① Text, 10.6

The stop band attenuation gives $20 \log_{10} \delta_s = -28\,dB$,

so $\delta_s = 0.0398$

$$\Omega_{p_1} = 2\pi \frac{f_{p_1}}{f_s} = 2\pi \frac{6500}{24000} = 0.542\pi \text{ rad}$$

$$\Omega_{s_1} = 2\pi \frac{f_{s_1}}{f_s} = 2\pi \frac{8000}{24000} = 0.667\pi \text{ rad.}$$

$$\omega_{p_1} = 2f_s \tan\left(\frac{\Omega_{p_1}}{2}\right) = 54791.4 \text{ rad/s}$$

$$\omega_{s_1} = 2f_s \tan\left(\frac{\Omega_{s_1}}{2}\right) = 83239.1 \text{ rad/sec}$$

Filter order:

$$n \geq \frac{\log_{10}\left(\frac{1}{\delta_s^2} - 1\right)}{2\log_{10}\left(\frac{\omega_{s_1}}{\omega_{p_1}}\right)} = \frac{\log_{10}\left(\frac{1}{.0398^2} - 1\right)}{2\log_{10}\left(\frac{83239.1}{54791.4}\right)} = 7.7$$

so order $= \boxed{8}$

analog: $$|H(\omega)| = \frac{1}{\sqrt{\left(\frac{\omega}{\omega_{p_1}}\right)^{2n} + 1}} = \frac{1}{\sqrt{\left(\frac{\omega}{54791.4}\right)^{16} + 1}}$$

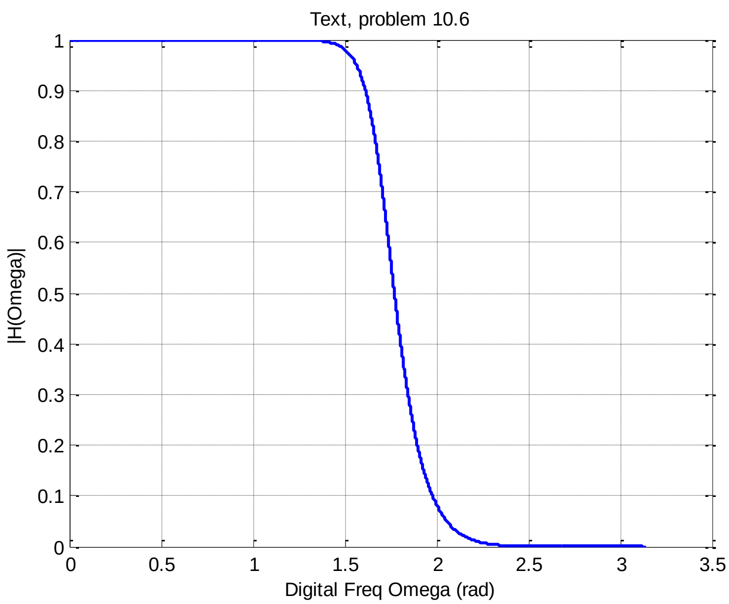
digital $$|H(\Omega)| = \frac{1}{\sqrt{\left(\frac{2f_s \tan\left(\frac{\Omega}{2}\right)}{54791.4}\right)^{16} + 1}} = \frac{1}{\sqrt{\left(\frac{48000 \tan\left(\frac{\Omega}{2}\right)}{54791.4}\right)^{16} + 1}}$$

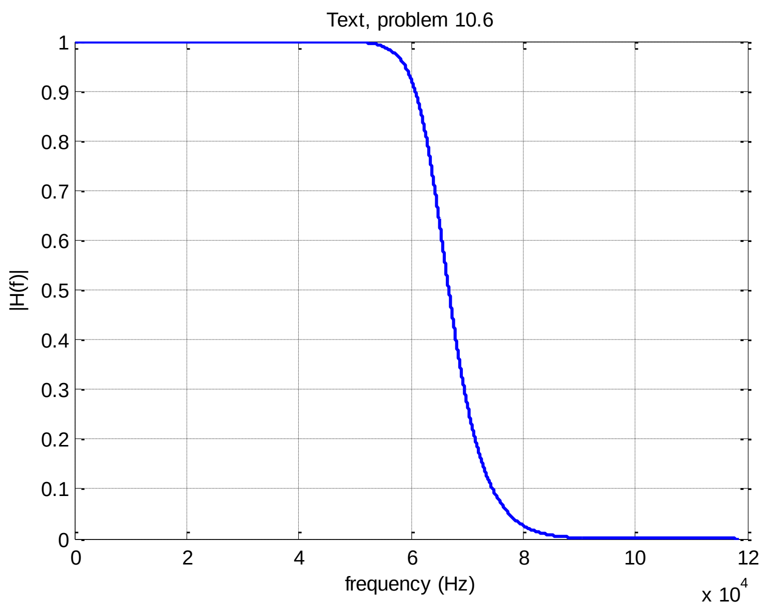
TOPS

% ps06 solutions

```
% Text, problem 10.6
omega = 0:.001:pi;
Hmag=1./sqrt((48000*tan(omega/2)./54791.4).^16+1);
figure,plot(omega,Hmag,'linewidth',2),xlabel('Digital Freq Omega (rad)'),
ylabel('|H(Omega)|'),title('Text, problem 10.6'), grid on

fs=24000;
f=fs*omega/2*pi;
figure,plot(f,Hmag,'linewidth',2),xlabel('frequency (Hz)'),
ylabel('|H(f)|'),title('Text, problem 10.6'), grid on
```




Text, problem 10.6
|H(f)|
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
2
4
6
8
10
12
frequency (Hz)
x 10^4

(2) Text, 10.12

(a) FIR - requires Hanning window w/ $N = 3.32 \frac{F_s}{TW}$

$$N = 3.32 \frac{(8000)}{1500-1000} = 53.1 \longrightarrow 53 \text{ terms, } 53 \text{ filter coefficients}$$

(b) stop band attenuation gives $20 \log_{10} \delta_s = -44 \text{ dB}$,

so $\delta_s = 0.00631$

$$\Omega_{p_1} = 2\pi \frac{f_{p_1}}{f_s} = 2\pi \frac{1000}{8000} = 0.25\pi \text{ rad}$$

$$\Omega_{s_1} = 2\pi \frac{f_{s_1}}{f_s} = 2\pi \frac{1500}{8000} = 0.375\pi \text{ rad}$$

$$\omega_{p_1} = 2f_s \left(\frac{\Omega_{p_1}}{2}\right) = 6627 \text{ rad/sec}$$

$$\omega_{s_1} = 2f_s \left(\frac{\Omega_{s_1}}{2}\right) = 10691 \text{ rad/sec}$$

order: $$n \geq \frac{\log_{10}\left(\frac{1}{\delta_s^2} - 1\right)}{2 \log_{10}\left(\frac{\omega_{s_1}}{\omega_{p_1}}\right)} = \frac{\log_{10}\left(\frac{1}{.00631^2} - 1\right)}{2 \log_{10}\left(\frac{10691}{6627}\right)} = 10.6$$

this requires an order of 11
for this order, $2(11)+1 = 23$ filter coeffs required

(3) DFT, iDFT

TOPS 35502

Problem 3.

```
function X=DFT(x)
%
%   function X=DFT(x)
%
%   This function computes the discrete Fourier transform of signal x.
N=length(x);
X=zeros(size(x));
% In the following loop, we will compute values of the DFT X[k]
%     as k goes from 0 up to N-1. Don't forget that MATLAB indices
%     must start at 1, not zero, so the computations are for X(k+1).
%
for k=0:N-1
  X(k+1)= sum( x.* exp(-j*2*pi*k/N.*(0:N-1)));
end
```

```
function x=iDFT(X)
%
%   function x=iDFT(X)
%
%   This function computes the inverse discrete Fourier transform of
X.
N=length(X);
x=zeros(size(X));
% In the following loop, we will compute values of the DFT x[n]
%     as n goes from 0 up to N-1. Don't forget that MATLAB indices
%     must start at 1, not zero, so the computations are for x(n+1).
%
for n=0:N-1
  x(n+1)= sum( X.* exp(j*2*pi*n/N.*(0:N-1)));
end
x=x/N;
```

(4) $x[n] = 5\delta[n] + 2\delta[n-1] - 2\delta[n-2] + 4\delta[n-3]$

index	$x[n]$	$X[k]$
0	5	9
1	2	$7+j2$
2	-2	-3
3	4	$7-j2$

(5)

index	$X[k]$	$x[n]$
0	21	1
1	$-1+j1.73205$	3
2	$-6+j3.464106$	5
3	-1	2
4	$-6-j3.464106$	4
5	$-1-j1.73205$	6

(6) Text, 11.13

(a) $\Delta f = \frac{f_s}{N} = \frac{40{,}000}{32} = 1250\,Hz$

for a 6 kHz signal, the closest peak is at

index $k = \frac{6000}{1250} = 4.8 \longrightarrow 5$

so frequency is $5(\Delta f) = \boxed{6250\ Hz}$

(b) For $N=64$, $\Delta f = \frac{f_s}{N} = \frac{40000}{64} = 625\ Hz$

largest peak will be at index $= \frac{6000}{625} = 9.6 \rightarrow 10$

so freq is at $10(625) = \boxed{6250\,Hz}$