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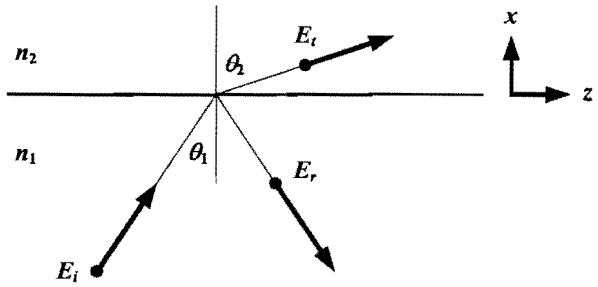
1. (15 pts) The figure shows the reflection and refraction of light at an interface between two different materials, each having an index of refraction as shown, where $n_1 > n_2$.

Given:

$n_1 \sin\theta_1 = n_2 \sin\theta_2$ $\qquad \sin\theta_c = \dfrac{n_2}{n_1}$

$\sin^2\theta + \cos^2\theta = 1$ $\qquad j = \pm\sqrt{-1}$

$$E_t(x,z) = \hat{y}E_t e^{-jk_o n_2 (x\cos\theta_2 + z\sin\theta_2)}$$



wording

For $\theta_1 > \theta_c$, $\sin\theta_2 > 1$, total internal reflection (TIR) occurs, and the refracted field is $E_t(x,z) = \hat{y}E_t e^{-\alpha x} e^{-j\beta z}$, where $\beta = k_o n_2 \sin\theta_2 = k_o n_1 \sin\theta_1$. Derive a **simplified expression** for the absorption coefficient α of the evanescent field in terms of the incident angle θ_1.

$$e^{-jk_o n_2 x \cos\theta_2} = e^{-\alpha x} \quad \text{so} \quad \alpha = jk_o n_2 \cos\theta_2$$

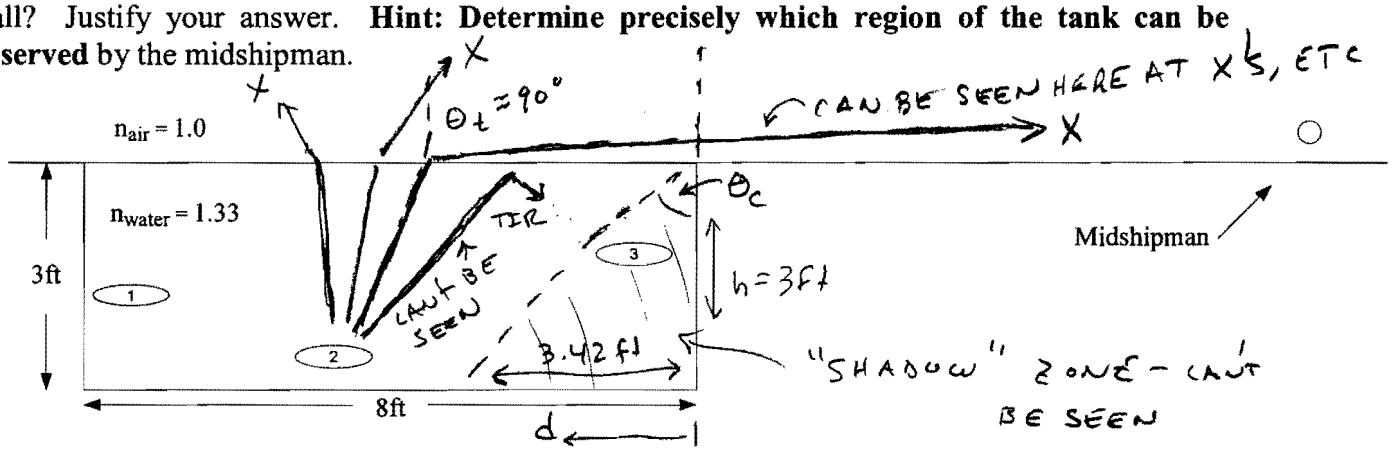
$$\cos\theta_2 = \sqrt{1 - \sin^2\theta_2}$$

$$\alpha = jk_o n_2 \sqrt{1 - \sin^2\theta_2} = k_o n_2 \sqrt{-1}\sqrt{1 - \sin^2\theta_2}$$

$$= k_o n_2 \sqrt{\sin^2\theta_2 - 1} = k_o \sqrt{(n_2 \sin\theta_2)^2 - n_2^2}$$

$$\boxed{\alpha = k_o \sqrt{(n_1 \sin\theta_1)^2 - n_2^2}}$$

2. (10 pts) A midshipman has gone to the Baltimore Aquarium on a Saturday afternoon, but after a busy week of exams he has fallen asleep while leaning against a wall. A good distance down the wall from the midshipman is a fish tank that is 3ft deep and 8ft long that contains three fish. If he were to wake up, which of the three fish, if any, could he see from his position near the wall? Justify your answer. **Hint: Determine precisely which region of the tank can be observed** by the midshipman.



IF TANK WELL LIGHTED, EACH FISH REFLECTS LIGHT IN ALL DIRECTIONS — SO REAL QUESTION IS WHETHER NEAR CORNER OF TANK CUTS OFF LIGHT COMING FROM A GIVEN FISH. SINCE MID IS NEAR WALL, LIGHT MUST REFRACT NEAR $\theta_t = 90°$ FOR HIM TO SEE A FISH

$\Rightarrow \theta_i \approx \theta_c$.

$$\theta_c = \sin^{-1}\left(\frac{n_{air}}{n_{water}}\right) = \sin^{-1}\left(\frac{1.0}{1.33}\right) = 48.75°$$

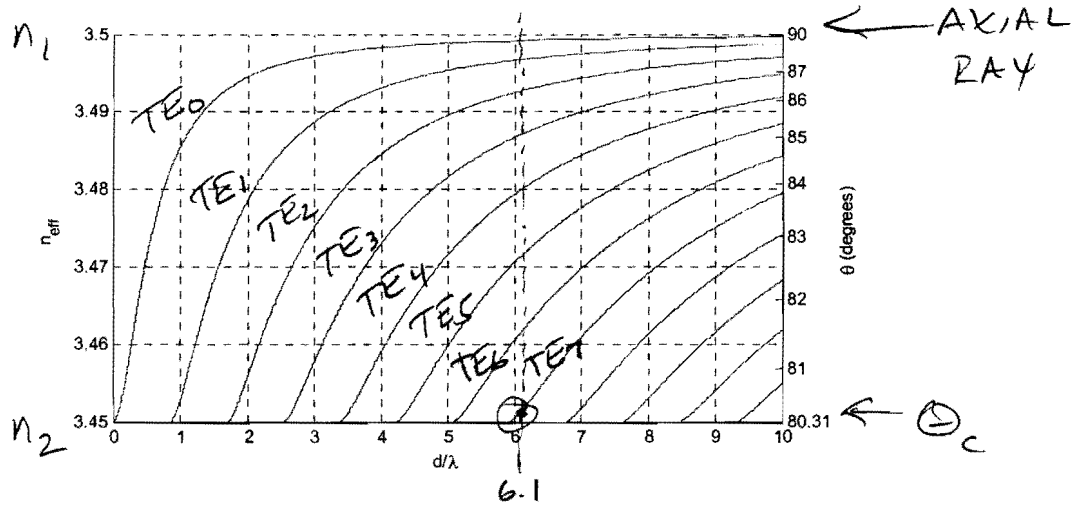
$$\tan\theta_c = \tan 48.75° = \frac{d}{h} \Rightarrow d = 3\tan 48.75 = 3.42\text{ft}$$

USING FISH 2 AS EXAMPLE - RAY JUST LESS THAN θ_c REFRACTS AT $\theta_t \approx 90°$

- RAY JUST OVER θ_c REFLECTS TOTALLY.

FISH 1 & 2 CAN BE SEEN

3. (35 pts) The mode chart for the active area of an AlGaAs LED is shown below. The range of the effective index of refraction in the guide is defined as $n_2 < n_{eff} < n_1$. The wavelength emitted by the LED is 820 nm. The thickness of the active area is 5 μm. The drive current is 100 mA. The carrier lifetime is 3 ns.



a) (5pts) **Label only the TE_m modes that actually propagate** in this LED along each corresponding curve on the mode chart.

$$\frac{d}{\lambda} = \frac{5\,\mu m}{0.82} = 6.1 \Rightarrow 8 \text{ MODES}$$

b) (5pts) Which TE_m mode propagates near the critical angle?

TE_7

c) (10pts) Solve for the numerical aperture and estimate the angular beam divergence of the light as it exits the LED into air ($n_o = 1$).

$n_1 = 3.5$

$n_2 = 3.45$

$$NA = \sqrt{n_1^2 - n_2^2} = \sqrt{3.5^2 - 3.45^2} = \boxed{0.589}$$

$$= n_o \sin \alpha_{o\,max} = \sin \alpha_{o\,max}$$

$$\alpha_{o\,max} = \sin^{-1}(0.589) = 36.1^\circ$$

$$2\alpha_{o\,max} = \boxed{72.2^\circ}$$

d) (10pts) If 10% of the electron-hole recombinations in the active area result in an emitted photon, solve for the optical power emitted by the LED.

$$P = \eta W_g(eV) \cdot I = 0.1\left(\frac{1.24}{0.82}\right)(100m) = \boxed{15.1\,mW}$$

$\eta = 0.1$ or 10%

$W_g = \frac{1.24}{\lambda(\mu m)}$

$I = 100\,mA$

$\lambda = 0.82\,\mu m$

e) (5pts) At what modulation frequency is the emitted optical power is reduced by 1.5dB? (See hint below.)

$$f_{1.5dB}(opt) = f_{3dB}(elec) = \frac{1}{2\pi\tau}$$

$$= \frac{1}{2\pi(3ns)}$$

$$= \boxed{53.1\,MHz}$$

Hint: If optical power P is linearly proportional to drive current I, then:

$$10\log\left(\frac{P_2}{P_1}\right) = \frac{1}{2} \times 20\log\left(\frac{I_2}{I_1}\right).$$

$-1.5dB = \frac{1}{2}(-3dB)$

↳ occurs @ $f_{3dB}(elec)$

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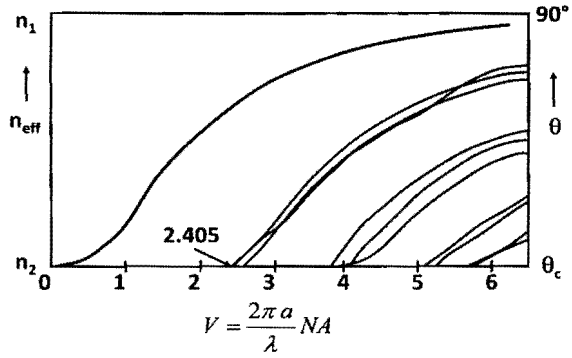
$a = 4\,\mu m$

4. (40pts) A certain optical fiber has a core diameter of 8 μm, a core index of refraction of 1.48, and a numerical aperture of 0.12. The mode chart is as shown below. The dispersion zero wavelength is 1310 nm, and the slope at the zero dispersion wavelength is -0.085 ps/nm^2-km. The source wavelength is 1.55 μm with spectral width 125 GHz.

a) (5pts) For what range of wavelengths does the fiber support only single mode propagation?



SMF FOR

$$V = \frac{2\pi a}{\lambda} NA < 2.405$$

OR $$\lambda > \frac{2\pi a NA}{2.405} = \frac{2\pi(4)(0.12)}{2.405}$$

$$\boxed{\lambda > 1.25\,\mu m}$$

b) (10pts) Solve for the **mode field diameter** of the fundamental LP_{01} mode.

$$w = a\left(0.65 + 1.619 V^{-3/2} + 2.879 V^{-6}\right)$$

$$V = \frac{2\pi(4)}{1.55} \cdot 0.12 = 1.95$$

$$w = 4\left(0.65 + 1.619(1.95)^{-1.5} + 2.879(1.95)^{-6}\right)$$

$$= 5.19\,\mu m$$

$$MFD = 2w = 2(5.19) = \boxed{10.38\,\mu m}$$

c) (15pts) Solve for the dispersive pulse spread/unit length in this fiber at the source wavelength.

$\lambda = 1.55\,\mu m > 1.25\,\mu m \Rightarrow$ SMF

Gave Defn of Mo

$$M = \frac{M_o}{4}\left(\lambda - \frac{\lambda_o^4}{\lambda^3}\right) = \frac{-0.085\ ps/nm^2\text{-}km}{4}\left(1550nm - \frac{1310nm^4}{1550nm^3}\right)$$

$$= -16.1\ ps/nm\text{-}km$$

$$\Delta\left(\frac{\tau}{L}\right) = -M\,\Delta\lambda \qquad \Delta\lambda = \Delta f \cdot \frac{\lambda^2}{c} = 125\,GHz \cdot \frac{(1550\times10^{-9})^2}{3\times10^8}$$

$$= 1.0\ nm$$

$$\Delta\left(\frac{\tau}{L}\right) = -16.1\ \frac{ps}{nm\text{-}km} \cdot 1nm = \boxed{-16.1\ ps/km}$$

d) (10pts) If the loss at the source wavelength is 0.2dB/km, solve for the maximum length of the fiber if the source power is -5dBm and the receiver sensitivity is -25dBm.

$$dBm_r = dBm_t + dB_s$$

$$-25dBm = -5dBm + dB_s$$

$$dB_s = -25 - (-5) = -20dB \Rightarrow 20dB\ loss$$

$$L = \frac{20dB}{0.2dB/km} = \boxed{100km}$$

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Equation sheet:

$$\omega = 2\pi f = \frac{2\pi c}{\lambda_o} = k_o c$$

$$NA = n_o \sin\alpha_{o\max} = \sqrt{n_1^2 - n_2^2} \cong n_1\sqrt{2\Delta}$$

$$V = \frac{2\pi a}{\lambda} NA$$

$$|\Delta\lambda| = \Delta f \frac{\lambda^2}{c}$$

$$W_g(\text{eV}) = \frac{1.24}{\lambda(\mu\text{m})}$$

$$P = \eta W_g(\text{eV}) I$$

$$f_{3dB} = \frac{1}{2\pi\tau} \quad \text{(electrical)}$$

$$P_{SP}(\omega) = \frac{\eta W_g I}{\sqrt{1+(\omega\tau)^2}}$$

$$dB = 20\log\left(\frac{I_2}{I_1}\right)$$

$$dB = 10\log\left(\frac{P_2}{P_1}\right)$$

$$\Delta\left(\frac{\tau}{L}\right) = \frac{n_1\Delta}{c} \quad \text{(MMF)}$$

$$\Delta\left(\frac{\tau}{L}\right) = -M\Delta\lambda \quad (S\text{MF})$$

$$M = \frac{M_0}{4}\left(\lambda - \frac{\lambda_0^4}{\lambda^3}\right)$$

$$\frac{w}{a} = 0.65 + 1.619(V)^{-1.5} + 2.879(V)^{-6}$$

$$dBm_r = dBm_t + dB_s$$