① Text, 6.33

$$H_1(z) = \frac{1-0.9z^{-1}}{1-0.5z^{-1}+0.3z^{-2}} = \frac{z^2-0.9z}{z^2-0.5z+0.3}$$

$$H_2(z) = \frac{0.8}{1+0.7z^{-1}-0.2z^{-2}} = \frac{0.8z^2}{z^2+0.7z-0.2}$$

(a) $$Y_1(z) = H_1(z)\,u(z) = \frac{z(z-0.9)}{z^2+0.7z-0.2}\cdot\frac{z}{z-1}$$

using Final value Thm,

$$\lim_{n\to\infty} g[n] = \lim_{z\to 1}(z-1)Y_1(z) = \lim_{z\to 1}\frac{z^2(z-0.9)}{z^2-0.5z+0.3}$$

$$= \frac{1(1-0.9)}{1-0.5+0.3} = \boxed{0.125}$$

$$Y_2(z) = H_2(z)\,u(z) = \frac{0.8z^2}{z^2+0.7z-0.2}\;\frac{z}{z-1}$$

$$\lim_{n\to\infty} g[n] = \lim_{z\to 1}(z-1)Y_2(z) = \lim_{z\to 1}\frac{0.8z^3}{z^2+0.7z-0.2}$$

$$= \frac{0.8}{1+0.7-0.2} = \boxed{0.5333}$$

(b) for $H_1(z)$, poles at $0.25 \pm j0.4873 = 0.5477\angle 1.0968$

for $H_2(z)$, poles at -0.9179 and 0.2179

The 2^{nd} filter has a pole closer to unit circle, which increases settling time.

∴ <u>1^{st} filter</u> settles more quickly.

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② Text, 7.3

$$2y[n]-y[n-1]+3y[n-2]=x[n]-4x[n-1]$$

$$y[n]-0.5y[n-1]+1.5y[n-2]=0.5x[n]-2x[n-1]$$

$$Y(z)\left(1-0.5z^{-1}+1.5z^{-2}\right)=X(z)\left(0.5-2z^{-1}\right)$$

$$H(z)=\frac{Y(z)}{X(z)}=\frac{0.5-2z^{-1}}{1-0.5z^{-1}+1.5z^{-2}} \qquad \text{let } z=e^{j\Omega}$$

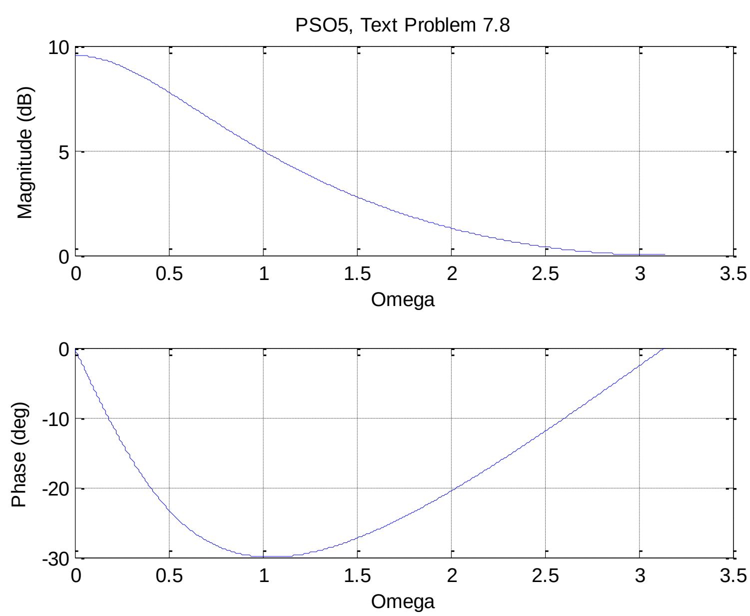
$$H(\Omega)=\frac{0.5-2e^{-j\Omega}}{1-0.5e^{-j\Omega}+1.5e^{-j2\Omega}}$$

③ Text, 7.8

$$H(\Omega)=\frac{3}{2-e^{-j\Omega}} \qquad 0\le\theta\le\pi \text{ rads}$$

Gain plot in dB, Phase plot in deg.

```
% Text, problem 7.8.
Omega=0:.001:pi;
H=3./(2-exp(-j*Omega));
figure(1),subplot(2,1,1),plot(Omega,20*log10(abs(H))),xlabel('Omega'),
ylabel('Magnitude (dB)'),grid on, title('PSO5, Text Problem 7.8')
subplot(2,1,2),plot(Omega,angle(H)*180/pi),xlabel('Omega'),
ylabel('Phase (deg)'),grid on
```



(4) Text, 7.12

$$y[n] = 1.3y[n-1] - 0.7y[n-2] + x[n] - 0.5x[n-1]$$

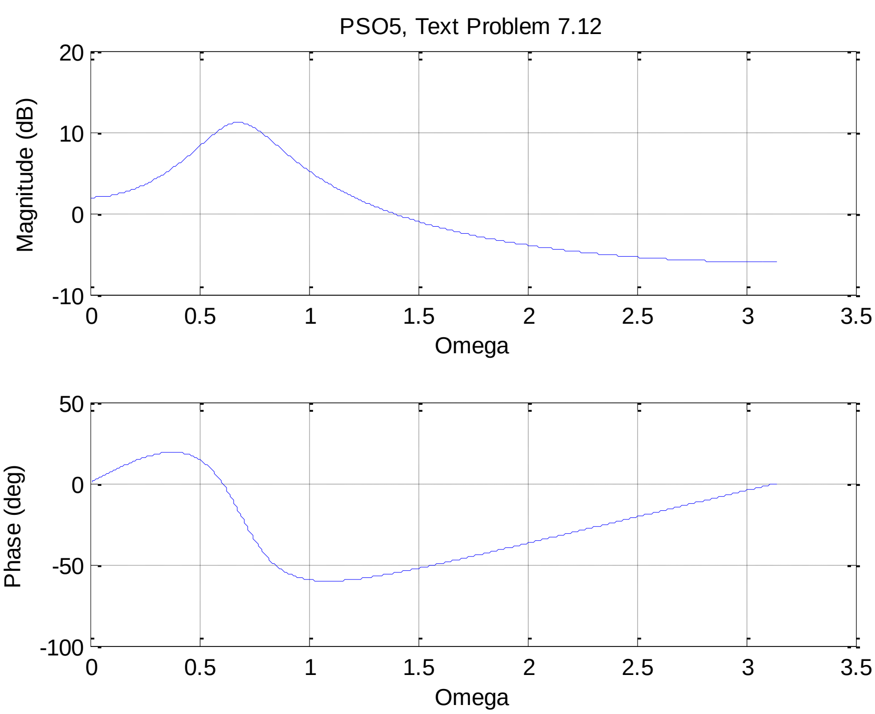
$$y[n] - 1.3y[n-1] + 0.7y[n-2] = x[n] - 0.5x[n-1]$$

$$Y(z)\left(1 - 1.3z^{-1} + 0.7z^{-2}\right) = X(z)\left(1 - 0.5z^{-1}\right)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 - 0.5z^{-1}}{1 - 1.3z^{-1} + 0.7z^{-2}}$$

$$H(\Omega) = \frac{1 - 0.5e^{-j\Omega}}{1 - 1.3e^{-j\Omega} + 0.7e^{-j2\Omega}}$$

```
% Text, problem 7.12.
Omega=0:.001:pi;
H=(1-0.5*exp(-j*Omega))./(1-1.3*exp(-j*Omega)+0.7*exp(-j*2*Omega));
figure(2),subplot(2,1,1),plot(Omega,20*log10(abs(H))),xlabel('Omega'),
ylabel('Magnitude (dB)'),grid on, title('PSO5, Text Problem 7.12')
subplot(2,1,2),plot(Omega,angle(H)*180/pi),xlabel('Omega'),
ylabel('Phase (deg)'),grid on
```



(5) Text, 7.14

$x[n] = 2\cos\left(\frac{4\pi}{9}n + 20^\circ\right)$, short form: $2\angle 20^\circ$

$\Omega = \frac{4\pi}{9} = 1.396$ rad

From magnitude plot, $20\log_{10}|H(1.396)| \approx 2.1$ dB

so $|H(1.396)| = 10^{\frac{2.1}{20}} = 1.2735$

From phase plot, $\angle H(1.396) = -40^\circ$

short form of freq response is $1.2735\angle -40^\circ$

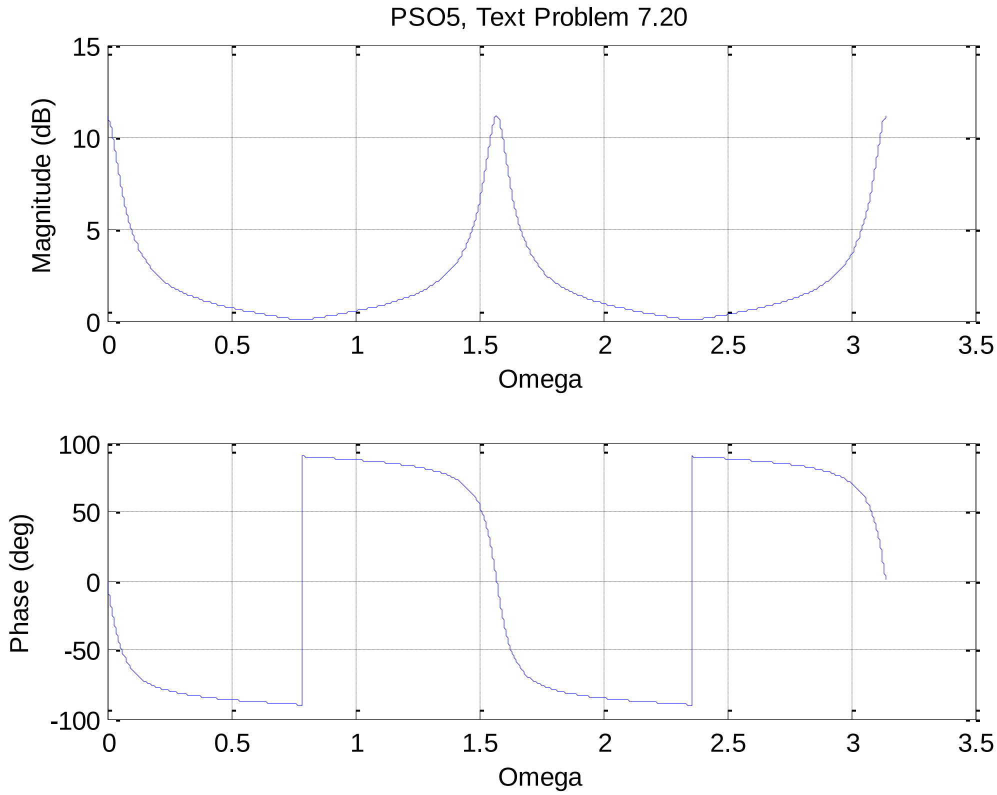
Output of filter is $2.547\angle -20$

or $2.547\cos\left(\frac{4\pi}{9}n - 20^\circ\right)$

(6) Text, 7.20

- code and plot on next page
- note: shape of filter: <u>comb filter.</u>

```
% Text, problem 7.20.
Omega=0:.001:pi;
H=(1+exp(-j*4*Omega))./(1-0.82*exp(-j*4*Omega));
figure(3),subplot(2,1,1),plot(Omega,(abs(H))),xlabel('Omega'),
ylabel('Magnitude (dB)'),grid on, title('PSO5, Text Problem 7.20')
subplot(2,1,2),plot(Omega,angle(H)*180/pi),xlabel('Omega'),
ylabel('Phase (deg)'),grid on
```

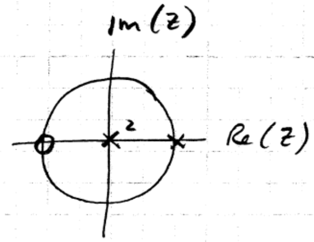


(7) Text, 7.26

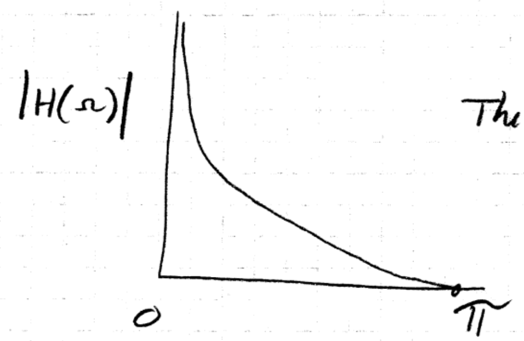
$$H(z) = \frac{z+1}{z^2(z-1)}$$

zero at $z=-1$

2 poles at $z=0$, pole at $z=1$



- poles at $z=0$ won't affect magnitude of freq response, since $|e^{-j\Omega}|=1$
- zero at $z=-1$ means $|H(\Omega)|=0$ (at $\Omega=\pi$)
- pole at $z=1$ means $|H(\Omega)|=\infty$ (at $\Omega=0$)



This is a low pass filter.