1.(a) $y[n] = \frac{1}{3}\left[x[n] + x[n-1] + x[n-2]\right]$

$$Y(z) = \frac{1}{3}\left[X(z) + z^{-1}X(z) + z^{-2}X(z)\right]$$

$$= \frac{1}{3}\left[1 + z^{-1} + z^{-2}\right]X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{3}\left(1 + z^{-1} + z^{-2}\right) = \frac{z^2 + z + 1}{3z^2}$$

ROC: all $|z| > 0$

(b) $y[n] = x[n-1] - .4\, y[n-1]$ note: causal system.

$$y[n] + .4\, y[n-1] = x[n-1]$$

$$Y(z) + .4\, z^{-1} Y(z) = X(z) z^{-1}$$

$$Y(z)\left(1 + .4\, z^{-1}\right) = X(z) z^{-1}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{z^{-1}}{1 + .4\, z^{-1}} = \frac{1}{z + .4}$$

ROC $|z| > .4$ (causal system)

(c) $y[n] = x[n] - 5y[n-1]$

note: Causal system

$y[n] + 5y[n-1] = x[n]$

$Y(z) + 2z^{-1}Y(z) = X(z)$

$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1+5z^{-1}} = \frac{z}{z+5}$

ROC: $|z| > 5$ (causal system)

(d) $y[n] + .8y[n-1] + .25y[n-2] = x[n] + x[n-2]$

$Y(z) + .8z^{-1}Y(z) + .25z^{-2}Y(z) = X(z) + z^{-2}X(z)$

$Y(z)\left[1 + .8z^{-1} + .25z^{-2}\right] = X(z)\left[1 + z^{-2}\right]$

$H(z) = \frac{Y(z)}{X(z)} = \frac{1+z^{-2}}{1+.8z^{-1}+.25z^{-2}} = \frac{z^2+1}{z^2+.8z+.25}$

poles at

$z = \frac{-.8 \pm \sqrt{(.8)^2 - 4(1)(.25)}}{2}$

$= \frac{-.4 \pm \sqrt{-.36}}{2}$

$= -.4 \pm j.3$

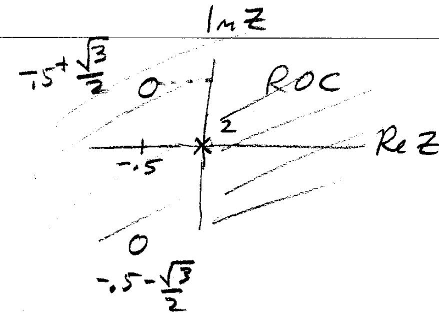
magnitude of the poles is 0.5

so ROC: $|z| > 0.5$

TOPS.

(2) Poles/Zeros/ROC/stable

(a) $H(z) = \dfrac{z^2 + z + 1}{3z^2}$



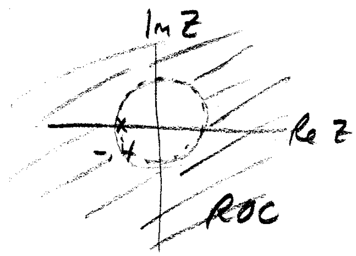
poles: 2 at $z = 0$

zeros at: $z = \dfrac{-1 \pm \sqrt{1^2 - 4}}{2}$

$= -.5 \pm j\dfrac{\sqrt{3}}{2}$

stable: ROC includes unit circle. $\Rightarrow$ yes

(b) $H(z) = \dfrac{1}{z + .4}$

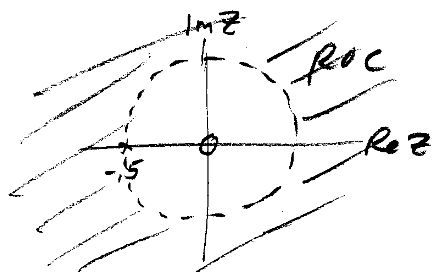


zeros – none

pole – $z = -.4$

stable – yes, unit circle in ROC

(c) $H(z) = \dfrac{z}{z + .5}$

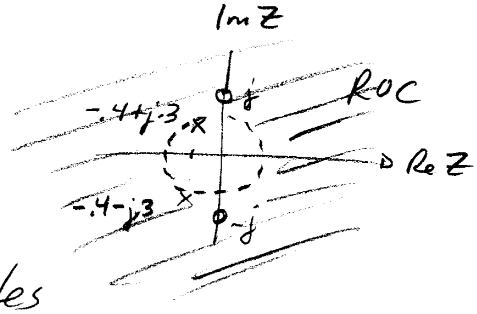


zero: $z = 0$

pole: $z = -.5$

stable – yes; ROC includes unit circle.

(d) $H(z) = \dfrac{z^2 + 1}{z^2 + .8z + .25}$



zeros: $z = \pm j$

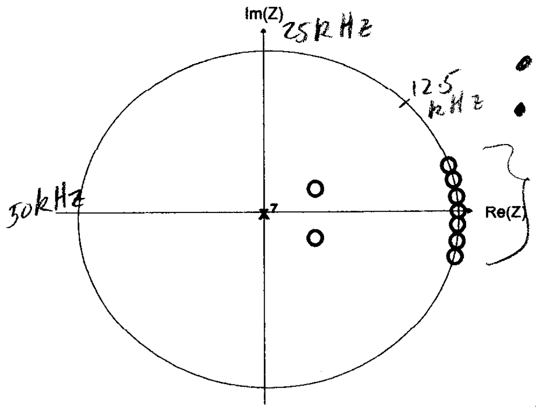
poles: $z = -.4 \pm j.3$
magnitude $= 0.5$

ROC: $|z| > 0.5$

stable – yes ROC includes unit circle

Note: to help answer question 3, I used fdatool and placed poles/zeros approximately in the locations of each frequency filter.

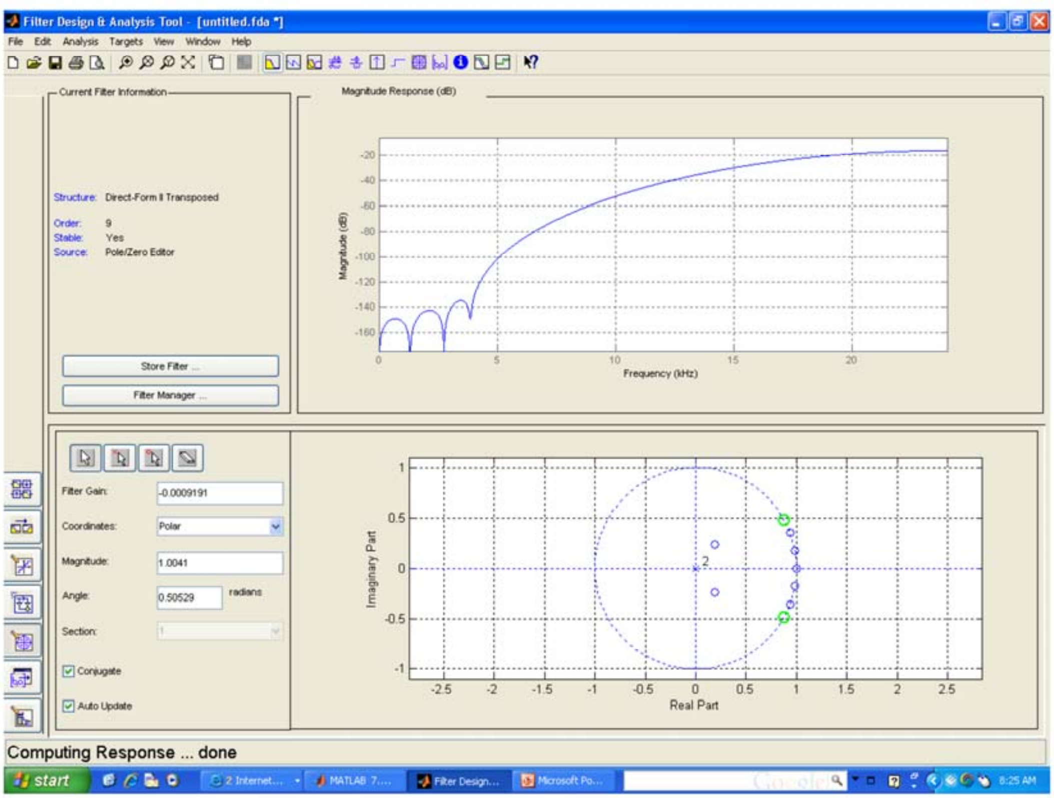
3. For the causal system pole zero plots that follow, classify the systems as to the following. Note that the unit circle is plotted on the Z-plane for each pole-zero plot.
   a. FIR or IIR? Why?
   b. Stable or not stable? Why?
   c. HPF or LPF or BPF or BRF? Why?

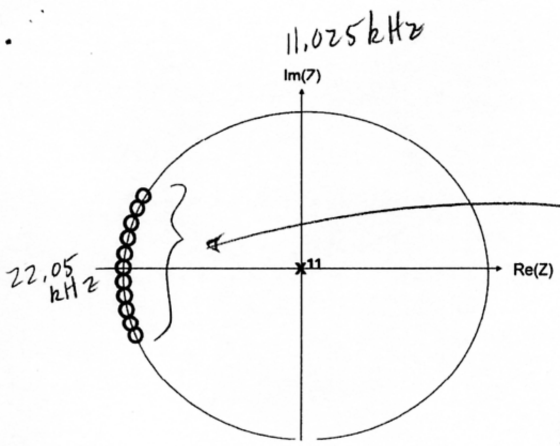


- FIR: poles only at $z=0$
- stable: all poles are inside unit circle.
- these zeroes at low freqs indicate a HPF

System A

$f_s = 100$ kHz
freqs $>\sim 6$ kHz pass

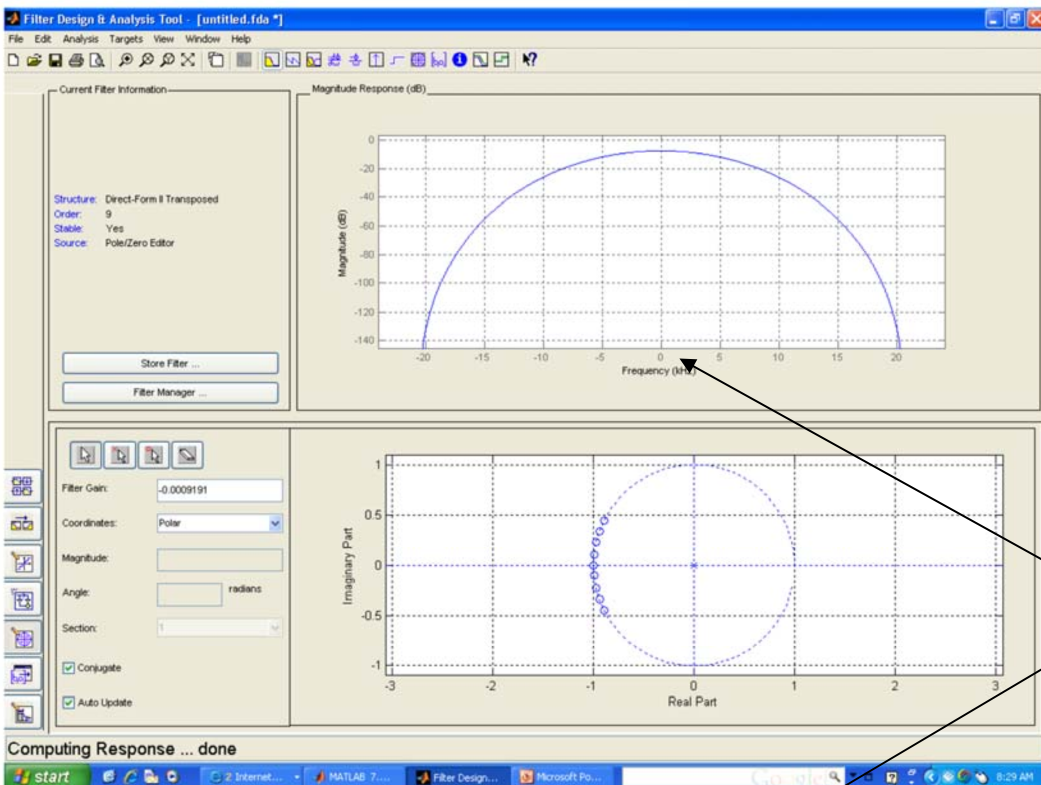




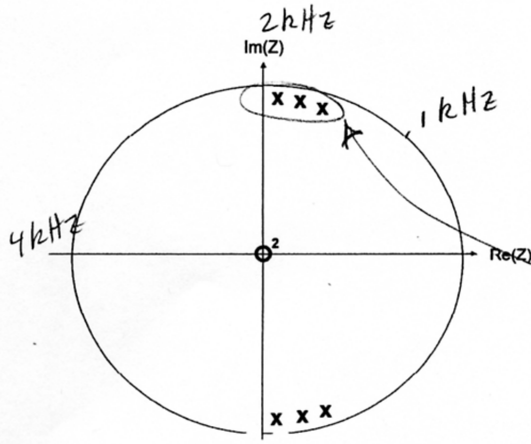
- FIR - poles only at $z=0$
- stable - all poles inside unit circle
- LPF - these zeros reduce the magnitude of $H(f)$ at high freqs.

System B

$f_s = 44.1$ kHz
freqs < ~17 kHz pass



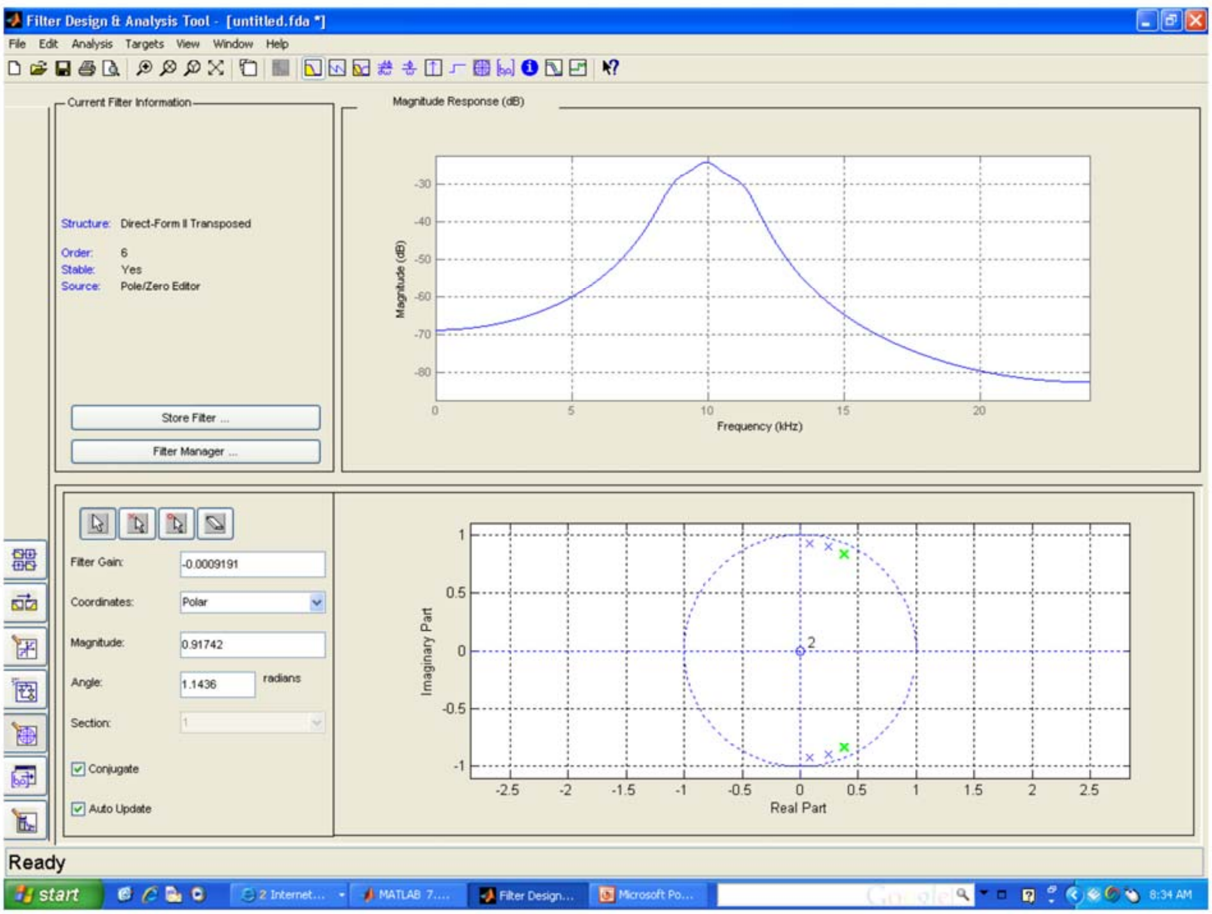
Note: 0 Hz is in the center of the frequency plot

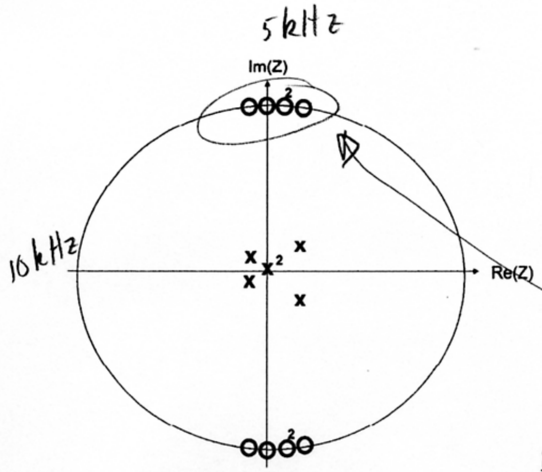


- IIR - poles not at $z=0$
- stable - poles are inside unit circle.
- the poles location will boost freqs around $f_s/4$, so this is a Band pass filter

System C $f_s = 8$ kHz

freqs 1.5–2 kHz are amplified

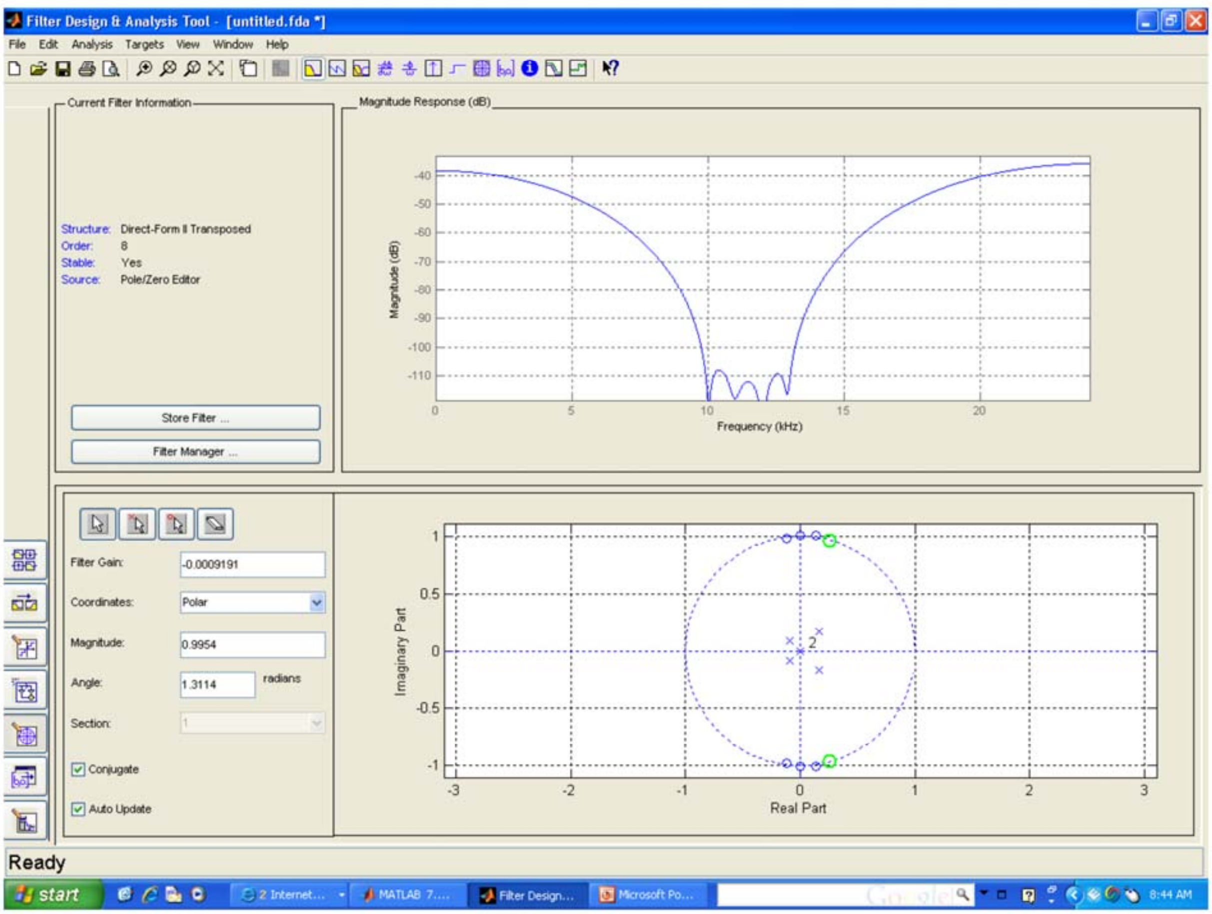


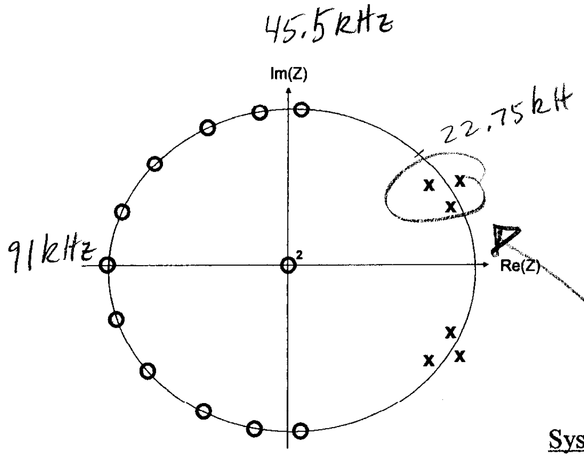


- IIR poles not at $z=0$
- stable – all poles inside unit circle
- Band reject filter – freqs close to $f_s/4$ will be suppressed by these zeros

<u>System D</u> $f_s = 20$ kHz

freqs ~ 4.5–5.5 kHz are rejected.





- IIR - poles not at $z=0$
- not stable - 2 poles outside unit circle
- Band pass filter - freqs close to $f_s/8$ will be boosted in amplitude by these poles

System E

$F_s = 192$ kHz

Freqs $>\sim 45.5$ kHz are attenuated alot, while freqs 15-20 kHz are amplified.