# Kings and Vikings: On the Dynamics of Competitive Agglomeration

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**Abstract**:

This paper studies the Viking age – the roughly 300 year period beginning in 800 AD – from the perspective of the economics of conflict. The Viking age is interesting because throughout the time period, the scale of conflict increased – small scale raiding behaviour eventually evolved into large scale clashes between armies. With this observation in mind, we present a theoretical model describing the incentives both the defending population and the invading population had to agglomerate into larger groups to better defend against attacks, and engage in attacks, respectively. The result is what might be called a theory of competitive agglomeration. We also apply our model in assessing the factors behind the onset of Vikings raids at the end of the 8th century.

**Jel Codes**: D74, H56, N40, O12

[†] The title of this paper is inspired by the title of P. H. Sawyer's (1994) book "Kings and Vikings: Scandinavia and Europe AD 700 – 1100, " which we felt to be an apt description of the approach taken in this paper.

*"For what are thieves' purchases but little kingdoms, for in theft the hands of the underlings are directed by the commander, the confederacy of them is sworn together, and the pillage is shared by law amongst them? And if those ragamuffins grow up to be able to keep enough forts, build habitations, possess cities, and conquer adjoining nations, then their government is no more called thievish, but graced with the eminent name of a kingdom, given and gotten, not because they have left their practices, not because they have left their practices, but because they may now use them without danger of law."*

*- St. Augustine, City of God, iv.4.,* quoted in Sawyer (1994, p. x)

## I. Introduction

Since the early 1990s an important literature on the causes and intensity of conflict has emerged. Initiated by seminal work of Hirshleifer (1991, 1995) and Grossman (1991; see also Grossman and Kim 1995), several analysts have considered how rational (albeit myopic) agents allocate their endowments across productive and appropriative activities to maximize their payoffs. Since conflict, in various forms, has played a major role throughout human history – shaping the development trajectory of civilization – improving our understanding of the nature of conflict appears extremely useful.

A slightly awkward feature of the conflict literature is its focus on static modeling. While, arguably, there are examples where opponents contest a prize during one instant and return to their usual business (or similarly, examples where conflict may be approximated by a near 'steady state outcome' with ongoing and unchanging conflict), there are many other cases where the *nature* of conflict evolves over time. Conflicts may evolve because technologies and relative prices change due to some endogenous or exogenous process, or because the incentives to behave in a certain fashion are subject to change. In this paper we adopt the latter perspective, and discuss the incentives for agents to agglomerate into larger groups. Agglomeration may enable parties to better defend themselves from aggressors, or to better exploit their weaker neighbors. The result is what can be called a theory of competitive agglomeration.

Our approach was motivated by the insight that dealing with issues of provision of defense and exploitation of neighbors was one of the primary reasons for governments and larger political groups to form in the first place. In this way, our theory contributes to the literature on the optimal size of political jurisdictions, providing an alternative angle to explain the development of nation states (e.g. Alesina and Spolaore 1997, 2005a,b). Like these theories, scale economies are at the heart of our story, and optimal nation size is determined by balancing benefits and costs of increasing the jurisdiction. Unlike other theories, however, our dynamic theory of agglomeration springs directly from the nature of the conflict technology, and not from an assumption about fixed costs in the provision of certain public goods. In fact, we show that the degree of agglomeration – a measure of the optimal size of a jurisdiction – depends upon the technology by which defenders and invaders interact. In essence we aim to provide a natural link between two strands of literature – analytical work on the size of nations (e.g. Friedman 1977, Bolton and Roland 1997, Alesina and Spolaore 1997) and on arms races (Schelling 1960, Sandler and Hartley 1995).

While we hope our theory is useful in understanding the basic forces driving agglomeration in the face of conflict in a general setting, we shall discuss our theory in terms of a particularly dramatic episode in history: the era of the Viking invasions. The Viking invasions are an interesting and in many ways ideal episode within which to study agglomeration: only through effectively forming larger groups could European settlements deal with Viking raids, while at the same time Viking invasions evolved over time from small, disorganized attacks into large, organized invasions. Our theory also allows us to consider various hypotheses as to how the invasions got started in the first place (see section II for details). In this paper we focus on these early days and on the development towards the high of the Viking invasions. While we don't analyze explicitly the end of the Viking era, the model can be used to shed some light on this matter as well.

Our model differs from much of the previous literature on economic conflict for a number of reasons. First and foremost, as mentioned above, we focus on agglomeration. Second, we explicitly consider the dynamics of this process. Others in the conflict literature (most notably perhaps, Skaperdas 1992 and Hirshleifer 1995) have considered "reaction functions" of players – an approach that is at least implicitly dynamic insofar

that it allows analyzing the behavior of the system when it is out-of-equilibrium. But we extend this approach and trace an explicit expression for the system's dynamics by introducing a measure of friction or inertia. Time becomes a variable in the model and the system is amenable to conventional stability analysis and calibration exercises.[1] Third, in our asymmetric model aggression can only flow one way – from a potential aggressor (the Vikings) to a potential victim (the British). This unidirectional focus implies comfortable middle ground between existing vertical models of conflict (e.g. insurrections and rebellion: the ruler versus the people – see for example Grossman 1991) and horizontal models of conflict (where tribes or agents compete for each other's output – e.g. Baker 2003), and also allows analyzing the decision of aggressors to participate in conflict. Finally, unlike most models of conflict our specification of the conflict process results in a model that is fully tractable. This applies both to a 'stripped down' version of the model as presented in the main text to illustrate the main story with as little distractions as possible, and to more elaborate specifications provided in three separate appendices that serve as robustness analyses.

The paper is organized as follows. In section II we provide a short history of the Vikings, fleshing out some of the more pertinent stylized facts and honing in on some of the gruesome details. Section III introduces notation and sketches the bare bones of the basic model. In section IV we consider the relatively easy case where villages can 'agglomerate' in coalitions to improve their oomph in contests, but where the level of their contribution to the coalition is fixed. In section V we allow villages to optimally choose their contribution (bearing in mind that investing in conflict effort comes at a cost). This allows us to trace out the full dynamic system, and analyze its properties. Section VI, finally, concludes.

## II. The Vikings are coming!

While our model can be applied to other interactions between populations of aggressors and defenders, the Viking age (roughly, the 300-year period beginning in c.

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[1] The analysis is not fully dynamic in the sense that we do not start out with equations of motion to keep track of the evolution of some state variables. See Maxwell and Reuveny (2005) for the only analysis in the conflict literature that we aware of to follow this route. They develop a model where bands of myopic agents share access to a common pool. Both human fertility and resource dynamics are defined by differential equations, allowing a full analysis of the system's dynamics.

800 AD) is a convenient stage upon which to discuss the basic logic of our model for two reasons. First, the historical events surrounding the arrival and onset of Viking invaders are well documented. Second, the intentions of the parties involved (at least in the early part of the Viking age) were clear: Viking invaders were interested in acquiring wealth of the societies they victimized,[2] while the main interest of potential victims was in stopping the Viking raiders from doing this.[3]

Like most historical epochs, pinning down an exact beginning and ending date for the Viking age is difficult, but historians agree that a good starting point is the raid on Lindesfarne, on the coast of Northumbria, on June 8th, 793. Shortly thereafter, Vikings turned up in a variety of places in the British Isles, including Wearmouth (794), Iona (795), North Ireland (795), Scotland and the Isle of Man (797); and branched out to other coastal regions of Europe, appearing off the coast of Frisia and Aquitaine (799), the Faroes (800), and, once again, Iona (802).[4] Why did the Viking onset begin at the precise moment it did? A host of explanations have been proposed (which we comment on below), including poor climate and deteriorating hunting and fishing conditions, growing internal strife, population pressure, increased commerce and prosperity in Northwestern Europe, advances in boat-building and sailing technology, and even boredom at home (Griffith 1995, Sawyer 1997).

Initial raids were directed primarily at coastal targets and conducted by relatively small fleets; while raids were certainly harrowing experiences for the victims,[5] setbacks were not uncommon among raiders groups.[6] Over time Viking attacks evolved into

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[2] Griffith (1995: 22) writes: "…the primary purpose of the marauding armies emanating from Scandinavia during the Viking era was probably seen to be less a matter of fighting battles than of pure economic activity – i. e. raising money by the easiest means."

[3] Much recent scholarship has been devoted to stressing other aspects of the Viking expansion by focusing on the trading, farming, and colonization activities that occurred simultaneously with Viking raids.

[4] These dates are drawn from Poertner (1971).

[5] How harrowing is a subject of some debate. The details of raids are primarily known through the writings of Churchmen, who were certainly not in a position to provide a balanced account of raids. It is hard to know how much credence should be given to barbaric behaviors allegedly practiced by the Vikings such as the "blood-eagle" – the practice to crack a victim's ribcage with an axe, pull out the lungs, and flap them like a pair of wings above one's head. Recent opinion has swayed in the other direction; for example, Keynes (1997: 49) writes "…it is now more fashionable to regard [the Vikings] as maligned and misunderstood." Sawyer (1995) is one among those who argue that Vikings weren't any more violent than other medieval peoples.

[6] Sawyer (1994: 81) reports "…the raiders who attacked Jarrow in 794 suffered casualties, and the Franks prevented the raiders of 820 from doing much damage until they reached the coast. The Irish also had their successes – in 811 in Ulster, and in 812 in both Connaught and Kerry."

bolder, better-organized and larger enterprises,[7] so much so that by the mid 9th century, Viking armies were conducting organized, large-scale invasions. Sawyer (1995: 81) writes: "In the summer of 834, the great market of Dorestad, some 80km from the sea, was attacked…and a new phase of Viking activity in Western Europe began." Large Viking forces appeared in Sheppey, on the Thames river, along the coast by Antwerp, in the interior of Ireland, and in the Bristol Channel; Dorestad was attacked again in 836. Viking fleets raided the Seine river basin in 841, and Hamburg in 845, led by the Danish King, Horik (Sawyer 1995). Viking forces attacked Nantes in 842, Seville in 844, and even Pisa in 859 (Roberts 1993).

How did the victims respond to these attacks? A broad trend towards agglomeration into larger and better-organized political units slowly took shape across Europe. On mainland Europe, for example, Louis the Pious, the Frankish king during the early part of the 9th century, took initial steps in organizing coastal defenses, although these efforts were cut short by his death. Some years later (in 862), Charles put into motion a program of building bridges and fortresses at critical points along rivers.

When the Vikings first attacked Lindesfarne, England was composed of a multitude of small kingdoms (Abels 1988b).[8] Early Viking raids focused on monasteries and churches (compounding the horror of the victims), and were typically undertaken by small fleets. As the Vikings grew better organized, even to the extent of constructing semi-permanent bases in the British Isles, attacks grew larger. In 851 a 350-ship force attacked Canterbury and London. Raiding forces did suffer setbacks; Aethelstan (ruler of Kent and other parts of Southeastern England), defeated the aforementioned force at sea (Kirby 1992),[9] suggesting that the size, organization, and skill of defenders had kept pace with the size of the raiding parties. The success was apparently short-lived, as new and larger Viking armies arrived in England, which "seriously began to threaten the

[7] Sawyer (1994) attributes the initial impetus towards larger, more ambitious attacks to lapses in defenses caused by disputes about the division of the Frankish empire after the death of Louis the Pious. In chapter 6 of his book, Sawyer (1994) describes the Viking knack for showing up in areas afflicted by civil strife and disorganization. See also Sawyer (1997) and Griffith (1995) on the increasing scale of Viking attacks.
[8] Kirby (1992) refers to the historical tradition of "heptarchy" of English Kingdoms existing prior to the Viking age, but argues that in fact this was almost certainly a gross overstatement of the degree of centralization in pre-Viking England.
[9] Kirby (1992: 172) writes that the Anglo-Saxon chronicle records this loss as "the greatest slaughter of a heathen army ever yet heard of."

capabilities of the Anglo-Saxon kingdoms." (Kirby 1992: 172). The large and nomadic "Great Army," (see Keynes 1997 for a good account of the Great Army's activities) which arrived in England in 865, took on a sort of momentum all its own, conquering substantial parts of England, and crossing over to France later in the 9th century, where it only disbanded some 30 years after forming.

In spite of the growing organizational capabilities of Viking raiders, defensive forces over time learned how to deal with the problems posed by large-scale Viking invasions, largely by combining into larger administrative units capable of maintaining concerted defenses. Sawyer (1997: 10) writes: "By 870 there had been profound changes in Frankia and in England." The achievements of Alfred the Great and the West Saxon dynasty in the late 9th century were particularly notable; under Alfred, a centralized system of forts and coastal defenses (the burghal system) were constructed. The fortresses overlooked virtually the entire landscape of the West Saxon kingdom, and served both as fortified points for defense of the local populace, blocked passage to rivers, and also served as launching points for permanently garrisoned defensive armies; the system also aided in consolidating centralized rule.[10] On the continent, similar developments occurred. Viking armies were defeated in pitched battles in Saucourt (881), and near Louvain in 891. Sawyer (1997: 14) writes that "for most of the tenth century, opportunities for Vikings were limited…because the best targets were defended by fortifications or organized armies," noting further that "only large-scale invasions offered any hope of significant success."

As Viking raids evolved into larger-scale affairs, so evolved the scale of society in Scandinavia. Sawyer (1994) discusses the possibility that there were fewer kingdoms in Scandinavia at the end of the first millennium than there had been 200 years earlier when raiding activity began, noting further that while early Viking activity was largely the work of raiders, "…in the late tenth and eleventh centuries Scandinavian Kings led

[10] These brief remarks certainly do not do justice to the many organizational changes instituted by Alfred. Abels (1988b: 79) writes: "The creation of the burghal system marks a watershed in the history of Anglo-Saxon governance. Despite formidable obstacles, the West Saxon dynasty of the late 9th and early 10th century managed to oversee the construction of a network of fortified towns…"; the system and its development is a subject of some interest covered in great detail in Abels (1988b); see also Abel's (1988a) book on Alfred the Great.

Viking raids…"[11] (Sawyer 1994: 144-5). In England, the centralized controls and institutions enacted by Alfred were extended and further developed under Edward and Aethelstan; indeed, a variety of innovations appear for the first time in 10th century English law. The trend towards larger political units was reflected in the newly emergent, broader definition of kingship.[12] Indeed, a sense of common identity had slowly emerged among the English in the face of the Viking raids, most dramatically reflected in the law codes of Wulfstan II, which declared that the English should follow one faith, under the leadership of one king (Keynes 1997).

The 11th century, the final years of the Viking age, was characterized by large-scale invasions that more closely resembled exercises in empire building than raiding, and the native territories of both victims of raiding and the raiders themselves had evolved into larger kingdoms. Both Norway and Denmark had evolved into relatively unified kingdoms, albeit in fits and starts. The Danish kings Sven Forkbeard and Canute (Knut) the Great succeeded in building large, if short-lived, kingdoms, and William the Conqueror, the son of Vikings who had taken up residence in Normandy, conquered England (Poertner, 1971). The process which began with small scale raids had gone a long way in creating incentives for Europeans to organize themselves into larger countries, and therefore exercised a profound impact on the future shape of Europe. In the next section, we describe a simple formal model of this process. The cornerstone of the model is the idea that conflict involves some degree of scale economies.

## III. The model

The agents in the model are divided into two separate populations: we shall call them Vikings and English.[13] For some historical reason (unspecified in the model) we assume that the former might prey on the latter, but that the reverse cannot happen.[14] Consider a total population of $n$ distinct Viking villages, each of which makes a decision about whether to engage in raiding or to pursue a peaceable occupation, such as farming.

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[11] As examples, Sawyer (1994: 144-5) notes the excursions of Sven Forkbeard, and adds of Knut: "He had a large fleet which he used to molest the world."

[12] See Abels (1988a) for a detailed analysis of these institutions.

[13] An interesting extension, which we do not entertain for reasons of brevity, would be to consider the interaction between a raiding population and two defending populations, e. g., the British and Franks.

[14] Counterattacks did in fact occur, particularly when raiders spent extended periods of time in the host country, but were in any case rare.

Let the number of raiding villages be given by $n_v$ and the number of villages engaging in farming as $n_f$, so that $n = n_v + n_f$.[15]

When engaging in raiding, Vikings randomly select an English target village. To successfully raid the target village, the raiders must overcome whatever defensive force the respective English village has in place; we refer to the size of the defensive force as $D$. In conducting raids, Vikings may wish to combine forces with other, like-minded Viking villages. Agglomeration into a larger raiding group conveys a distinct advantage – the massing of forces thus achieved admits a larger probability of successfully executing a raid. The downside is that a successful raiding group must split the returns thereby gained among its members.

To make this idea concrete, let $v$ denote the number of raiding villages participating in a given raiding coalition, and let $g$ denote the forces contributed by each village to the coalition. Then, the size of the raiding force amassed is $G = gv$. In much of what follows, we shall assume that each group simply contributes all of its labor to raiding, which results in $G = v$.[16] Note that since all groups are identical, they make identical decisions in equilibrium, so it follows that across the population of villages engaging in raiding, the number of distinct raiding coalitions is $V = n_v / v$.

The likelihood a raid is successful is determined by a contest success function, which maps the relative size of the invading force and the size of the defensive force faced into a success probability. We assume that the contest success function is of the following form:

$$s(G,D) = \frac{G^\rho}{G^\rho + D^\rho}. \qquad (1)$$

The function (1) is standard in the literature. For clarity, it is worth emphasizing the role of the parameter $\rho$, which Hirshleifer (1995) refers to as a decisiveness parameter. Most of our results shall pertain to the case in which $\rho > 1$. The assumption

[15] An alternative approach, which generates virtually identical results, is to allow all groups to contribute some positive fraction of their labor to raiding and some to farming.

[16] This assumption, and generally, the assumption that raiding groups contribute a fixed amount to the raiding party is with little loss of generality. As detailed in the appendix, allowing for variable contributions from raiding groups to the joint effort does not substantially change our qualitative results.

$\rho > 1$ implies a certain degree of "decisiveness" in outcomes – conflicts do not evolve towards compromises where both parties share the contested prize, but are fought until either the British or the Vikings prevail (respectively retaining the prize or taking most of it). For $\rho \to \infty$ the contest function approaches a step function so that the slightly more powerful party wins the conflict with near certainty. This is a plausible approximation as far as we can infer from historical sources.[17] This specification allows for increasing returns to force size to, for example, Vikings so long as $G \leq D$. Once the point $G = D$ is reached, diminishing returns to force size set in (see also Skaperdas 1992). A successful raid results in winnings of size $\pi$.[18] Using $G = v$, the expected returns per village participating in the typical raiding coalition can be written as:

$$x_v = \frac{r\pi}{v} \frac{v^\rho}{v^\rho + D^\rho}. \tag{2}$$

In (2), *r* denotes the number of raids the coalition becomes engaged in per period, which we take to be exogenous, determined by the state of shipping technology. So *r* is not a choice variable but the result of the interplay between technology – how fast can the Vikings travel between Scandinavia and British coasts? – and the time constraint.[19] Note that the first part of expression (2) includes the idea that expected winnings are divided by the number of villages in the raiding group.[20] Success probabilities and payoffs as a function of conflict effort (given defense level *d*) are depicted in Figure 1. The inflection point (where the second derivative of the conflict function is 0) occurs at $d[(\rho-1)/(\rho+1)]^{1/\rho}$, while the maximum group returns occur at the point $d[(\rho-1)]^{1/\rho}$. This

[17] Hirshleifer (2000) discusses conditions and historical episodes under which one might expect the decisiveness parameter to be relatively high (such as in Naval battles) or relatively low.

[18] Since the contested prize, $\pi$, is treated as an exogenous parameter, the model is on the interface of conflict models and rent seeking models. As discussed by Neary (1997), most conflict models have a general equilibrium nature where agents contest the output they first produce themselves. However, since British villages cannot opt out of the game unilaterally, the model is of the conflict and not of the rent-seeking type. Also note that treating $\pi$ as a parameter implies ignoring the fact that the British will have to accumulate wealth after being raided, so that in reality $\pi(n_v, v, D)$. Such a model could, in theory, be solved for the length of the optimal raiding cycle, not unlike optimal cutting rotations in forestry (think of the traditional Faustmann model). However, such a model is intractable when we allow for endogenous agglomeration, and detracts a bit from the main point that we wish to get across. Therefore, we leave this as an interesting option for future research in the context of a simpler model.

[19] In light of the evidence described in the previous section of the paper, it would seem important that a larger group should be able to conduct more raids. Allowing for this possibility makes things a bit messier, without fundamentally changing our results. We describe the impact of variable raids in the appendix.

[20] Note that we do not formally include any transactions costs associated with group formation. This is done purely for simplicity. However, the impact on the results we obtain is relatively straightforward.

means the inflection point is always to the left of the maximum return point – you always want to be a little bit larger than the opponent.

Optimal raiding group size is chosen to maximize the returns of the average member. Differentiating (2) with respect to $v$ gives the following:

$$\frac{\partial x_v}{\partial v} = (\rho - 1)\frac{r\pi}{v}\frac{v^{\rho-1}}{v^{\rho} + D^{\rho}} + \rho v^{\rho}\frac{r\pi}{v}\frac{v^{\rho-1}}{(v^{\rho} + D^{\rho})^2} = 0\,. \qquad (3)$$

Solving equation (3) for $v$ gives the following:

$$v = (\rho - 1)^{\frac{1}{\rho}} D\,. \qquad (4)$$

Equation (4) describes the optimal size of the raiding group as an increasing function of the size of the defense force faced. Note that $v$>1 (so that more than one group bands together) requires $\rho > 1$, implying that agglomeration into a larger raiding group will only takes place if the degree of decisiveness in conflict is sufficiently high.[21] Plugging $v$ from (4) into (2) gives:

$$x_v^* = \frac{r\pi}{D}\frac{(\rho - 1)^{1-\frac{1}{\rho}}}{\rho} \qquad (5)$$

Equations (4) and (5) imply that as the size of the defense, *D*, confronted grows, it becomes desirable for groups to form larger coalitions in raiding, while at the same time each member of the coalition receives less from raiding activities. Thus, a larger defensive force deters raiding behavior, while at the same time encouraging those groups that do engage in raiding to agglomerate into larger raiding units. One can already see how our model, *in equilibrium*, might allow for seemingly anomalous behaviors to occur simultaneously; it is plausible, for instance that a larger fraction of the raiding population might engage in peaceable activities while at the same time, raids that do occur will be larger and more sophisticated, in that they involve more highly agglomerated groups of attackers. (Out of equilibrium dynamics, as explored in the next section, are consistent with a broader range of outcomes, including increases in both raid size, $v$, as well as raiding activity, $n_V$, in general.) To determine explicitly the size of the raiding population,

[21] If $\rho < 1$, the incentives for invaders are to break up into raiding parties as small as possible.

suppose that (agricultural) production in Scandinavia is governed by a production function of the form:

$$Q = An_f^{1-\alpha} R^{\alpha}, \tag{6}$$

where $R$ is the common resource base used in production. Home production is subject to crowding; in accordance with this, suppose that there is free access to the means of production in Scandinavia, so each group gets its average product from land production. Using the identity $n = n_v + n_f$, we have:

$$x_f = \frac{Q}{n_f} = A\left(\frac{R}{n - n_v}\right)^{\alpha}. \tag{7}$$

In equilibrium, returns from engaging in raiding and engaging in farming must be equal, so using (3) and (5) we have the equilibrium condition:

$$A\left(\frac{R}{n - n_v}\right)^{\alpha} = \frac{r\pi}{D}\frac{(\rho - 1)^{1-\frac{1}{\rho}}}{\rho}. \tag{8}$$

Solving (8) for $n_v$ gives:

$$n_v = n - R\left(\frac{\rho AD}{r\pi(\rho - 1)^{1-\frac{1}{\rho}}}\right)^{\frac{1}{\alpha}}. \tag{9}$$

One interpretation of (9) is as a sufficient condition for the onset of raiding activity, given that the English are unprepared for raiding. For example, assume the English have some initial amount of defense $\underline{D}$. Upon substituting this into the above yields the following sufficient condition for raiding to begin:

$$n_v = \frac{n}{R} > \left(\frac{\rho A\underline{D}}{r\pi(\rho - 1)^{1-\frac{1}{\rho}}}\right)^{1/\alpha}. \tag{10}$$

From (10) one can see that raiding is more likely to commence the greater the population of the raiding country (an increase in $n$), the fewer the resources or poorer the environment in Scandinavia (a fall in $R$), the worst the technology/productivity of land on the home front (a decrease in $A$), the greater the profitability of raiding (an increase in

$\pi$), and the easier it is to conduct raids (an increase in $r$). One might interpret historians' "land thirst" and "better shipping technology" arguments as a decrease in *R* or an increase in *r*, respectively; further, as described an increase in population $n$ also makes raiding more likely, in line with the hypothesis that overpopulation was a cause of increased raiding. Finally, raising α (or lowering the value marginal product of labor in domestic production) also provides an impetus for raiding.

We can use (9) to solve for a critical level of defenses that would deter conflict. Upon setting $n_v$=0 we can also solve for this threshold level as: $\overline{D} = n^{\alpha} r\pi(\rho-1)^{(\rho-1)/\rho}(R\varphi A)^{-1}$. That is, if the British are able to muster defense levels equal to $D \geq \overline{D}$ then raiding would becomes and unprofitable activity and Danes would specialize in domestic production.

Let us now consider the decisions of the defending population. They must decide how much to invest in defending their resources, and the group size they should agglomerate into in order to best deal with invaders. What we have in mind is a situation in which a cluster of English villages decides to combine each of their individual defensive resources (their local armies) into one large force, which is deployed to the necessary place when any village in the coalition of defensive villages is raided. Define *e* as the size of the defending coalition, or the number of villages pooling forces.

In this case, there is an obvious possibility that group members may attempt to free-ride on the efforts of others in the coalition. The defense problem is different than the offender's problem in that members of a coalition have an incentive to cheat and free-ride; one example might be refusing to provide defending forces when a fellow coalition member is attacked. There must be some sort of institution in place that "coerces" members of a defensive coalition to participate in the collective defense of its members.[22] Suppose that the total costs of managing a coalition of size $e$ can be written as $C_e(e), C' > 0, C'' > 0$. The total administrative costs are borne equally among all participating villages so costs per village of managing the coalition are $c_e(e) = C_e(e)/e$; under the assumptions governing the shape of the cost function, $c_e(e)$ is also increasing

[22] Transactions costs are much less important to raiders, it can be argued, because if they do not participate as described by group rules, they can be excluded from winnings. There is, put another way, a much smaller intertemporal participation problem.

in the size of the coalition. Denote the losses (assets stolen and property damaged) experienced by the typical village in the event of a successful attack as $\pi$ (also the amount seized by attackers in a successful raid) and the size of the average invading force as $G$, which is taken as given by defenders when the size of the coalition is determined. Suppose that the typical defending group experiences $\tilde{r}$ raids. Per village, total expected costs (defense, managing the coalition, and raiding) are as follows:

$$L_e = \pi\tilde{r}\frac{G^\rho}{G^\rho + (ed)^\rho} + c_e(e). \tag{12}$$

Defenders choose $e$ to minimize (6) given $d$ (the defense contribution per village), which we normalize to unity (again, see the appendix for a more elaborate specification with free choice of both $e$ and $d$). It is easiest to transform this into a problem in which defenders choose only one choice variable. The first- order condition is:

$$\tilde{r}\pi\frac{\rho e^\rho v^\rho}{e(v^\rho + e^\rho)^2} - c_e'(e) = 0. \tag{13}$$

If attacks are randomly distributed among English villages, and there are $n_v / v$ distinct raiding coalitions which each carry out $r$ raids, we have $\tilde{r} = rn_v/(vn_e)$. Thus, (13) can be rewritten as:

$$c'(e) = r\frac{n_v}{vn_e}\varphi\frac{\rho e^\rho v^\rho}{e(v^\rho + e^\rho)^2}. \tag{14}$$

Equation (14) permits a closed-form solution only for specific functional forms. In the case that the total costs of maintaining a defensive coalition of size $e$ are given by $C(e) = ce\ln(e)$ — so that $C' > 0$ and $C'' > 0$ still hold — and administrative costs are borne equally by all villages, then $C(e)/e = c\ln(e)$ defines administrative costs per village, and we can write (14) as

$$c = r\frac{n_v}{vn_e}\varphi\frac{\rho e^\rho v^\rho}{(v^\rho + e^\rho)^2} \tag{15}$$

Solving (15) for $e$ gives:

$$e = v\left(\frac{\sqrt{\rho\pi rn_v(\rho\pi rn_v - 4cvn_e)} + \rho\pi rn_v - 2cvn_e}{2cvn_e}\right)^{\frac{1}{\rho}} \tag{16}$$

Figure 2 provides a graph of the reaction function, showing the optimal level of agglomeration $e$ as a function of $n_v$ and $v$. It is evident that the optimal level of defense is increasing in both the number of raids and the number of Viking villages involved in the attacks.

We shall see that using (4), (9), and (16), we can describe a simple dynamic model of the process of agglomeration in conflict. Before doing so, it is worthwhile to consider what the simultaneous solution of (4), (9), and (16) tells us about the size and scale of defense and raiding activities. A closed-form solution can be obtained for the special case when $\alpha = 1$, or when the opportunity cost of raiding effort is invariant with respect to the number of Vikings in the peaceful occupation. Then we get:

$$e^* = \frac{\pi rn(\rho - 1)^{1-\frac{1}{\rho}}}{\rho(cn_e + RA)}; \quad v^* = \frac{\pi rn(\rho - 1)}{\rho(cn_e + RA)}, \quad n_v = \frac{ncn_e}{cn_e + RA}. \qquad (17)$$

A first observation from (17) is the way in which the costs of agglomeration in the defending country (which might be taken to represent the talents of a particular king) influence the equilibrium levels of agglomeration in the defending country, and simultaneously determine the size of raiding groups in the invading country. As administrative costs increase, the size of defending groups falls, the number of Vikings participating in raiding rises, while the size of the raiding group falls. Population increases in the invading country also ultimately cause increases in the organization of defenders *and raiding groups*. The logic is that defenders must become more organized to deal with the increased threat of raiding; Vikings then face an incentive to increase the size of their armies to overcome the more concerted defenses. One can also see from (17) that equilibrium agglomeration of raiders and defenders increases as the winnings to successful raids occur (an increase in $\pi$). An interpretation of this result is that increased commercial activity in Northern Europe increased incentives for raiding, which in turn resulted in larger English political units and larger Viking armies.

**4. Adding Dynamics**

The solutions described in equation (17) summarize the equilibrium of the Nash game between invaders and their victims. An equally interesting issue concerns the

development trajectories towards the equilibrium – do we obtain intertemporal patterns that fit stylized facts well so that, over time, coalitions of Vikings and English grew larger?

Putting the model above in a dynamic context implies, strictly speaking, that we can distinguish between three different state variables: Viking coalition size ($v$), Defenders' coalition size ($e$) and the number of Viking villages engaged in raiding ($n_v$).[23] While it is possible to track such a model over time, it is also cumbersome and defies straightforward representation in a simple (two dimensional) phase plane. For this reason, and because the qualitative results are unaffected, we simplify the model by assuming that the Vikings are able to *instantaneously* cluster into invasive groups of the optimal size. Hence, $v=v^*(e)$, as derived in (4), holds always. Note that this does not mean that coalition size is stable: it varies over time as the size of the defensive coalition changes (also note that the number of coalitions varies, as discussed below).

Now, turn to the dynamic equations of the model. First, consider the clustering process of the Vikings. It is reasonable to assume that people respond to profit differentials between agriculture and raiding by switching from less to more profitable occupations, but it is also reasonable to assume that such responses only occur with a time lag. There may be many reasons why switching is not immediate, some rooted in psychology and others caused by matters like incomplete information, transaction costs, and so on. Given the choice of the optimal coalition size, the dynamics of the labor allocation choice may be described by the following *ad hoc* specification:

$$\frac{dn_v}{dt} = \phi\left[x_v - x_f\right] = \phi\left[\frac{r\pi}{e}\frac{(\rho-1)^{1-\frac{1}{\rho}}}{\rho} - A\left(\frac{R}{n-n_v}\right)^{\alpha}\right], \qquad (18)$$

where $\phi$ is an adjustment or sluggishness parameter, measuring the speed with which "switching" occurs in response to profit differentials. In equilibrium, returns from engaging in raiding and engaging in farming must be equal, and equation (9) simply provides the $dn_v/dt=0$ isocline.

[23] Note that the number of "victim villages"— $n_e$ – is exogenous and fixed: the British cannot choose to opt out of the game – although they certainly would have liked to!

Similarly, we may expect that defenders will cluster in defensive coalitions if this is profitable for them, and that coalitions continue to grow as long as having additional members increases the payoff of all members. Again, the pace at which this happens is arbitrary, and we may specify the dynamics as:

$$\frac{de}{dt} = -\xi\left[\frac{dL_e}{de}\right], \tag{19}$$

where ξ is an adjustment or sluggishness parameter. In equilibrium, the marginal gain of having an additional member is zero, and the *de/dt*=0 isocline is simply:

$$e = \frac{(\rho-1)\pi r}{\rho c n_e (\rho-1)^{1/\rho}} n_v = \Omega n_v \,. \tag{20}$$

Figure 3 combines the isoclines (9) and (20) in a phase plane, and also displays how Viking coalition size develops (the lower quadrant). From Figure 3, three results stand out. First, starting from a 'decentralized beginning' without significant cooperation between like-minded villages – i.e. close to the origin in the southwest part of the phase plane – we see that $n_v$, $v$ and $e$ all grow over time. The number of Viking villages 'going a-Viking' increases because raiding is a relatively profitable occupation, and these extra efforts are 'matched' to a certain extent by the English who cluster in more powerful defense coalitions. In response, the Vikings also cluster in larger groups, providing a further impetus for the English to expand coalition size.

Second, the number of Viking villages engaged in the raids need not monotonously increase over time. Figure 3 provides one such non-monotonous trajectory: while coalition size on both sides of the battle field continues to increase, profits from raiding fall so that some Danes find it in their interest to return to farming. However, alternative outcomes are also feasible. For different initial values or parameters the system displays a monotonous approach or cyclical behavior – the equilibrium may be a node or focus. In case of "cycles" the size of Danish and English coalitions goes up and down – suggesting an ebb-and-tide pattern of unilateral conflict. From a theoretical viewpoint, therefore, a rich set of results is feasible, and it is unfortunate that the historical record is not detailed enough to select the most appropriate outcome. We

believe that both the node and focus outcome may be consistent with what is known about this era.[24]

Third, while strictly speaking beyond the boundaries of the model, we can use the model to say something about the final phase of the Viking era – the transition from raiding and marauding to other activities such as trade. It is evident that effort will shift from raiding to farming if (due to some exogenous process such as the conversion to Christianity) the benefits from pillaging and destruction, as represented by the parameter $\pi$, become smaller. This rotates the $dn_V/dt=0$ isocline clockwise. Similarly, effort will be re-allocated away from raiding if English' institutions develop so that coordination costs $c$ are reduced (rotating the $de/dt=0$ isocline clockwise). For sufficiently large changes in $\pi$ and $c$ pillaging all but disappears. On a more speculative note, it could be argued that after the stage was set for agglomeration into sufficiently large "states," the perspective of the decision makers changed. That is, the affairs and interests of the nation-state, rather than the benefits of a bunch of independent armies, took primacy. This could induce greater emphasis on matters like mutually beneficial trade.[25]

**5. Discussion and conclusions**

Decisiveness in conflict and the Vikings forged modern Europe by setting in motion forces that led to agglomeration and larger nation-states; on both sides. For example, it is unlikely that Alfred could have conducted his centralization plan without the interventions of the Vikings, and the gradual escalation in the size and nature of conflict during the early Viking age certainly aided in setting the stage for the achievements of Knut (Canute the Great), and William the Conqueror.

The idea that war and defense are linked to the size of nations is not new. Alesina and Spolaore (2005a,b), for example, argue that the size of countries is determined by a need to balance "heterogeneity costs" (associated with jurisdictional expansion) and scale

[24] The empirical facts are rather obscure on this matter. For example, it is hard to determine whether more or less Viking villages got involved in the raids over time as the Viking era drew to a close. While we know that raiding coalitions got larger throughout most (but perhaps not all) of the Viking era, this could be more than offset by a decline in the number of coalitions.

[25] Alternatively, one could introduce a third activity (such as "trading") into the model from the outset. While perhaps not competitive at early stages, trading may become more attractive over time as seafaring technologies improved through some learning-by-doing process (lowering transaction costs of this activity, and allowing it to eventually dominate the others).

economies that come with the provision of public goods provision such as defense effort. They show that the number of countries should increase in response to a reduction in the probability of conflict but that, in turn, the increase in number of independent nations may result in a greater number of conflicts.[26] We have shown that fixed costs of public good provision are not necessary to achieve consolidation in larger political units. Instead, we look at the conflict technology, which introduces a competitive aspect to agglomeration that we feel is novel. It links the inherently static literature on optimal nation size to the dynamic literature on arms races.

Of course the model is a highly stylized and incomplete representation of reality. A number of extensions are feasible. For example, it would be interesting to include population dynamics, in particular because changes in the population affect potential strength on the battlefield. Alternatively, explicitly accounting for the wealth generation process of the victims may be a useful avenue of future research – accounting for the fact that increasing defensive efforts reduces the speed with which assets may be accumulated, and accounting for the fact that the time lag between successive attacks affects the available loot. Third, one could consider the challenges facing multiple populations (such as Franks and English) confronted by a common threat might be interesting, as well as more explicitly considering game theoretic aspects of the agglomeration problem. However, work along these lines suffers from the drawback that it becomes intractable quickly.

[26] They also demonstrate that this implies that the peace dividend may be smaller than perhaps anticipated.

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**Appendix**

*A. Varying invasive effort levels.*

We first describe how varying invasive effort levels alter the basic nature of the problem. The main result of interest is that allowing invading groups to vary their level of activity alongside the agglomeration decision does not qualitatively change the participation function and the returns from engaging in conflict, so long as one includes some degree of increasing costs for each group to raiding. The logic behind this result is that no individual raiding group would wish to agglomerate into a larger group if it could increase its size without disproportionately increasing its costs.

Along these lines, consider the modified returns-to-raiding function after (2);

$$x_v = \frac{r\pi}{v}\frac{(gv)^\rho}{(gv)^\rho + D^\rho} - c(g)\,. \tag{A.1}$$

To take a specific example, suppose that $c(g) = c_g g^2$. Differentiating (A.1) with respect to both $g$ and $v$ (which implies that invasive effort can be chosen so as to maximize the returns from raiding of the average group member), and solving the resulting first-order conditions for $g$ and $v$ gives the solutions:

$$g = \frac{1}{2}\frac{r\pi(\rho-1)^{1-\frac{1}{\rho}}}{c\rho D}; \quad v = 2\rho\frac{D^2 c(\rho-1)^{\frac{2}{\rho}-1}}{r\pi} \tag{A.2}$$

The solutions in (A.2) imply that the overall size of the raiding group, $G = gv$, can be readily shown to be the same as that given in equation (4). Substituting the solutions in (A.2) back into (A.1) gives:

$$x_v = \frac{1}{4}\frac{r^2\pi^2(\rho-1)^{2\left(1-\frac{1}{\rho}\right)}}{\rho^2 c D^2} \tag{A.3}$$

The expression in (A.3) has the same qualitative properties as the return function used in equation (9) to derive equilibrium raiding participation; thus, the qualitative characteristics are not altered by the simplifying assumption that raiding effort is fixed. It is, however, of some interest to note that closed form solutions for reaction functions are still obtainable in this case.

*B. Varying levels of defensive effort*

Consider now a case in which we allow defending groups to invest in variable amounts of defense, for a given amount of raids. This gives a total raiding cost function of the form:

$$L_e = \pi\tilde{r}\frac{G^\rho}{G^\rho + (ed)^\rho} + c_e(e) + c_d(d), \tag{A.4}$$

Which means that we have the following first-order conditions:

$$\frac{\partial L_e}{\partial e} = -\pi\tilde{r}\rho\frac{(ed)^\rho}{e}\frac{G^\rho}{[G^\rho + (ed)^\rho]^2} + c'_e(e) = 0, \tag{A.5}$$

$$\frac{\partial L_e}{\partial e} = -\pi\tilde{r}\rho\frac{(ed)^\rho}{d}\frac{G^\rho}{[G^\rho + (ed)^\rho]^2} + c'_d(d) = 0. \tag{A.6}$$

Together, these two conditions imply that the following must hold:

$$c'_d(d)d = ec'_e(e), \tag{A.7}$$

Note that in the case we explored in the text, where $c_e(e) = c_e \ln(e)$, this would imply that $c'_d(d)d = c_e$, so that $d$ would be set at a constant level, regardless of the level of agglomeration across groups. In a more general case, results still would not substantially alter our approach. For example, if we supposed that $c_d(d) = c_d d^{\lambda_d} \lambda_d^{-1}$, and $c_e(e) = c_e e^{\lambda_d} \lambda_e^{-1}$, (A.7) implies that $c_e e^{\lambda_d} = c_d d^{\lambda_d}$, or $d = e^{\lambda_e/\lambda_d}(c_e/c_d)^{1/\lambda_d}$. Plugging this into (A.5) gives the following:

$$\frac{\partial L_e}{\partial e} = -\pi\tilde{r}\rho\frac{((c_e/c_d)e^{1+\lambda_e/\lambda_d})^\rho}{e}\frac{G^\rho}{[G^\rho + ((c_e/c_d)e^{1+\lambda_e/\lambda_d})^\rho]^2} + c'_e(e) = 0 \tag{A.8}$$

Expression (A.8) is a more complex version of (13), though it has the same basic properties.

*C. Larger groups can conduct more raids.*

In the historical record, it appears that some of the larger Viking armies stayed together for substantial amounts of time; therefore, a logical extension is to consider a situation in which a larger army can stay together for longer periods of time and conduct more raids over this period of time. This would also allow the size of the prize to expand

with the number of groups. Along these lines, one might replace expression (2) with something along the lines of:

$$x_v = \frac{rv^{\gamma}\pi}{v}\frac{v^{\rho}}{v^{\rho}+D^{\rho}} \tag{A.9}$$

In expression (A.9), the term $rv^{\gamma}$, $0 \le \gamma < 1$ describes how the number of raids expands as group size expands, where now $r$ denotes the number of raids that can be conducted by a raiding group composed of a single village. Under these circumstances, the optimal raiding coalition size is given by:

$$v^{*} = \left(\frac{\gamma+\rho-1}{1-\gamma}\right)^{\frac{1}{\rho}} D. \tag{A.10}$$

Plugging this back into (A.9) gives:

$$x_v^{*} = \left(\frac{\gamma+\rho-1}{1-\gamma}\right)^{\frac{\gamma+\rho-1}{\rho}} \frac{(1-\gamma)}{\rho}\frac{r\pi}{D^{1-\gamma}} \tag{A.11}$$

The raiding returns described in (A.11) have the same basic functional form as the returns described in (A.5), except for being a bit less responsive to changes in $D$.

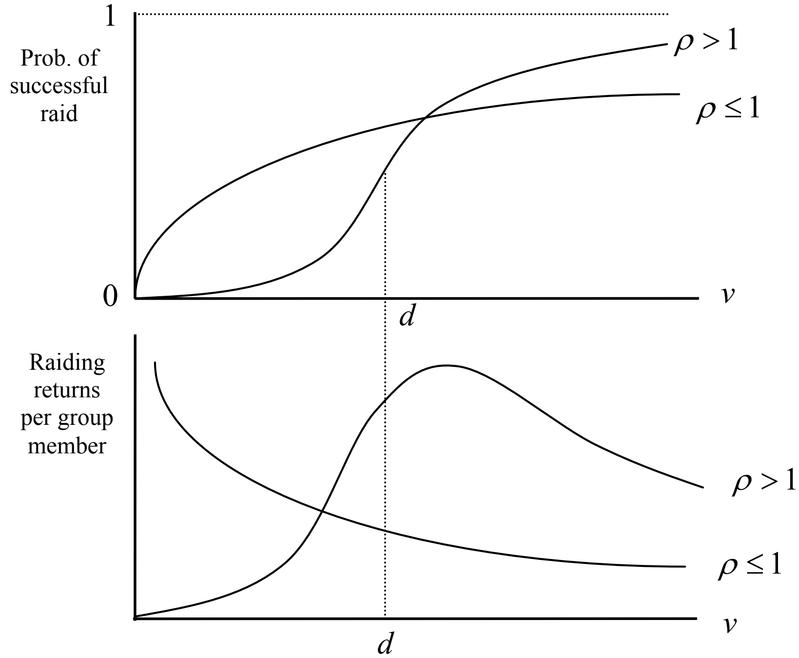


*Figure 1: Conflict technology and payoffs*

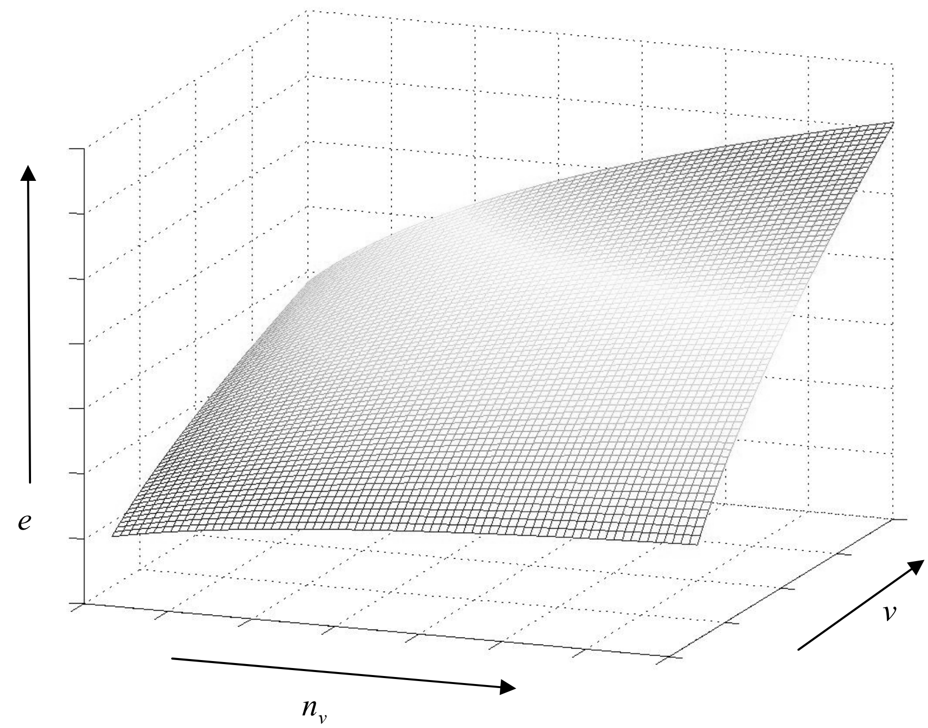


*Figure 2: The reaction function of the defending population*

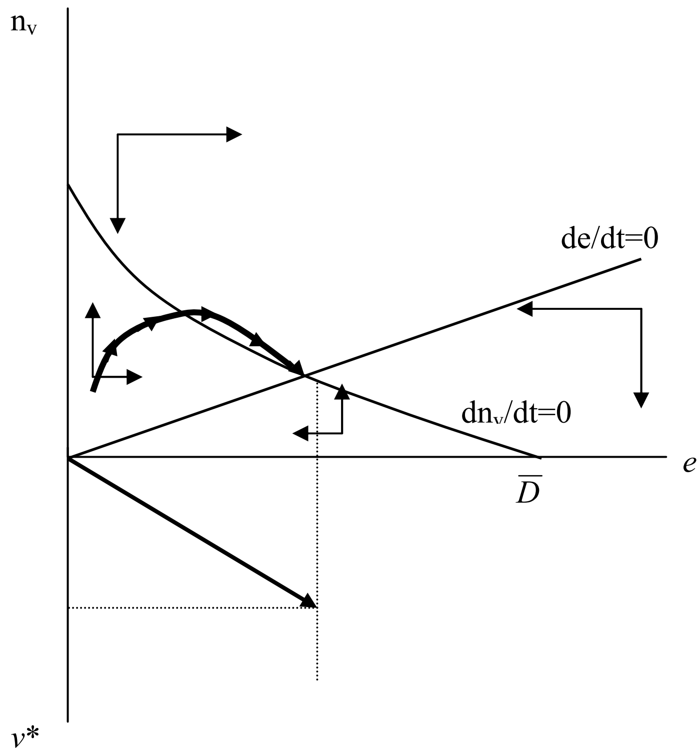


*Figure 3: Dynamics of Viking raiding effort and coalition size, and prey coalition size: One possible trajectory.*