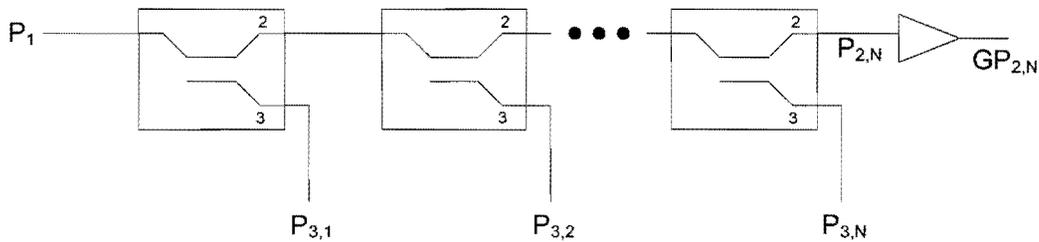


1. (25pts) A fiber bus is used to broadcast an input signal with total optical power P_1 to nodes using 1x2 splitters as shown below. The throughput and tap losses from input port 1 to output ports 2 and 3, respectively, on each splitter are $L_{THP} = 3.2\text{dB}$ and $L_{TAP} = 3.4\text{dB}$, respectively. An erbium doped fiber amplifier after the Nth coupler exactly compensates loss in the first N nodes, providing additional throughput power for nodes further down the bus. Assume fiber loss and connector loss is negligible between nodes.



a) (15 pts) If the receiver design limits the dynamic range to $DR \leq 10\text{dB}$, where $DR = 10 \log(P_{3,1} / P_{3,N})$, find the maximum number of nodes N before an amplifier is needed, and find the amplifier gain in dB, $G_{dB} = 10 \log(P_1 / P_{2,N})$.

$$\begin{aligned}
 \text{in dBm} \quad P_{3,1} &= P_1 - L_{TAP} \\
 P_{3,N} &= P_1 - (N-1)L_{THP} - L_{TAP}
 \end{aligned}
 \left. \vphantom{\begin{aligned} P_{3,1} \\ P_{3,N} \end{aligned}} \right\} \begin{aligned}
 DR &= P_{3,1} - P_{3,N} \\
 &= P_1 - L_{TAP} - (P_1 - (N-1)L_{THP} - L_{TAP}) \\
 &= (N-1)L_{THP} = (N-1)(3.2) < 10 \text{ dB} \\
 \Rightarrow \boxed{N=4}
 \end{aligned}$$

$$G_{dB} = P_1 - P_{2,N} = 4(L_{THP}) = 4(3.2) = \boxed{12.8 \text{ dB}}$$

b) (10pts) Solve for the excess loss in dB for each coupler, where $L_E = 10 \log\left(\frac{P_1}{P_2 + P_3}\right)$.

$$\frac{P_2}{P_1} = 10^{-3.2/10} = 0.4786$$

$$\frac{P_3}{P_1} = 10^{-3.4/10} = 0.4571$$

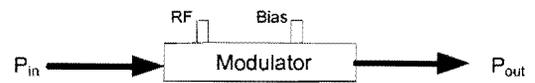
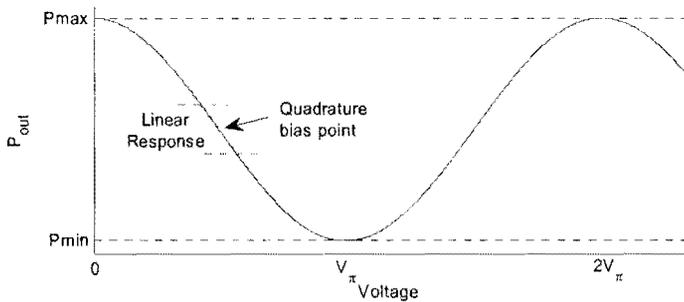
$$\begin{aligned}
 L_E &= -10 \log\left(\frac{P_2}{P_1} + \frac{P_3}{P_1}\right) \\
 &= -10 \log(0.4786 + 0.4571) \\
 &= \boxed{0.289 \text{ dB}}
 \end{aligned}$$

2. (15pts) A laser diode has a threshold current of 25 mA. The slope of the laser diode response above threshold is 0.4 mW/mA. The carrier lifetime is 2 ns. If the laser is modulated with a drive current of $i(t) = 35 + 5\sin(2\pi 100\text{MHz} \cdot t)$ mA, solve for the emitted optical power $P(t)$.

$$\left. \begin{array}{l} I_{TH} = 25 \text{ mA} \\ I_{DC} = 35 \text{ mA} \\ I_{SP} = 5 \text{ mA} \\ \alpha_1 = 0.4 \text{ mW/mA} \end{array} \right\} \begin{array}{l} P_{DC} = \alpha_1 (I_{DC} - I_{TH}) = 0.4 (35 - 25) = 4 \text{ mW} \\ P_{SP} = \frac{\alpha_1 I_{SP}}{\sqrt{1 + \omega^2 \tau^2}} = \frac{0.4 (5)}{\sqrt{1 + (2\pi 100\text{M} \times 2\text{n})^2}} = 1.245 \text{ mW} \end{array}$$

$$P(t) = 4 + 1.245 \sin(2\pi 100\text{MHz} \cdot t) \text{ mW}$$

3. (10pts) Instead of directly modulating the drive current for the laser diode in problem 2, assume the drive current is held constant at 35mA and an external modulator is used for which $V_\pi = 6\text{V}$.



a) (4pts) Based on the modulator response shown above, what bias voltage should be applied to the modulator to provide a linear response?

$$V_{bias} = \frac{V_\pi}{2} = \boxed{3 \text{ V}}$$

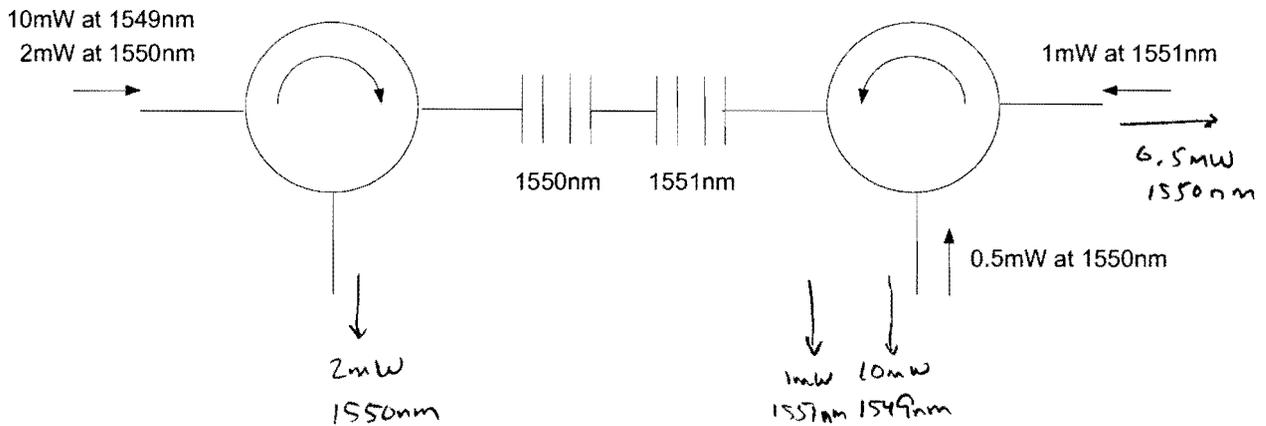
b) (3pts) For a digital link, what peak-to-peak RF voltage minimizes the bit error rate?

$$V_{RF \text{ p.k.p.k}} = V_\pi = \boxed{6 \text{ V}}$$

c) (3pts) For an analog link, what peak-to-peak RF voltage minimizes total harmonic distortion?

V_{RF} as small as possible

4. (8pts) Circulators and two fiber Bragg gratings (FBGs) are arranged in a WDM device as shown below. The input power level on 4 input channels is labeled, along with the corresponding wavelength. The reflection wavelength of each FBG is as labeled. Indicate **clearly** on the diagram where the power on each channel is output from the device.



5. (12pts) The EDFAs used in a certain WDM optical network have a gain bandwidth of 30nm near 1550nm. If the network uses a dense WDM channel spacing of 50GHz, and if each channel supports a data rate of 2.5Gb/s, what is the total aggregate data throughput that can be supported on each optical fiber in this network?

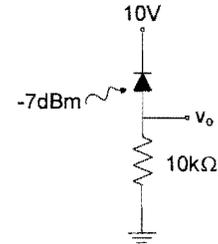
$$\Delta\lambda = \Delta f \frac{\lambda^2}{c} = (50\text{GHz}) \frac{(1550\text{nm})^2}{3 \times 10^8} = 0.4 \text{ nm}$$

$$R = \frac{30 \text{ nm}}{0.4 \text{ nm}} \times 2.5\text{G} = \boxed{187.5 \text{ Gb/s}}$$

6. (30pts) The responsivity of a certain PIN photodetector is 0.8 A/W at a room temperature of 300 K . The PIN receiver includes a $10 \text{ k}\Omega$ load resistor and a 10 V DC source as shown below. Assume that the dark current is negligible. The wavelength of the incident light is 1310 nm .

a) (22pts) A signal with an average optical power of -7 dBm at a 20% modulation depth is incident on the detector. Determine the electrical signal to noise ratio (SNR) in dB in the receiver. The noise bandwidth is 100 MHz .

$$\begin{aligned} \rho &= 0.8 \text{ A/W} & R_L &= 10 \text{ k}\Omega & \lambda &= 1310 \text{ nm} \\ T &= 300 \text{ K} & V_B &= 10 \text{ V} & P_{\text{inc}} &= -7 \text{ dBm} = 10^{-0.7} \\ & & & & &= 200 \mu\text{W} \\ m &= 0.2, \Delta f &= 100 \text{ MHz} \\ \text{PIN} &\rightarrow M=1 & J &= \frac{q}{h\nu} \rho P = \rho P = 0.8 P \end{aligned}$$



$$S = \frac{M^2}{2} \cdot M^2 (\rho P)^2 R_L = \frac{(0.2)^2}{2} \cdot (1)^2 (0.8 \times 200 \mu)^2 (10 \text{ k}) = 5.12 \times 10^{-6}$$

$$N = M^2 2eR_L \Delta f (\rho P) + 4kT \Delta f$$

$$= 2(1.6 \times 10^{-19})(10 \text{ k})(100 \text{ M})(0.8 \times 200 \mu) + 4(1.38 \times 10^{-23})(300)(100 \text{ M})$$

$$= 52.86 \times 10^{-12} \text{ W}$$

$$\frac{S}{N} = \frac{5.12 \times 10^{-6}}{52.86 \times 10^{-12}} = 96860 \text{ or } 10 \log 96860 = \boxed{49.86 \text{ dB}}$$

b) (8pts) The SNR can be improved if the incident optical power is increased. Determine the maximum optical power that can be incident on the detector without saturating the receiver.

$$V_o = I \cdot R_L < V_B = 10 \text{ V}$$

$$I_{\text{max}} = \frac{10 \text{ V}}{R_L} = \frac{10 \text{ V}}{10 \text{ k}} = 1 \text{ mA}$$

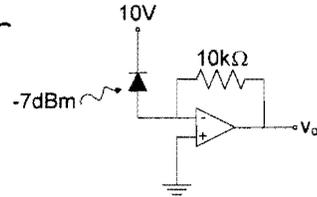
$$P_{\text{max}} = \frac{I_{\text{max}}}{\rho} = \frac{1 \text{ mA}}{0.8} = \boxed{1.25 \text{ mW}}$$

c) BONUS (15pts) For video applications, a SNR of 60dB or higher may be required. Based on the results in parts a) and b), determine which of the following two design options is best to meet this need. Explain and provide a numerical justification for your answer.

i) Replace the PIN diode with an APD to improve sensitivity.

ii) Add a transimpedance amplifier to the PIN receiver to improve dynamic range.

THE MAX. SNR OCCURS AT QUANTUM LIMIT -
FOR P AS LARGE AS POSSIBLE (SEE PART b)



$$\frac{S}{N} = \frac{m^2}{2} \cdot \frac{\eta P}{2hf\Delta f} \quad f = \frac{c}{\lambda} = \frac{3 \times 10^8}{1.31 \times 10^{-6}}$$

$$\eta = \rho \cdot W_g (\text{eV}) = \rho \frac{1.24}{\lambda} = 0.8 \cdot \frac{1.24}{1.31} = 0.757$$

$$\frac{S}{N} = \frac{(0.2)^2}{2} \cdot \frac{0.757 (1.25 \text{ m}) (1.31 \times 10^{-6})}{2 (6.626 \times 10^{-34}) (3 \times 10^8) 100 \text{ M}} = 623600$$

$$= 58 \text{ dB} < 60 \text{ dB}$$

STILL LESS
THAN NEEDED

⇒ CAN'T GET TO 60 dB SNR USING AN APD

∴ USE A TRANSIMPEDANCE AMPLIFIER - ALLOWS YOU
TO INCREASE POWER BEYOND 1.25 mW

Equations:

$$\text{dB} = 10 \log_{10} \left(\frac{P_1}{P_2} \right)$$

$$P_{DC} = a_1 (I_{DC} - I_{TH})$$

$$P_{SP} = \frac{a_1 I_{SP}}{\sqrt{1 + (\omega\tau)^2}}$$

$$\frac{P_{out}(t)}{P_{in}} = \frac{1}{2} \left[1 + \cos \left(\pi \left(\frac{V_{dc} + V_{rf}(t)}{V_{\pi}} \right) \right) \right]$$

$$\frac{S}{N} = \frac{m^2}{2} \frac{M^2 (\eta e P / hf)^2 R_L}{M^2 2eR_L \Delta f (I_D + \eta e P / hf) + 4kT \Delta f}$$

$$\frac{S}{N} = \frac{m^2}{2} \frac{\eta P}{2hf \Delta f} \quad (\text{quantum limited SNR})$$

$$\text{BER} \cong \frac{\exp(-Q^2/2)}{Q\sqrt{2\pi}}$$

$$Q = \frac{I_1 - I_0}{\sigma_1 + \sigma_0}$$

$$\sigma^2 = 2eI \Delta f + \frac{4kT \Delta f}{R_L}$$

$$I = \rho P = \frac{\eta e}{hf} P = \frac{\eta P}{W_g \text{ (in eV)}} = \eta P \frac{\lambda \text{ (in } \mu\text{m)}}{1.24}$$

$$\text{THD} = \frac{\text{electrical power in harmonics}}{\text{electrical power in fundamental}}$$

$$\Delta f = \Delta \lambda \frac{c}{\lambda^2}$$

$$k = 1.38 \times 10^{-23} \text{ J/K}$$

$$h = 6.626 \times 10^{-34} \text{ J-s}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$