Electric Circuit Exercises
Setting Up Electric Circuit ODE and Verifying Solution

In each of the following problems you are to: [a] use Kirchhoff's law to write down the Initial Value Problem — ODE and initial condition(s) — for the given circuit ; [b] show that the given function is the solution to the IVP . (The quantities R, L, C, and E below do not necessarily represent realistic physical electric circuits.)

1. [**a**] $L = 25$ henries, $R = 50$ ohms, no capacitor, constant EMF $E(t) = 100$ volts, initial current is 0 amps when the switch is closed; [**b**] $I(t) = 2 - 2e^{-2t}$

2. [**a**] No inductor, $R = 30$ ohms, $C = 0.01$ farads, constant EMF $E(t) = 50$ volts, initial charge is 0 coulombs when the switch is closed; [**b**] $Q(t) = \frac{1}{2} - \frac{1}{2}e^{-10t/3}$

3. [**a**] $L = 25$ henries, $R = 25$ ohms, no capacitor, decaying EMF $E(t) = 50e^{-2t}$ volts, initial current is 0 amps when the switch is closed; [**b**] $I(t) = 2e^{-t} - 2e^{-2t}$

4. [**a**] No inductor, $R = 25$ ohms, $C = 0.01$ farads, decaying EMF $E(t) = 50e^{-2t}$ volts, initial charge is 0 coulombs when the switch is closed; [**b**] $Q(t) = e^{-2t} - e^{-4t}$

5. [**a**] $L = 15$ henries, $R = 40$ ohms, no capacitor, oscillatory EMF $E(t) = 50\cos(6t)$ volts, initial current is 0 amps; [**b**] $I(t) = \frac{1}{97}\left(20\cos(6t) + 45\sin(6t) - 20e^{-8t/3}\right)$

6. [**a**] No inductor, $R = 25$ ohms, $C = 0.005$ farads, oscillatory EMF $E(t) = 50\cos(6t)$ volts, initial charge is 0 coulombs; [**b**] $Q(t) = \frac{1}{25}\left(4\cos(6t) + 3\sin(6t) - 4e^{-8t}\right)$

7. [**a**] A simple series circuit consists of 25 ohm resistor, a 25 henry inductor, and an EMF $E(t) = 20e^{-3t} + 10e^{-6t}$ volts; the initial current is 0 amps when the switch is closed; [**b**] $I(t) = \frac{1}{25}\left(12e^{-t} - 10e^{-3t} - 2e^{-6t}\right)$

8. [**a**] A simple series circuit consists of a 20 ohm resistor, a 0.01 farad capacitor, and an EMF $E(t) = 20e^{-3t} + 10e^{-6t}$ volts; the initial charge on the capacitor is 0 coulombs when the switch is closed; [**b**] $Q(t) = \frac{1}{2}\left(e^{-3t} - e^{-6t}\right)$

9. [**a**] A simple series circuit consists of 25 ohm resistor, a 25 henry inductor, and a decaying oscillatory EMF $E(t) = 50e^{-3t}\cos(4t)$ volts; the initial current is 0 amps when the switch is closed; [**b**] $I(t) = \frac{1}{5}\left(e^{-t} - e^{-3t}\cos(4t) + 2e^{-3t}\sin(4t)\right)$

10. [**a**] A simple series circuit consists of a 25 ohm resistor, a 0.01 farad capacitor, and a decaying oscillatory EMF $E(t) = 50e^{-2t}\cos(6t)$ volts; the initial charge on the capacitor is 0 coulombs when the switch is closed; [**b**] $Q(t) = \frac{1}{10}e^{-2t}\left(\cos(6t) + 3\sin(6t) - e^{-2t}\right)$

11. [**a**] A simple series circuit consists of a 1 henry inductor, 12 ohm resistor, a 0.01 farad capacitor, and a constant EMF $E(t) = 100$ volts; the initial charge on the capacitor is 0 coulombs and the current is 0 amps when the switch is closed; **[b]** $Q(t) = 1 - e^{-6t}\,(\cos(8t) + (3/4)\sin(8t))$

12. **[a]** A simple series circuit consists of a 2 henry inductor, 60 ohm resistor, a 0.004 farad capacitor, and a constant EMF $E(t) = 50$ volts; the initial charge on the capacitor is 0 coulombs and the current is 0 amps when the switch is closed; **[b]** $Q(t) = \frac{1}{5} - \frac{1}{4}e^{-5t} + \frac{1}{20}e^{-25t}$

Electric Circuit Exercises
Solving Electric Circuit ODEs

(The quantities R, L, C, and E below do not necessarily represent realistic physical electric circuits.)

13. A simple series circuit consists of a 50 ohm resistor, a 25 henry inductor, and a constant EMF $E(t) = 100$. If the initial current is 0 amps when the switch is closed:
[a] write down the (DE and initial condition) mathematics model from Kirchhoff's law;
[b] determine the current $I(t)$ for any time $t > 0$;
[c] compute the current at $t = 0.01$;
[d] compute the current at $t = 1.0$;
[e] find the "steady-state" current $\lim_{t\to\infty} I(t)$;
[f] approximate the time(s) when the current is at 90% of steady-state.

14. Repeat the preceding problem for a simple series circuit that consists of a 100 ohm resistor, a 25 henry inductor, a constant EMF $E(t) = 50$, and an initial current of 0 amps.

15. A simple series circuit consists of a 30 ohm resistor, a 0.01 farad capacitor, and a constant EMF $E(t) = 50$. If the initial charge on the capacitor is 0 coulombs when the switch is closed:
[a] write down the (DE and initial condition) mathematics model from Kirchhoff's law;
[b] determine the charge $Q(t)$ for any time $t > 0$;
[c] compute the charge at $t = 0.01$;
[d] compute the charge at $t = 1.0$;
[e] find the "steady-state" charge $\lim_{t\to\infty} Q(t)$;
[f] approximate the time(s) when the charge is at 90% of steady-state.

16. Repeat the preceding problem for a simple series circuit that consists of a 50 ohm resistor, a 0.005 farad capacitor, a constant EMF $E(t) = 100$, and an initial charge of 0 coulombs.

17. A simple series circuit consists of a 25 ohm resistor, a 25 henry inductor, and a decaying EMF $E(t) = 50\,e^{-2t}$. If the initial current is 0 amps when the switch is closed:
[a] write down the (DE and initial condition) mathematics model from Kirchhoff's law;
[b] determine the current $I(t)$ for any time $t > 0$;
[c] compute the current at $t = 0.01$;
[d] compute the current at $t = 1.0$;

[**e**] approximate what the maximum current is and the time when it is achieved.

18. Repeat the preceding problem for a simple series circuit that consists of a 40 ohm resistor, a 10 henry inductor, a decaying EMF $E(t) = 50\,e^{-4t}$, and an initial current of 0 amps.

19. A simple series circuit consists of a 25 ohm resistor, a 0.01 farad capacitor, and a decaying EMF $E(t) = 50e^{-2t}$. If the initial charge on the capacitor is 0 coulombs when the switch is closed:
[**a**] write down the (DE and initial condition) mathematics model from Kirchhoff's law;
[**b**] determine the charge $Q(t)$ for any time $t > 0$;
[**c**] compute the charge at $t = 0.01$;
[**d**] compute the charge at $t = 1.0$;
[**e**] approximate what the maximum charge is and the time when it is achieved.

20. Repeat the preceding problem for a simple series circuit that consists of a 25 ohm resistor, a 0.01 farad capacitor, a decaying EMF $E(t) = 50e^{-4t}$, and an initial charge of 0 coulombs.

21. A simple series circuit consists of a 40 ohm resistor, a 15 henry inductor, and an oscillatory EMF $E(t) = 50\cos(6t)$. If the initial current is 0 amps when the switch is closed:
[**a**] write down the (DE and initial condition) mathematics model from Kirchhoff's law;
[**b**] determine the current $I(t)$ for any time $t > 0$;
[**c**] compute the current at $t = 0.01$;
[**d**] compute the current at $t = 1.0$;
[**e**] identify the "steady-state" current as an oscillatory function.

22. Repeat the preceding problem for a simple series circuit that consists of a 25 ohm resistor, a 15 henry inductor, an oscillatory EMF $E(t) = 50\cos(3t)$, and an initial current of 0 amps.

23. A simple series circuit consists of a 25 ohm resistor, a 0.005 farad capacitor, and an oscillatory EMF $E(t) = 50\cos(6t)$. If the initial charge on the capacitor is 0 coulombs when the switch is closed:
[**a**] write down the (DE and initial condition) mathematics model from Kirchhoff's law;
[**b**] determine the charge $Q(t)$ for any time $t > 0$;
[**c**] compute the charge at $t = 0.01$;
[**d**] compute the charge at $t = 1.0$;
[**e**] identify the "steady-state" charge as an oscillatory function.

24. Repeat the preceding problem for a simple series circuit that consists of a 25 ohm resistor, a 0.005 farad capacitor, an oscillatory EMF $E(t) = 50\cos(3t)$, and initial charge 0 coulombs.

25. A simple series circuit consists of a 25 ohm resistor, a 25 henry inductor, and an EMF $E(t) = 20e^{-3t} + 10e^{-6t}$. If the initial current is 0 amps when the switch is closed:
[**a**] write down the (DE and initial condition) mathematics model from Kirchhoff's law;
[**b**] determine the current $I(t)$ for any time $t > 0$;

[**c**] compute the current at $t = 0.01$;
[**d**] compute the current at $t = 1.0$;
[**e**] approximate what the maximum current is and the time when it is achieved.

26. Repeat the preceding problem for a simple series circuit that consists of a 20 ohm resistor, a 5 henry inductor, an EMF $E(t) = 20e^{-t} + 10e^{-4t}$, and an initial current of 0 amps.

27. A simple series circuit consists of a 20 ohm resistor, a 0.01 farad capacitor, and an EMF $E(t) = 20e^{-3t} + 10e^{-6t}$. If the initial charge on the capacitor is 0 coulombs when the switch is closed:
[**a**] write down the (DE and initial condition) mathematics model from Kirchhoff's law;
[**b**] determine the charge $Q(t)$ for any time $t > 0$;
[**c**] compute the charge at $t = 0.01$;
[**d**] compute the charge at $t = 1.0$;
[**e**] approximate what the maximum charge is and the time when it is achieved.

28. Repeat the preceding problem for a simple series circuit that consists of a 20 ohm resistor, a 0.01 farad capacitor, an EMF $E(t) = 20e^{-t} + 10e^{-4t}$, and an initial charge of 0 coulombs when the switch is closed.

29. A simple series circuit consists of a 25 ohm resistor, a 25 henry inductor, and a decaying oscillatory EMF $E(t) = 50e^{-3t}\cos(4t)$. If the initial current is 0 amps when the switch is closed:
[**a**] write down the (DE and initial condition) mathematics model from Kirchhoff's law;
[**b**] determine the current $I(t)$ for any time $t > 0$;
[**c**] compute the current at $t = 0.01$;
[**d**] compute the current at $t = 1.0$;
[**e**] approximate what the maximum current is and the time when it is achieved.

30. Repeat the preceding problem for a simple series circuit that consists of a 15 ohm resistor, a 30 henry inductor, a decaying EMF $E(t) = 50e^{-2t}\cos(6t)$ and an initial current of 0 amps.

31. A simple series circuit consists of a 25 ohm resistor, a 0.01 farad capacitor, and a decaying oscillatory EMF $E(t) = 50e^{-2t}\cos(6t)$. If the initial charge on the capacitor is 0 coulombs when the switch is closed:
[**a**] write down the ODE and initial condition mathematics model from Kirchhoff's law;
[**b**] determine the charge $Q(t)$ for any time $t > 0$;
[**c**] compute the charge at $t = 0.01$;
[**d**] compute the charge at $t = 1.0$;
[**e**] approximate what the maximum charge is and the time when it is achieved.

32. Repeat the preceding problem for a simple series circuit that consists of a 25 ohm resistor, a 0.01 farad capacitor, a decaying oscillatory EMF $E(t) = 50e^{-4t}\cos(3t)$, and a charge of 0 coulombs when the switch is closed.

33. A simple series circuit consists of a 50 ohm resistor, a 25 henry inductor, and an EMF $E(t)$ that is a constant 100 volt battery that drives the circuit for one second and then is disconnected. If the initial current is 0 amps when the switch is closed, then determine the current $I(t)$ for any time $t > 0$. [Suggestion: first solve the problem for an EMF that is constantly 100 volts, then determine the current $I(1)$ and use this value as the initial value for the same circuit with an EMF of 0 volts. Piece these two solutions together to get the current for the given circuit.]

34. A simple series circuit consists of a 50 ohm resistor, a 0.01 farad capacitor, and an EMF $E(t)$ that is a constant 25 volt battery that drives the circuit for one second and then is disconnected. If the initial charge on the capacitor is 0 coulombs when the switch is closed, then determine the charge $Q(t)$ for any time $t > 0$. [Suggestion: first solve the problem for an EMF that is constantly 25 volts, then determine the charge $Q(1)$ and use this value as the initial value for the same circuit with an EMF of 0 volts. Piece these two solutions together to get the charge for the given circuit.]

35. A simple series circuit consists of a 100 ohm resistor, a 25 henry inductor, and an EMF $E(t)$ that is a constant 50 volt battery that drives the circuit for two seconds and then is disconnected. If the initial current is 0 amps when the switch is closed, then determine the current $I(t)$ for any time $t > 0$.

36. A simple series circuit consists of a 20 ohm resistor, a 0.01 farad capacitor, and an EMF $E(t)$ that is a constant 50 volt battery that drives the circuit for two seconds and then is disconnected. If the initial charge on the capacitor is 0 coulombs when the switch is closed, then determine the charge $Q(t)$ for any time $t > 0$.

37. Use an appropriate integrating factor to solve the following ODE:

[**a**] $\dfrac{dy}{dx} - y = 2e^{3x}$

[**b**] $x\,y' + 2\,y = x\,\cos\left(x^3\right)$

[**c**] $L\,I'(t) + R\,I(t) = V$ where L, R, and V are positive constants.

38. A simple series circuit consists of a varying resistance of $50/(1+0.5t)$ ohms, a 25 henry inductor, and a constant EMF $E(t) = 100$. If the initial current is 0 amps when the switch is closed: [**a**] write down the (DE and initial condition) mathematics model from Kirchhoff's law; [**b**] determine the current $I(t)$ for any time $t > 0$.

39. A simple series circuit consists of a 50 ohm resistor, a varying capacitance of $0.005 + 0.01t$ farads, and a constant EMF $E(t) = 100$. If the initial charge on the capacitor is 0 coulombs when the switch is closed: [**a**] write down the (DE and initial condition) mathematics model from Kirchhoff's law; [**b**] determine the charge $Q(t)$ for any time $t > 0$;