EE461 Microprocessor-based Digital Design

Memory Error Detection and Correction

1. In these problems you will be considering the design and use of a SECDED Hamming Error-Correction Code. The purpose of the code is to permit transmission of 6-bit messages by adding check and parity bits to them. Explain all your answers using clear English, not just symbols and numbers.

   (a) How many error-detection and error-correction bits must the code contain in all? What is the length of the resultant code words, including both message bits and error-handling bits combined?

   SOLUTION

   Excluding the overall parity bit, we require that an (n — 1)-bit code contain k code bits and $m = n—(k+1)$ message bits. In this problem, m =6.

   We also require that

$$n—1 = m+k \le 2^k —1$$
$$2_k \ge 7+k$$

   and

$$n—1 = m+k > 2^{k-1} — 1$$
$$2_{k-1} < 7+k.$$

   It is easy to see that k = 4 because only this value satisfies both inequalities. If we now include the overall parity bit, we require a total of five error-handling bits and a combined code-word length of $n = m + k + 1 = 6 + 4 + 1 = 11$.

(b) You have received the code word $5C5_{16}$. State whether the contained 6-bit message has zero, one, or two errors. If it has zero or one errors, give the (corrected) message in hexadecimal.

The parity bit p_0 is incorrect and the indicated error location is bp1. Therefore, there is a one-bit error in the hamming code, and the message is 101100.

(c) You have received the code word $45C_{16}$. State whether the contained 6-bit message has zero, one, or two errors. If it has zero or one errors, give the (corrected) message in hexadecimal.

SOLUTION

The parity bit is correct, and the hamming code calculation leads to bp9. Therefore, there is a 2-bit error.