

EE432 In-class exercise-11/14/11

Example 10.7. A low pass Butterworth IIR filter is to have a gain of -3dB at a frequency of 1 kHz, and a stop band attenuation of 30 dB at a frequency of 12 kHz. Find the difference equation and plot the frequency response for the filter if $f_s = 25$ kHz.

Step 1: the analog edge frequencies are...

$$f_{p1} = \quad \quad \quad f_{s1} =$$

The attenuation at the edge of the stop band is $-20 \log_{10} \delta_s =$

Step 2: the digital edge frequencies are...

$$\Omega_{p1} = 2\pi \frac{f_{p1}}{f_s} = \quad \quad \quad , \quad \Omega_{s1} = 2\pi \frac{f_{s1}}{f_s} = \quad \quad \quad$$

Step 3: the pre-warped analog edge frequencies are...

$$\omega_{p1} = 2f_s \tan\left(\frac{\Omega_{p1}}{2}\right) = 2(\quad) \tan\left(\frac{\quad}{2}\right) = \quad \quad \quad$$

$$\omega_{s1} = 2f_s \tan\left(\frac{\Omega_{s1}}{2}\right) = 2(\quad) \tan\left(\frac{\quad}{2}\right) = \quad \quad \quad$$

Step 4: find δ_s to find the order of the required filter...

$$\text{Since } -20 \log_{10} \delta_s = 30 \text{ dB, } \delta_s = \quad \quad \quad =$$

Step 5: find the order of the filter...

$$n \geq \frac{\log_{10}\left(\frac{1}{\delta_s^2} - 1\right)}{2\log_{10}\left(\frac{\omega_{s1}}{\omega_{p1}}\right)} = \frac{\log_{10}\left(\frac{1}{\delta_s^2} - 1\right)}{2\log_{10}\left(\frac{1}{\delta_s^2} - 1\right)} =$$

Step 6: Find the transfer function of the digital filter (H(z)) and the difference equation...

The transfer function for the analog 1st order Butterworth filter (pre-warped) would be:

$$H(s) = \frac{\omega_{p1}}{s + \omega_{p1}} =$$

Now apply the bilinear transformation...let $s = 2f_s \frac{z-1}{z+1}$

$$H(z) =$$

So the difference equation is...

$$y[n] =$$

And the frequency response is...

$$H(\Omega) =$$