SM311O Syllabus

Text: Advanced Engineering Mathematics with *Mathematica* and *MATLAB*, Volume II, by R. Malek–Madani, Addison–Wesley, 1998.

Materials on the web:

1. "Internal Forces in Fluids (a kinder and gentler approach)," located on my homepage. This material is needed during the last two weeks of the semester.
2. "Navier–Stokes Equations in a rotating frame." This material is needed during the last two weeks of the semester.

Material on Reserve at the Nimitz Library:

1. Introductory Dynamical Oceanography, 2nd Edition, by Stephen Pond and George L. Pickard.
2. Geophysical Fluid Dynamics, by Joe Pedlosky.
3. An Introduction to Dynamic Meteorology, by James R. Halton.

Weekly Assignments

Week 1
(a) Section 8.2 (Vectors)
   i. **Ideas and Terminology**: Scalars (denisty, pressure, salinity), Vectors (velocity, acceleration, pressure gradient, vorticity, angular velocity), dimensions.
   ii. **Problems**: Page 7. (**Do by hand, i.e. without using a computer. You can of course compare your results to what you might get by using a software package or TI92.**)
      A. **1(a)**, **1(b)**, **1(c)**, **1(h)**,
      B. **3(a)** (choose 5 equally distanced points on the interval $(0, 1)$ and compare your hand-drawn figure with the one you may obtain using a software package), **3(f)** (choose three points in each quadrant of the circle),
      C. **4** (Please correct the units of $\mathbf{\Omega}$ to read rad/sec) Also show that $\mathbf{a} = -\Omega^2\mathbf{R}$, where $\mathbf{R}$ is the projection of $\mathbf{r}$ on the xy-plane. Draw a diagram to show the direction along which this vector points at the pole, latitudes 60 and 45 degrees, and at the equator. Assuming the radius of earth is $6,000$ kilometers, determine the magnitude of $\mathbf{a}$ at the equator.
      D. Prove the following identity: Let $\mathbf{a}(t) = a_1(t)\mathbf{i} + a_2(t)\mathbf{j} + a_3(t)\mathbf{k}$ and $\mathbf{b}(t) = b_1(t)\mathbf{i} + b_2(t)\mathbf{j} + b_3(t)\mathbf{k}$. Prove that $\frac{d}{dt}(\mathbf{a} \times \mathbf{b}) = \mathbf{a}' \times \mathbf{b} + \mathbf{a} \times \mathbf{b}'$.

(b) Section 8.3 (Curves and Surfaces)
   i. **Ideas and Terminology**: Degrees of Freedom (curves have one and surfaces have two), parametrization, tangent and acceleration vectors.
   ii. **Problems**: Page 22. Do by hand. Compare your hand-drawn graphs with ones you may obtain using a software package.
      A. **1**,

B. **2(c)**, **2(f)**,
C. **3(a)**, **3(b)**, **3(c)**, **3(d)**,
D. **8(a)**, **8(b)**,
E. **9(b)**, **9(c)**, **9(d)**,
F. **11(b)**.

(c) Section 8.5 (Partial Derivatives)

i. **Ideas and Terminology**: Partial derivatives, rate of change, Dimensions of ρ_t and v_x.

ii. **Problems**: Page 39. Do by hand first and then compare your answer against what you may obtain from TI-92 or another software.

A. **1(a)**, **1(c)**, **1(d)**, **1(f)** (compute u_{xx} and u_{yy}. What is u_{tt} equal to?), **1(i)**, **1(j)**, **1(k)**, **1(n)**,
B. **6**,
C. **7**,
D. **8**,
E. **9**.

Week 2 (a) Section 8.6 (Gradient)

i. **Ideas and Terminology**: Gradient, directional derivative, direction of steepest descent, level curves, normal vectors to surfaces. The gradient of a function p at a point P is perpendicular to the level curve of p that passes through P and it points in the direction of increasing level curves. Hence, when p is the pressure in a flow, $-\nabla p$ is the force experienced by P.

ii. **Problems**: Page 55.

A. **1(a)**, **1(b)**, **1(d)**,
B. **3** (use only the first three functions defined in Problem 1),
C. **4** (note that **e** should be a unit vector),
D. **5** (if you are doing this problem in *Mathematica*, make sure that you type the backquote ‘ when entering `VectorAalaysis`),
E. **6(a)**, **6(b)**.

(b) Section 8.8 (2-D Vector Fields)

i. **Ideas and Terminology**: The connection between a velocity field and its particle paths, differential equation solvers, vector fields A, B, and C.

ii. **Problems**: Page 69.

A. **4**,
B. **8**.

Week 3 (a) Section 8.9 (Divergence and Conservation of Mass)

i. **Ideas and Terminology**: Connection between density ρ and velocity $\mathbf{v}$, connection between incompressibility and divergence, connection between stream function ψ and a 2-D steady velocity field $\langle v_1, v_2\rangle$ (that $v_1 = \frac{\partial\psi}{\partial y}$, $v_2 = -\frac{\partial\psi}{\partial x}$).

ii. **Problems**: Page 77.

A. **1** (note that the vector field v in Problem 1(a) should have been in boldface),

B. **2** (note that Δ and ∇^2 are the same, are called the laplacian, and computed as $\nabla \cdot \nabla$),
C. **3**,
D. **5**,
E. **6**,
F. **8**,
G. **9**,
H. **10**.

Week 4 (a) Section 8.10 (Curl, Vorticity, and Laplace's Equation)

i. **Ideas and Terminology**: Connection between curl and vorticity, the significance of the identity $\nabla \times \nabla\phi = \mathbf{0}$ and the flow being irrotational, significance of $\Delta\psi = 0$ and the flow being incompressible and irrotational.

ii. **Problems**: Page 86.

A. **1**,
B. **2**,
C. **3** (What is the difference between (**3(c)** and (**3(e)**?),
D. **4**.

Week 5 Review and FIRST EXAM.

Week 6 (a) Section 8.11 (Line Integrals)

i. **Ideas and Terminology**: Line integrals over open and closed curves, circulation, dimension of $\oint_C \mathbf{v} \cdot d\mathbf{r}$, in 2-D the connection between average vorticity of particles in the region D enclosed by C and $\oint_C \mathbf{v} \cdot d\mathbf{r}$.

ii. **Problems**: Page 93.

A. **1**,
B. **2**,
C. **3** (try doing the integrals by hand. If don't succeed, use a symbolic or numerical integration package.),
D. **4**,
E. **5**,
F. **6**.

(b) Section 10.7 (Path Independence and the Existence of a Potential)

i. **Ideas and Terminology**: Potential, the identity $\nabla \times \nabla\mathbf{v} = \mathbf{0}$ and its relation to the existence of a potential, path independence, how to compute $\int_C \mathbf{v} \cdot d\mathbf{r}$ when $\mathbf{v}$ has a well-defined potential, simply connected regions.

ii. **Problems**: Page 209.

A. **1**,
B. **2**,
C. **3**,
D. **4**.

Week 7 (a) Section 9.2 (Flow in the Bay)

i. **Ideas and Terminology**: Separation of variables, boundary conditions, Laplace's equation, normal modes.
ii. **Problems**: Page 117.
A. **4**,
B. **5**,
C. **6**,
D. **9**,
E. **10**.

(b) Section 9.3 (Separation of Variables–Heat and Wave Equations)

i. **Ideas and Terminology**: Separation of Variables, general solution, initial–boundary value problem.
ii. **Problems**: Page 126.
A. **2**,
B. **3(a)**, **3(g)**
C. **4**,
D. **6**,
E. **15**,
F. **17**,
G. **19**.

Week 8 (a) Section 14.3 (Fourier Series)

i. **Ideas and Terminology**: Fourier sine and cosine series, inner product, the formula $f = \sum_{n=1}^{\infty} a_n \phi_n(x)$ and $a_n = \frac{(f, \phi_n)}{(\phi_n, \phi_n)}$.
ii. **Problems**: Page 452.
A. **3**,
B. **10**,
C. **11**,
D. **14**.

(b) Section 14.5 (Fourier Series and the Wave Equation):

i. **Ideas and Terminology**: The connection between boundary conditions and whether one uses Fourier sine or cosine series to obtain the solution to the initial–boundary value problem.
ii. **Problems**: Page 461.
A. **3**,
B. **4**,
C. **5**,
D. **14**,
E. **15**.

Week 9 Review and SECOND EXAM

Week 10 (a) Section 10.2 (Double Integrals)

i. **Ideas and Terminology**: Double integration with application to volumes and areas.

ii. **Problems**: Page 170.
   A. **2**,
   B. **3**,
   C. **4(a)**, **4(c)**,
   D. **5**,
   E. **6**,
   F. **7(a)**, **7(e)**,
   G. **8**.

(b) Section 10.3 (Surface Integrals)

   i. **Ideas and Terminology**: Surface and double integrals, flux of a fluid flow, the relation between the flux of a flow in a region and the values of the stream function on the contours corresponding to the boundary of the region.
   ii. **Problems**: Page 181.
      A. **1(a)**, **1(b)**, **1(c)**, **1(f)**, **1(g)**,
      B. **3**,
      C. **5**,
      D. **6**,
      E. **7**,
      F. **9**,
      G. **15**.

Week 11 (a) Section 10.6 (Stokes Theorem)

   i. **Ideas and Terminology**: Stokes Theorem, flux of vorticity, circulation
   ii. **Problems**: Page 204.
      A. **1(a)**, **1(b)**, **1(c)**,
      B. **2**,
      C. **3**,
      D. **4**.

Week 12 (a) Section 12.3 (Acceleration)

   i. **Ideas and Terminology**: The terms $\nabla \mathbf{v}$ and $\mathbf{v} \cdot \nabla \mathbf{v}$. When does acceleration have a potential?
   ii. **Problems**: Page 313.
      A. **1**,
      B. **2**,
      C. **4(a)**, **4(c)**,
      D. **5(a)**, **5(c)**.

(b) Read pages 6 through 8 of Halton's book (on reserve in the Nimitz Library) which touch on the concepts of viscous forces and stress.

(c) Section 12.5 and "Internal Forces in Fluids (a kinder and gentler approach," which you can find on my homepage (Internal Forces)

   i. **Ideas and Terminology**: Viscous and pressure gradient forces, stress.

ii. **Problems**: See manuscript on the web.

Week 13 (a) Section 12.7 (Navier-Stokes Equations)

i. **ideas and terminology**: $\rho(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v}) = -\nabla p + \mu \Delta \mathbf{v} + \mathbf{F})$

ii. **Problems**: Page 331.

A. **3** (Does it matter if the fluid is viscous or not to support this motion?),

B. **4**,

C. **5**,

D. **6**.

(b) Section 12.8.2 (Flow over an Oscillating Plate)

i. **Ideas and Terminology**: This example is a precursor to the Ekman Layer problem we will study later.

ii. **Problems**: Page 335.

A. **4**,

B. **5**.

Week 14 (a) Section 12.14(Acceleration in a Rotating Frame). **See "Navier–Stokes Equations in a rotating frame" on my homepage for more material on this subject**.

i. **Ideas and terminology**: Velocity in a rotating frame (equation (12.230)), acceleration in a rotating frame (equation (12.236)).

ii. **Problems**: Page 358. **1** and problems in "Navier–Stokes Equations in a rotating frame."

(b) Section 12.14 (Geostrophic Equations). **Also see "Navier–Stokes Equations in a rotating frame" on my hompage for more material on this subject**.

i. **Ideas and Terminology**: The geostrophic equations (equations (12.251) and (12.252).

ii. **Problems**: Will be assigned in class.

(c) Section 12.16 (Ekman Layer). **Also see "Navier–Stokes Equations in a rotating frame" on my hompage for more material on this subject**.

i. **Ideas and Terminology**: Solving the system of equations (12.296), determining the general solution of the 4-th order equation $u'''' + \alpha^2 u = 0$, where α is a constant.

ii. **Problems**: Will be assigned in class.

Week 15 Review and THRID EXAM