2.6905z^3 + 3.3491z^2 - 1.6502z + 1.6725}{z^6 - 1.15z^5 + 1.5425z^4 - 0.7350z^3 + 0.3825z^2}$$

b. Use MATLAB to plot its magnitude response and its pole-zero plot. Give the plots a title that includes your name and the problem #. Turn in these plots with your exam.

c. Is this filter stable? Why or why not?

Yes - all poles inside unit circle.

d. This filter was designed to remove (or at least severely attenuate) an annoying 550 Hz tonal from an analog signal. What should the sample frequency in the A/D be in order for this filter to remove the 550 Hz annoying signal?

most attenuated freq is at $0.625 \times f_N$

$$550\,Hz = 0.625 f_N$$

so

$$f_N = \frac{550}{0.625} = 880\,Hz$$

$$\text{so } f_s = 2f_N = \boxed{1760\,Hz}$$

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```
% prob 4
z1=0.25+j*0.9;,z2=0.25-j*0.9;
z3=1.13+j*0.8;,z4=1.13-j*0.8;
z5=exp(j*5*pi/8);,z6=exp(-j*5*pi/8);

num1=conv([1 -z1],[1 -z2]);
num2=conv([1 -z3],[1 -z4]);
num3=conv([1 -z5],[1 -z6]);
num=conv(num1,num2);,num=conv(num,num3)

p1=0.75*exp(j*pi/3);,p2=0.75*exp(-j*pi/3);
p3=0.2+j*0.8;,p4=0.2-j*0.8;
den1=conv([1 -p1],[1 -p2]);
den2=conv([1 -p3],[1 -p4]);
den3=conv([1 0],[1 0]);
den=conv(den1,den2);,den=conv(den,den3);
fvtool(num,den)
```