I. Practice Problem 2: R-L AC Circuit

Work on the questions for the given circuit; indicated links give (partial) solutions.

An R-L circuit consists of a $60\sin(4t)$ volt AC generator connected in series with a 12 ohm resistor and a 4 henry inductor.

Questions:

[a] Sketch the circuit diagram.

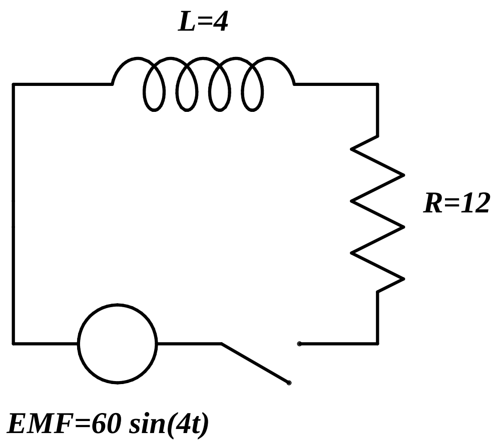
[b] Use Kirchhoff's law to write the Initial Value Problem; assume current starts to flow when the open switch is closed.

[c] Verify that $I(t) = -2.4\cos(4t) + 1.8\sin(4t) + 2.4e^{-3t}$ is the current in this circuit for $t \geq 0$.

[d] Graph $I(t)$.

[e] Compute the root-mean-square steady-state current for this circuit.

[a] Sketch the circuit diagram for the circuit with $R = 12\ \Omega$, $L = 4$ H, and $E(t) = 60\sin(4t)$ V.



[b] Use Kirchhoff's law to write the Initial Value Problem; assume current starts to flow when the open switch is closed.

$E_L + E_R = E$, with $E_L = L \cdot I'(t)$ and $E_R = R \cdot I(t)$, translates into

$$4\,I'(t) + 12\,I(t) = 60\sin(4t)$$

which simplifies to

$$I'(t) + 3\,I(t) = 15\sin(4t)\,, \qquad I(t) = 0 \quad \text{at} \quad t = 0$$

[c] Verify that $I(t) = -2.4\cos(4t) + 1.8\sin(4t) + 2.4e^{-3t}$ is the current in this circuit for $t \geq 0$.

If $I(t) = -2.4\cos(4t) + 1.8\sin(4t) + 2.4e^{-3t}$ then

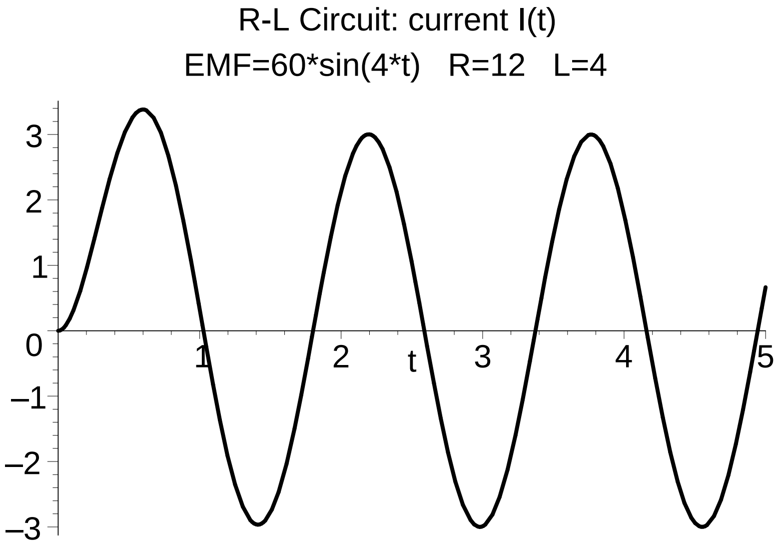
$$\begin{aligned} I'(t) &= -2.4\,(-4\sin(4t)) + 1.8\,(4\cos(4t)) + 2.4\left(-3e^{-3t}\right) \\ &= 9.6\sin(4t) + 7.2\cos(4t) - 7.2e^{-3t} \end{aligned}$$

and so

$$\begin{aligned} I'(t) + 3\,I(t) &= 9.6\sin(4t) + 7.2\cos(4t) - 7.2e^{-3t} + \\ &\qquad 3\left(-2.4\cos(4t) + 1.8\sin(4t) + 2.4e^{-3t}\right) \\ &= 9.6\sin(4t) + 7.2\cos(4t) - 7.2e^{-3t} - 7.2\cos(4t) + 5.4\sin(4t) + 7.2e^{-3t} \\ &= 15\sin(4t) \end{aligned}$$

and therefore $I(t)$ does satisfy the ODE. Also, $I(0) = -2.4\cos(0) + 1.8\sin(0) + 2.4e^{0} = -2.4 + 0 + 2.4 = 0$ and $I(t)$ thus satisfies the IC.

[d] Graph $I(t)$.

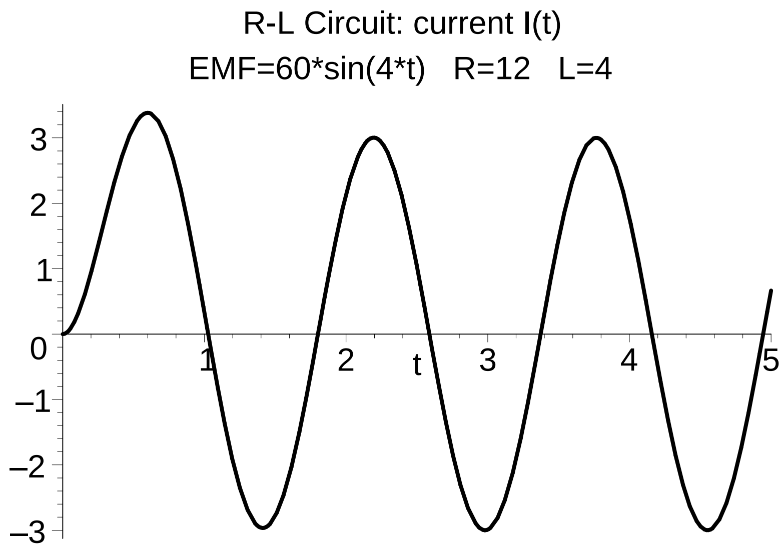


[e] Compute the root-mean-square steady-state current for this circuit.

The *transient current* is the part of the current

$$I(t) = -2.4\cos(4t) + 1.8\sin(4t) + 2.4e^{-3t}$$

that "dies off" over time — namely $2.4e^{-3t}$ which goes to zero fairly quickly. The *steady-state current* is that part of $I(t)$ that is completely oscillatory — namely $-2.4\cos(4t) + 1.8\sin(4t)$. After a couple of seconds, $I(t) \approx -2.4\cos(4t) + 1.8\sin(4t)$, as seen in the graph of $I(t)$.



The steady-state current $-2.4\cos(4t) + 1.8\sin(4t)$ has period $T = 2\pi/4 = \pi/2$ and so the root-mean-square steady-state current would be computed as

$$\sqrt{\frac{2}{\pi}\int_0^{\pi/2} (-2.4\cos(4t) + 1.8\sin(4t))^2 \, dt} = \frac{3}{\sqrt{2}} \approx 2.121 \text{ amps}$$