EE432: Digital Signal Processing Fall 2011

Project 06: Z-Transforms & Transfer Functions

Assigned: Tues 10/11/11 **Due: Tues 10/18/11**

This project is intended to give you some practice working with Z-transforms, and their relationship to difference equations, impulse responses and transfer functions. Note: all plots should be stem plots.

I. Z-Transforms

1. Find the Z-transform and ROC of a signal given by $x[n] = 0.5\delta[n+1] + \delta[n] - \delta[n-2]$.

2. Find the Z-transform and ROC of a signal given by: $y[n] = (-0.25)^n u[n] + \left(-\frac{2}{3}\right)^n u[n]$.

3. Find the Z-transform and ROC of a signal given by: $x[n] = (0.85)^n \cos\left(\frac{2\pi n}{3}\right) u[n]$.

4. Find the signal whose Z-transform is $X(z) = \frac{2z^2 - 5z}{z^2 + .4z + .03}$, with ROC |z| > 0.3. Hint : use PFE.

II. Transfer Functions

1. For each difference equation below, do the following: (1) find the transfer function (by hand in the space below); (2) use MATLAB's *impz* function to determine the impulse response, and then stem plot the systems' impulse responses; (3) use *roots* to find the poles and zeros of the transfer function; (4) use the *zplane* function to plot the poles and zeros of the transfer function; and finally, (5) determine if it is a BIBO stable system or not (state your reasoning in the space below). **Use a 2x2 subplot for the four plots in parts a-b, and another 2x2 subplot for the four plots in parts c-d to minimize the amount of paper you turn in. Label the plots with the problem #.**

   a. $y[n] = 0.1y[n-2] - 0.1x[n] + 0.25x[n-1]$

   $H(z)$ = ____

   BIBO stable? Why or why not?

   b. $y[n] = 0.5y[n-1] - y[n-2] - 0.6x[n] + 0.2x[n-3]$

   $H(z)$ = ____

   BIBO stable? Why or why not?

   c. $y[n] = x[n] - 0.3x[n-1] + 0.06x[n-3] + 0.2y[n-1] - 0.25y[n-2] + 0.25y[n-3]$.

   $H(z)$ = ____

   BIBO stable? Why or why not?

   d. $y[n] - 0.65y[n-8] = x[n]$.

   $H(z)$ = ____

   BIBO stable? Why or why not?

2. A transfer function filter has the following 6 poles and 6 zeros:

$$\text{Zeros: } \begin{cases} z = 0.2 \pm j0.95 \\ z = 1.1 \pm j0.83 \\ z = e^{\pm j\frac{7\pi}{16}} \end{cases}$$

$$\text{Poles: } \begin{cases} z = 0,0 \ \ \text{(2 poles)} \\ z = 0.85e^{\pm j\frac{2\pi}{3}} \\ z = 0.25 \pm j0.88 \end{cases}$$

Determine the system transfer function, and use *zplane* to plot the poles and zeros. Is the system BIBO stable? Why or why not? Turn in your code. Note: The *conv* function can be used to multiply polynomials together. For example, to multiply $(z^2 + 1)$ by $(z - 1)$, use:

```
>> newpoly=conv([1 0 1],[1 -1])

newpoly =

     1    -1     1    -1
```

which represents $z^3 - z^2 + z - 1$.

$H(z)$ = ____

BIBO stable? Why or why not?

3. Using MATLAB's *conv* function, multiply the following terms together to determine the polynomial. Turn in your code.

   a. $(x + 3)(x + 12) =$ ______________________________________

   b. $(x + 2)(x^2 + 3x - 6) =$ ______________________________________

   c. $(x+1.3)^4 =$ ______________________________________

   d. $(x^3 + 0.2x - 4)(x^5 - 0.8) =$ ______________________________________

4. Using MATLAB's *roots* function, find the poles and zeros of the following, then plot the roots with *zplane* in a 2x1 subplot (one subplot total for parts a-b) and indicate if these systems are BIBO stable or not. Turn in your code.

   a. $H(z) = (z^4 - 0.5z^3 + 2z - 0.5) / (z^5 - 0.3z^4 + 2z^2 + 1)$

      Poles: Zeros:

      BIBO stable?

   b. $H(z) = (z^2 - 0.7z) / (z^5 - 0.3z^4 + 0.2z^3 + 0.6)$

      Poles: Zeros:

      BIBO stable?

III. Plucked String Filter

A plucked string filter is a digital representation of the sound generated when a string from a guitar gets plucked. The transfer function of a plucked string filter is of the form:

$$H(z) = \frac{z^{L+1} + z^{L}}{2z^{L+1} - R^{L}z - R^{L}}$$

In this equation, *R* is the magnitude of a pole, and *L* represents the number of delays in the systems.

1. Write a MATLAB function called *pluckstring* that will take 2 inputs and produce 2 outputs. The two inputs are the value of *L* and the value of *R*. The two outputs are the *b* coefficients of the transfer function (a vector) and the *a* coefficients of the transfer function (another vector).

   Error checking: Your function must check to make sure that the input value of *L* is an integer > 1, and the value of *R* is < 1.0 (this will ensure the filter is stable).

2. Design a plucked string filter with L=100 and R=0.999: Determine the transfer function *H*(*z*) in positive powers of z, and the difference equation. Use *zplane* to plot the poles and zeros.

3. Random numbers are used as an input to the filter to generate the sound of the plucked string. We will use a sample frequency of 8000 Hz, and your filter from step 1. Create an input signal consisting of *L*=100 random numbers (use *randn* to create them) followed by enough zeros to make the signal last for at two seconds. Use *filter* to filter this input with your plucked string filter.

4. Use *wavplay* or *soundsc* to play your filter's output. Don't forget to use the correct sample frequency. Does it sound like a plucked string?

5. Determine the resonant frequency of the note that is played. The resonant frequency is given by:

   $$f_r = \frac{f_s}{L + 0.5} \text{ Hz.}$$

   Resonant frequency = ___________

   Plot the output note versus <u>time</u> (in sec) in a well annotated plot and turn it in with your report. Use *wavwrite* to write this out as a sound file and email it to the professor. Call the file "note1.wav".

6. Now try a different note. Create a filter as you did in step 1, but use L=120 and R=0.99999999999999999 (there are seventeen 9s). If your MATLAB won't allow you to have a value 17 places beyond the decimal point, you can use the most that your MATLAB will allow, just annotate that on your write-up. What will be the resonant frequency of the note? Create the correct input to the filter and the note, then use *wavwrite* to write out the note, and send it to the professor. Call the file "note2.wav". Plot the output note versus <u>time</u> (in sec) in a well annotated plot and turn it in with your report. What are the audible differences between note1 and note2?

**Turn in your answers to the problems in parts I-III,
your code, the stem plots called for, and the wav files from part III.**