(1) $x(t) = 4\cos 40\pi t - \sin 30\pi t + 3$

$$T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{40\pi} = \frac{1}{20}\,s \qquad T_2 = \frac{2\pi}{\omega_2} = \frac{2\pi}{30\pi} = \frac{1}{15}\,s$$

$$T_0 = \mathrm{Lcm}(T_1, T_2) = \frac{\mathrm{Lcm}\left(60\cdot\frac{1}{20},\ 60\cdot\frac{1}{15}\right)}{60} = \frac{\mathrm{Lcm}(3,4)}{60} = \frac{12}{60} = \frac{1}{5}$$

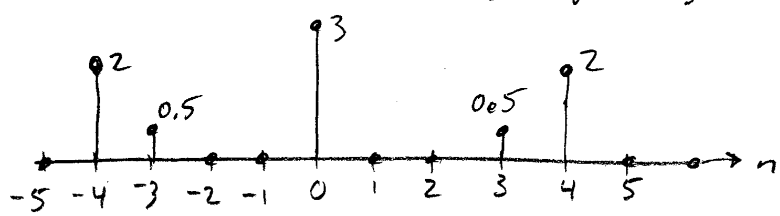
$$\text{so } \omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{\frac{1}{5}} = \underline{\underline{10\pi \text{ rps}}}$$

$$x(t) = 4\cdot\frac{1}{2}\left(e^{j40\pi t} + e^{-j40\pi t}\right) - \frac{1}{2j}\left(e^{j30\pi t} - e^{-j30\pi t}\right) + 3e^{j0}$$

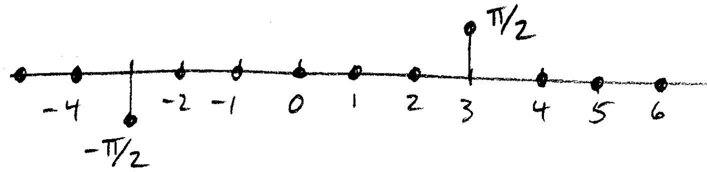
$$= 2e^{j4\cdot 10\pi t} + 2e^{j(-4)10\pi t} + 0.5j\,e^{j(3)10\pi t} - 0.5j\,e^{j(-3)10\pi t} + 3e^{j0}$$

$C_4 = 2 \qquad C_{-4} = 2 \qquad C_3 = 0.5j \qquad C_{-3} = -0.5j \qquad C_0 = 3$

$|C_n|$



$\angle C_n$



(2) $T = 2.5\,msec \Rightarrow \underline{\omega_0} = \frac{2\pi}{.0025} = \underline{800\pi}$

From plot,

$C_{-4} = 3e^{j\pi} = -3$

$C_{-2} = 0.5e^{j\pi/2} = 0.5j$

$C_2 = 0.5e^{-j\pi/2} = -0.5j$

$C_4 = 3e^{j\pi} = -3$

$$x(t) = C_4 e^{j(4)\omega_0 t} + C_{-4} e^{j(-4)\omega_0 t} + C_2 e^{j(2)\omega_0 t} + C_{-2} e^{j(-2)\omega_0 t}$$

$$= -3 e^{j3200\pi t} - 3e^{-j3200\pi t} + -.5j\, e^{j1600\pi t} + 0.5j\, e^{-j1600\pi}$$

$$= -6\left(\frac{1}{2} e^{j3200\pi t} + e^{-j3200\pi t}\right) + \frac{1}{2j}\left(e^{j1600\pi t} - e^{-j1600\pi t}\right)$$

$$\boxed{x(t) = -6\cos(3200\pi t) + \sin 1600\pi t}$$

③ Fourier transform of $f(t) = e^{-|2t|}$

The diligent student would have noticed that this is solved in example 3.9 on p40 of notes:

$$F(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t} dt = \int_{-\infty}^{0} e^{2t} e^{-j\omega t} dt + \int_{0}^{\infty} e^{-2t} e^{-j\omega t} dt$$

$$= \int_{-\infty}^{0} e^{(2-j\omega)t} dt + \int_{0}^{\infty} e^{-(2+j\omega)t} dt$$

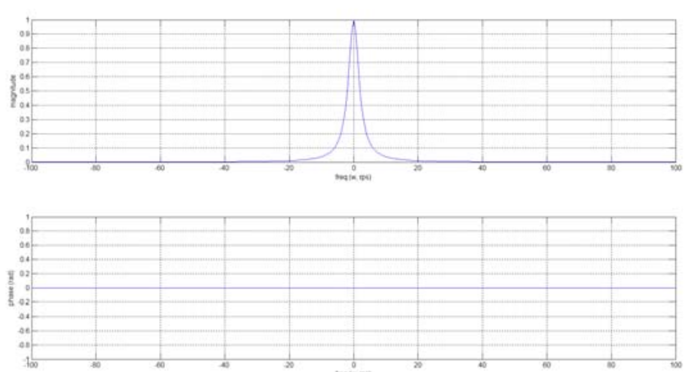
$$= \frac{1}{2-j\omega} e^{(2-j\omega)t}\Big|_{-\infty}^{0} - \frac{e^{-(2+j\omega)t}}{(2+j\omega)}\Big|_{0}^{\infty}$$

$$= \frac{1}{2-j\omega} + \frac{1}{2+j\omega}$$

$$= \frac{4}{4-j^2\omega^2} = \boxed{\frac{4}{4+\omega^2}}$$

```
% Problem 3
w=-100:.5:100;
figure(1),title('PS06, problem 3'),subplot(2,1,1)
plot(w,abs(4./(4+w.*w))),xlabel('freq (w, rps)'),ylabel('magnitude')
grid on
subplot(2,1,2)
plot(w,angle(4./(4+w.*w))),xlabel('freq (w, rps)'),ylabel('phase (rad)')
grid on
```

Bandwidth is $f_2 - f_1$, where f_1 is the freq where magnitude falls to 0.707 of its max value, and $f_2 = 0$.

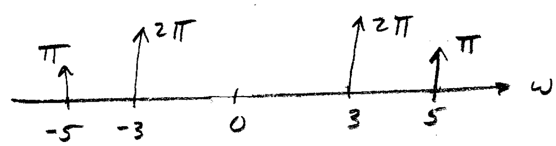


(4) Practice Problems 3.3

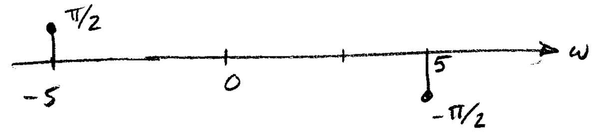
1. $f(t) = 2\cos 3t + \sin 5t$

$$F(\omega) = 2\pi\,\delta(\omega-3) + 2\pi\,\delta(\omega+3) + j\pi\,\delta(\omega+5) - j\pi\,\delta(\omega-5)$$

$|F(\omega)|$



$\angle F(\omega)$



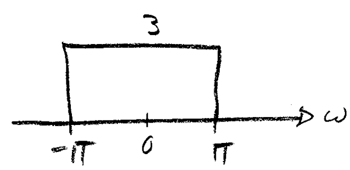
plot of $f(t)$ – see attached Matlab

3. $f(t) = 3\,\mathrm{sinc}(t-\pi)$

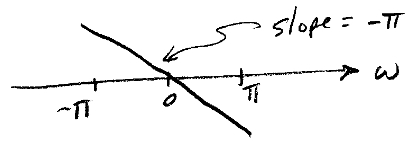
$$3\,\mathrm{sinc}(t) \longleftrightarrow 3\,\mathrm{rect}\left(\frac{\omega}{2\pi}\right)$$

$$3\,\mathrm{sinc}(t-\pi) \longleftrightarrow 3e^{-j\pi\omega}\,\mathrm{rect}\left(\frac{\omega}{2\pi}\right) = F(\omega)$$

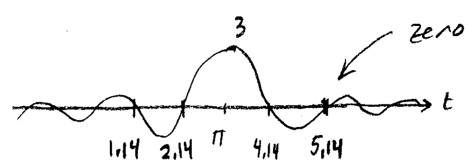
$|F(\omega)|$



$\angle F(\omega)$



$f(t)$



zero crossings are multiples of 1 sec from π sec.

(4) (con't)

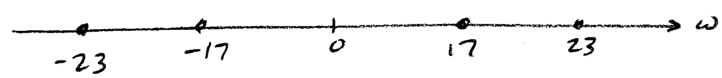
4. $f(t) = \cos 3t \cos 20t$

$$= \tfrac{1}{2}\cos(23t) + \tfrac{1}{2}\cos(17t)$$

$$F(\omega) = \tfrac{\pi}{2}\delta(\omega-23) + \tfrac{\pi}{2}\delta(\omega+23) + \tfrac{\pi}{2}\delta(\omega-17) + \tfrac{\pi}{2}\delta(\omega+17)$$

$|F(\omega)|$

$\angle F(\omega)$

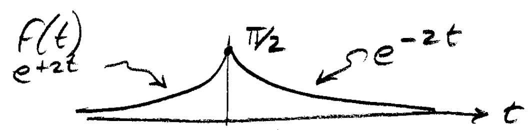


plot of $f(t)$ - see attached Matlab

7. $F(\omega) = \dfrac{2\pi}{\omega^2+4}$

since $e^{-a|t|} \longleftrightarrow \dfrac{2a}{\omega^2+a^2}$

$$\frac{\pi}{2}e^{-2|t|} \longleftrightarrow \frac{\frac{\pi}{\cancel{2}}\cancel{2}\cdot 2}{\omega^2+2^2} = \frac{2\pi}{\omega^2+2^2}$$



8. $F(\omega) = e^{-j\omega 4}\dfrac{3}{j\omega}$

$3u(t) \longleftrightarrow 3\pi\delta(\omega) + \dfrac{3}{j\omega}$

$e^{j\omega_0 t} \longleftrightarrow 2\pi\delta(\omega-\omega_0)$

so $e^{j0}=1 \longleftrightarrow 2\pi\delta(\omega)$

$1.5 \longleftrightarrow 3\pi\delta(\omega)$

8. $F(\omega) = e^{-j\omega 4}\,\dfrac{3}{j\omega}$

$$3u(t) \longleftrightarrow 3\pi\delta(\omega) + \frac{3}{j\omega}$$

$$e^{j\omega_0 t} \longleftrightarrow 2\pi\delta(\omega-\omega_0)$$

so

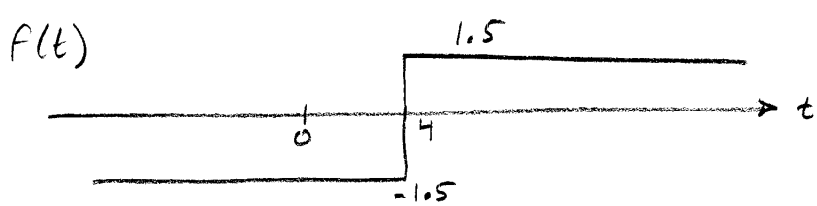
$$e^{j0} = 1 \longleftrightarrow 2\pi\delta(\omega)$$

so

$$1.5 \longleftrightarrow 3\pi\delta(\omega)$$

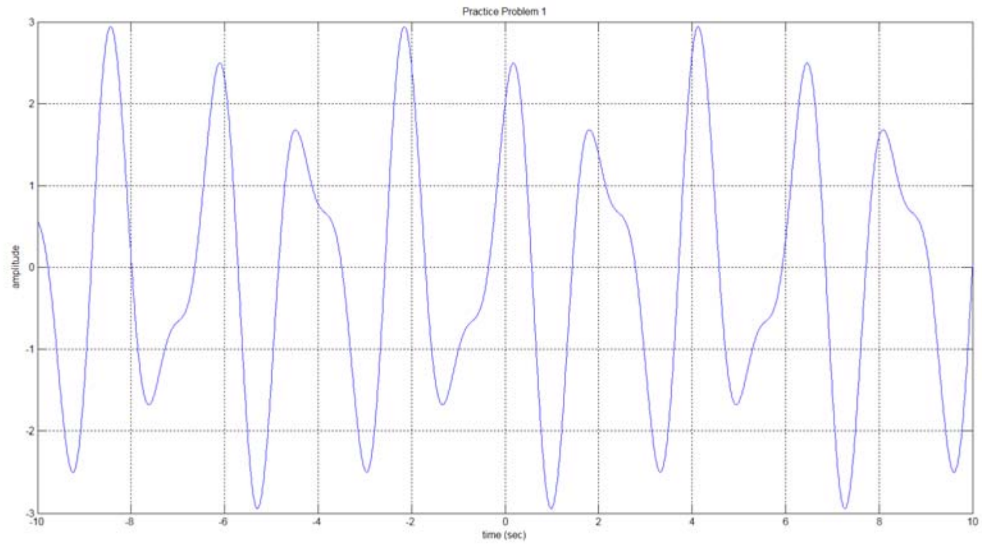
Then $3u(t) - 1.5 \longleftrightarrow 3\pi\delta(\omega) + \dfrac{3}{j\omega} - 3\pi\delta(\omega) = \dfrac{3}{j\omega}$

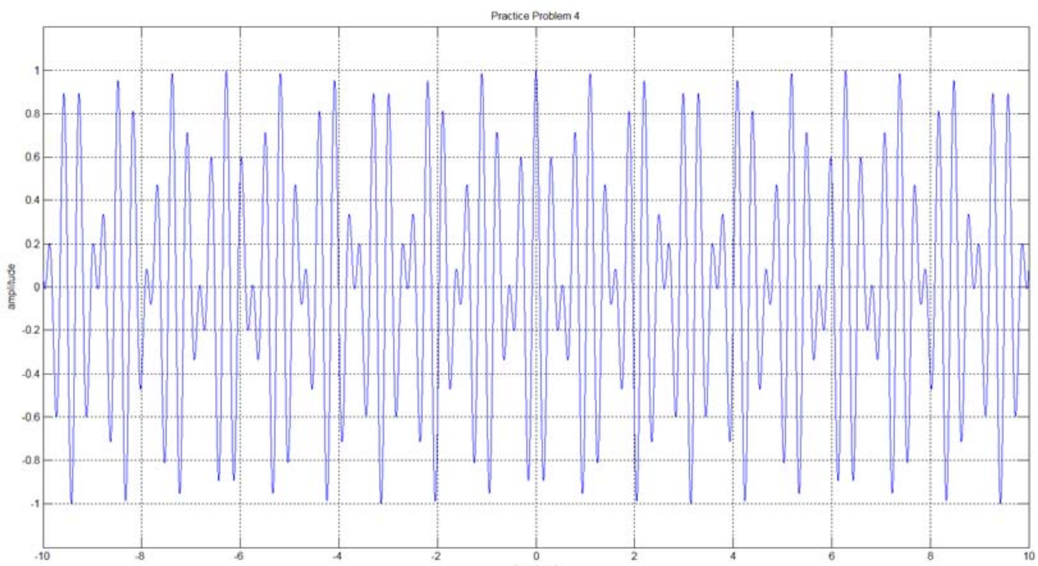
$$f(t) = 3u(t-4) - 1.5 \longleftrightarrow e^{-j\omega 4}\,\frac{3}{j\omega}$$




```
% Practice problems 3.3, # 1
t=-10:.01:10;
figure(1),plot(t,2*cos(3*t)+sin(5*t)),xlabel('time (sec)'),ylabel('amplitude')
grid on,title('Practice Problem 1')

% Practice problems 3.3, # 4
t=-10:.0001:10;
figure(1),plot(t,cos(3*t).*cos(20*t)),xlabel('time (sec)'),ylabel('amplitude')
grid on, axis([-10 10 -1.2 1.2]),title('Practice Problem 4')
```


Practice Problem 1
amplitude
time (sec)
3
2
1
0
-1
-2
-3
-10
-8
-6
-4
-2
0
2
4
6
8
10


Practice Problem 4
amplitude
time (sec)
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-10
-8
-6
-4
-2
0
2
4
6
8
10