Mathematics of Light

SM421A

Fall 2012

The goal of this course is to study what light is, and specifically how to make sense of what a light beam is, from a fundamental mathematical perspective. The prerequisites for the course are basic familiarity with multivariate calculus, elementary differential equations (e.g., SM212 or SM222), and a basic course on college physics (e.g., SP212). The first two weeks of the course will be dedicated to reviewing the concepts needed from mathematics and physics, before we proceed to concentrate on light itself. MATLAB will be used throughout the course to simulate properties of functions and to carry out data analysis.

Student projects constitute a significant aspect of this course. Each student will be assigned a basic article on the subject of light and mathematics. The last three weeks of the course are dedicated to student presentations of their respective articles.

Resources:

a) Optics, by Hecht. Copies will be placed on course reserve in the Nimitz Library
b) Laser Beam Propagation through Random Media, by Andrews and Phillips. Copies will be placed in the Nimitz Library
c) Handouts, shared in class or placed on website

Exams: Two in-class exams are scheduled for weeks 5 and 11.

Syllabus

Week 1:

a) Introduce the basic laws of electromagnetism (Coloumb, Ampere, etc.)
b) Review the basic properties of gradient, divergence and curl
c) Write simple programs in MATLAB to plot graphs of functions.

Week 2:

a) Introduce Maxwell's equation; derivation of the wave equation
b) Begin the derivation of the Paraxial Wave Equation (PWE)
c) Review writing for … end and if … then … else statements in MATLAB

Week 3:

a) Completion of the derivation of PWE; begin the analysis of finding exact solutions of PWE
b) MATLAB programs to plot solutions of PWE
c) Preparation of the first data collection experiment

Week 4:

a) Continuation of the study of PWE and completion of the derivation of lowest order Transversely Modulated solution T_00; Chapter 4 of Andrews and Phillips
b) First field measurement experiment (we will all follow Prof. Avramov-Zamurovic's lead here).

Week 5:

a) Discussion of the data and the data collection process
b) **First in-class exam**

Week 6:

a) Review of probability and distributions, in particular Gaussian, Poisson, Bernoulli and Binomial
b) MATLAB implementation of data collected
c) Selection of student course papers

Week 7:

a) An in-depth discussion of the student course papers. Discussion of the course writing assignment and presentation goals and parameters.
b) Begin review of random variables, concept of covariance and correlation
c) MATLAB implementation of statistical tools

Week 8:

a) Introduction to statistical turbulence; fluid versus optical turbulence; Chapter 5 of Andrews and Phillips
b) Introduction to construction of Probability Density Functions from data

Week 9:

a) Continuation of the concept of turbulence; examples of chaotic systems
b) Implementation of chaotic systems in MATLAB
c) Introduction to Galerkin Methods

Week 10:

a) Galerkin methods (cont'd)
b) Implementation of Galerkin methods in MATLAB

Week 11:

a) Review
b) **Second in-class exam**

Week 12:

a) Mathematics of the double-slit experiment; near-field versus far-field effects
b) MATLAB implementation

Week 13:

a) Preliminary student presentation of projects

Week 14:

a) Mathematical development of the Airy disk problem
b) MATLAB simulation of Airy disk and its analogues

Week 15:

a) Final student presentations.

Coordinator: Prof R. Malek-Madani rmm@usna.edu