① (a) $y[n] = \frac{1}{2}(x[n] + x[n-1])$

- FIR (output only depends on inputs)
- causal (does not depend on future inputs or output.
- $h[n] = \frac{1}{2}\delta[n] + \frac{1}{2}\delta[n-1]$
- <u>BIBO stable</u> $\sum_{n=-\infty}^{\infty} |h[n]| = \frac{1}{2} + \frac{1}{2} = 1 < \infty$

(b) $y[n] = x[n-1] - 0.25\, y[n-1]$

- IIR - output depends on previous outputs
- causal - does not depend on future inputs or outputs
- $h[n]$:

n	$x[n]=\delta[n]$	$x[n-1]$	$-.25\,y[n-1]$	$y[n]$
0	1	0	0	0
1	0	1	0	1
2	0	0	$-.25$	$-.25$
3	0	0	$(-.25)^2$	$(-.25)^2$
4	0	0	$(-.25)^3$	$(-.25)^3$

$$h[n] = (-.25)^{n-1}\, u[n-1]$$

- <u>BIBO stable</u> -

$$\sum_{n=-\infty}^{\infty} |h[n]| = 1 + .25 + (.25)^2 + \cdots$$

$$= \frac{1}{1-.25} = \frac{1}{3/4} = 4/3 < \infty$$

TOPS 35500

(c) $y[n] = x[n-1] - 2y[n-1]$

n	$x[n]=\delta[n]$	$x[n-1]$	$-2y[n-1]$	$y[n]$
0	1	0	0	0
1	0	1	0	1
2	0	0	-2	-2
3	0	0	$(-2)^2$	$(-2)^2$
4	0	0	$(-2)^3$	$(-2)^3$
				⋮

- $$h[n] = (-2)^{n-1} u[n-1]$$

- Not BIBO stable -

$$\sum_{n=-\infty}^{\infty} |h[n]| = 1 + 2 + 2^2 + 2^3 + \dots = \infty$$

- IIR - output depends on previous outputs
  $h[n]$ continues to be nonzero out to $n = \infty$
- causal - does not depend on future input or outputs

(d) $2y[n] + 6y[n-1] = x[n] - x[n-2]$

$$y[n] = -3y[n-1] + x[n] - x[n-2]$$

- IIR - output depends on previous outputs
- causal - does not depend on future inputs or outputs

n	$x[n] = \delta[n]$	$x[n-2]$	$-3\,y[n-1]$	$y[n]$
0	1	0	0	1
1	0	0	-3	-3
2	0	1	$(-3)^2$	$1 + (-3)^2 = 10$
3	0	0	$-3 + (-3)^3$	$-3 + (-3)^3 = -30$
4	0	0	$(-3)^2 + (-3)^4$	$(-3)^2 + (-3)^4 = 54$
				$\vdots$
				$h[n]$

- Not BIBO stable

$$\sum_{n=-\infty}^{\infty} |h[n]| = 1 + 3 + 10 + 30 + 54 + \dots = \infty$$

Problem 2:

```
n=1:100;
x=1/10*sinc((n-51)/10); % multiply by 1/10 to account for time-scale factor
w=blackman(length(x));
w=w.';  % the blackman function creates a window
        %   that is a column vector, but I want a row vector.
x=[x zeros(1,100)];  % pad with zeros
w=[w zeros(1,100)];
n1=0:length(x)-1;     % n1 is the length of the padded signals
figure(1),subplot(2,2,1), plot(n1,x),xlabel('n'),
    ylabel('amplitude'),title('x[n], Fig. 16-1c')
figure(1),subplot(2,2,2), plot(n1,w),xlabel('n'),ylabel('amplitude'),
    title('Blackman window, Fig. 16-1e')

N=length(x);    % # of points in the FFT = # samples input to FFT
xfft=fft(x);    % fft of original truncated + time shifted sinc
MagX=abs(xfft(1:N/2+1));  % just need the 1st N/2 + 1 points
f=linspace(0,0.5,length(MagX));  % create frequency vector (fraction of sample
freq)
figure(1),subplot(2,2,3),plot(f,MagX),xlabel('amplitude'),
    title('DFT/FFT of X, Fig. 16-1d')
xw=x.*w;    % multiply the padded x[n] by the padded window
xwfft=fft(xw);  % FFT of windowed, truncated, time shifted sinc
MagXW=abs(xwfft(1:N/2+1));
figure(1),subplot(2,2,4),plot(f,MagXW),xlabel('amplitude'), axis([0 0.5 0 1.5])
    title('DFT/FFT of Windowed X, Fig. 16-1g')
```