

① Text, 6.2c, e

$$(c) x[n] = 0.5^n u[n] \quad \underline{X}(z) = \frac{0.5z}{(z-1)^2} \quad |z| > 1$$

$$(e) x[n] = -(0.8)^n \sin\left(n \frac{3\pi}{4}\right) u[n] \quad \underline{X}(z) = \frac{-0.8z \sin\left(\frac{3\pi}{4}\right)}{z^2 - 2(0.8)z \cos\left(\frac{3\pi}{4}\right) + 0.8^2}$$

$$\underline{X}(z) = \frac{-0.5657z}{z^2 - 1.1314z + 0.64}$$

② Text, 6.5

$$(a) y[n] = x[n] + 0.4x[n-1] + 0.1x[n-2] + 0.2y[n-1]$$

$$(b) Y(z) = \underline{X}(z) + 0.4z^{-1}\underline{X}(z) + 0.1z^{-2}\underline{X}(z) + 0.2z^{-1}Y(z)$$

$$Y(z)(1 - 0.2z^{-1}) = \underline{X}(z)(1 + 0.4z^{-1} + 0.1z^{-2})$$

$$H(z) = \frac{Y(z)}{\underline{X}(z)} = \frac{1 + 0.4z^{-1} + 0.1z^{-2}}{1 - 0.2z^{-1}} = \frac{z^2 + 0.4z + 0.1}{z^2 - 0.2z}$$

③ Text, 6.6 a, c

$$(a) y[n] - 0.7y[n-1] = x[n] + 0.2x[n-1]$$

$$Y(z)(1 - 0.7z^{-1}) = \underline{X}(z)(1 + 0.2z^{-1})$$

$$H(z) = \frac{Y(z)}{\underline{X}(z)} = \frac{1 + 0.2z^{-1}}{1 - 0.7z^{-1}} = \frac{z + 0.2}{z - 0.7}$$

$$(c) \quad y[n] = 0.25 (x[n] + x[n-1] + x[n-2] + x[n-3])$$

$$Y(z) = 0.25 X(z) (1 + z^{-1} + z^{-2} + z^{-3})$$

$$H(z) = \frac{Y(z)}{X(z)} = 0.25 (1 + z^{-1} + z^{-2} + z^{-3})$$

$$= \frac{0.25 (z^3 + z^2 + z + 1)}{z^3}$$

④

④ Text, 6.7 a, b

$$(a) \quad H(z) = \frac{Y(z)}{X(z)} = \frac{1 - 0.4z^{-1}}{1 + 0.5z^{-1} + 0.75z^{-2}}$$

$$Y(z)(1 + 0.5z^{-1} + 0.75z^{-2}) = X(z)(1 - 0.4z^{-1})$$

$$y[n] = x[n] - 0.4x[n-1] - 0.5y[n-1] - 0.75y[n-2]$$

$$(b) \quad H(z) = \frac{Y(z)}{X(z)} = 1 - 0.4z^{-1} + 0.1z^{-2}$$

$$Y(z) = X(z) (1 - 0.4z^{-1} + 0.1z^{-2})$$

$$y[n] = x[n] - 0.4x[n-1] + 0.1x[n-2]$$

⑤ Text, 6.11

$$h_1[n] = 0.2^n (u[n] - u[n-3]) = \delta[n] + 0.2\delta[n-1] + 0.04\delta[n-2]$$

$$H_1(z) = 1 + 0.2z^{-1} + 0.04z^{-2}$$

$$h_2[n] = -3n(u[n] - u[n-4]) = -3\delta[n-1] - 6\delta[n-2] - 9\delta[n-3]$$

$$H_2(z) = -3z^{-1} - 6z^{-2} - 9z^{-3}$$

$$H(z) = H_1(z)H_2(z) = -3z^{-1} - 6.6z^{-2} - 10.32z^{-3} - 2.04z^{-4} - 0.36z^{-5}$$

(b) Text, 6.15c, k

$$(e) \quad X(z) = \frac{1}{(z-0.2)(z+0.4)} = \frac{A}{z-0.2} + \frac{B}{z+0.4}$$

$$A = \left. \frac{z-0.2}{z-0.2} \frac{1}{z+0.4} \right|_{z=0.2} = \frac{1}{0.6} = \frac{5}{3}$$

$$B = \left. \frac{z+0.4}{z+0.4} \frac{1}{z-0.2} \right|_{z=-0.4} = -\frac{1}{0.6} = -\frac{5}{3}$$

$$X(z) = \frac{5/3}{z-0.2} - \frac{5/3}{z+0.4}$$

$$x[n] = \frac{5}{3}(0.2)^{n-1}u[n-1] - \frac{5}{3}(-0.4)^{n-1}u[n-1]$$

note: $\alpha^n u[n] \leftrightarrow \frac{z}{z-\alpha}$
 $\alpha^{n-1} u[n-1] \leftrightarrow \frac{z^{-1}z}{z-\alpha} = \frac{1}{z-\alpha}$

$$(k) \quad X(z) = \frac{1-0.5z^{-1}}{1-z^{-1}+z^{-2}} = \frac{z^2-0.5z}{z^2-z+1} = \frac{z(z-0.5)}{z^2-z+1}$$

note: $\cos(\Omega_0 n) u[n] \leftrightarrow \frac{z(z-\cos \Omega_0)}{z^2-z\cos \Omega_0+1}$

$$\text{so } x[n] = \cos(\Omega_0 n) u[n]$$

$$\cos(\Omega_0) = 0.5, \text{ so } \Omega_0 = \frac{\pi}{3}$$

$$\text{so } x[n] = \cos\left(\frac{\pi}{3}n\right) u[n]$$

(7) Text 6.24

$H(z)$ has a zero at $z=0$

poles at $z = -0.8, -0.5 \pm j0.6$

unity gain

$$\begin{aligned}
 (b) \quad H(z) &= \frac{z}{(z+0.8)(z - (-0.5+j0.6))(z - (-0.5-j0.6))} \\
 &= \frac{z}{(z+0.8)(z^2+z+0.61)} \\
 &= \frac{z}{z^3+1.8z^2+1.41z+0.488}
 \end{aligned}$$

(a)

