EE432 In-class exercise-11/14/11

Example 10.7. A low pass Butterworth IIR filter is to have a gain of -3dB at a frequency of 1 kHz, and a stop band attenuation of 30 dB at a frequency of 12 kHz. Find the difference equation and plot the frequency response for the filter if f_s = 25 kHz.

Step 1: the analog edge frequencies are... (given)

f_{p1} = 1000 Hz $\qquad$ f_{s1} = 12000 Hz

The attenuation at the edge of the stop band is $-20\log_{10}\delta_s$ = 30 dB

Step 2: the digital edge frequencies are...

$$\Omega_{p1} = 2\pi\frac{f_{p1}}{f_s} = 0.08\pi \text{ rad}, \quad \Omega_{s1} = 2\pi\frac{f_{s1}}{f_s} = 0.96\pi \text{ rad}$$

Step 3: the pre-warped analog edge frequencies are...

$$\omega_{p1} = 2f_s\tan\left(\frac{\Omega_{p1}}{2}\right) = 2(25{,}000)\tan\left(\frac{0.08\pi}{2}\right) = 6316.5 \text{ rad/sec}$$

$$\omega_{s1} = 2f_s\tan\left(\frac{\Omega_{s1}}{2}\right) = 2(25{,}000)\tan\left(\frac{0.96\pi}{2}\right) = 794{,}727.2 \text{ rad/sec}$$

Step 4: find δ_s to find the order of the required filter...

Since $-20\log_{10}\delta_s$ = 30 dB, $\delta_s = 10^{-30/20} = 0.03162$

Step 5: find the order of the filter…

$$n \geq \frac{\log_{10}\left(\frac{1}{\delta_s^2}-1\right)}{2\log_{10}\left(\frac{\omega_{s1}}{\omega_{p1}}\right)} = \frac{\log_{10}\left(\frac{1}{(0.03162)^2}-1\right)}{2\log_{10}\left(\frac{794727.2}{6316.5}\right)} = 0.714$$

→ so let $n = 1$ and use a 1st order Butterworth filter

Step 6: Find the transfer function of the digital filter (H(z)) and the difference equation…

The transfer function for the analog 1st order Butterworth filter (pre-warped) would be:

$$H(s) = \frac{\omega_{p1}}{s+\omega_{p1}} = \frac{6316.5}{s+6316.5}$$

Now apply the bilinear transformation…let $s = 2f_s\frac{z-1}{z+1}$

$$H(z) = \frac{6316.5}{50000\frac{z-1}{z+1}+6316.5} = \frac{0.1122(1+z^{-1})}{1-0.7757z^{-1}}$$

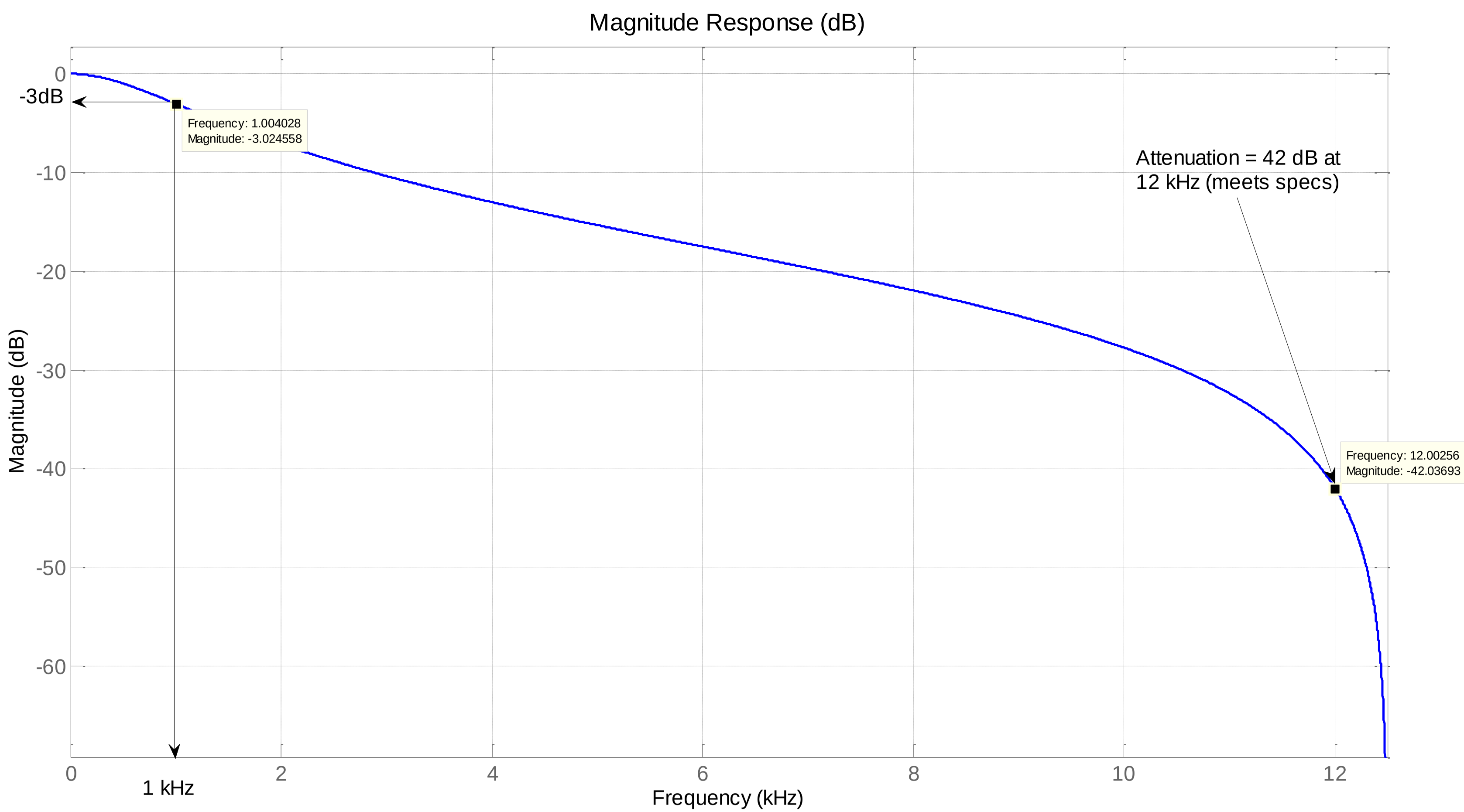
So the difference equation is…

y[n] = $0.1122\,x[n] + 0.1122\,x[n-1] + 0.7757\,y[n-1]$

And the frequency response is… let $z = e^{j\Omega}$ in the transfer function $H(z)$

$$H(\Omega) = \frac{0.1122(1+e^{-j\Omega})}{1-0.7757e^{-j\Omega}}$$

Frequency response plot on next page.


Magnitude Response (dB)
Magnitude (dB)
0
-3dB
-10
-20
-30
-40
-50
-60
Frequency: 1.004028
Magnitude: -3.024558
Attenuation = 42 dB at
12 kHz (meets specs)
Frequency: 12.00256
Magnitude: -42.03693
0
1 kHz
2
4
6
8
10
12
Frequency (kHz)

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