C1M2

Parametric Curves

Have you ever played with a toy called *"Etch-a-Sketch"*? One hand controls the x-axis while the other controls the y-axis. It is as if you are graphing $(x(t), y(t))$, $a \le t \le b$, which is exactly what happens when a function in the plane is defined parametrically. Be very careful where you place the right bracket,], when using Maple to plot parametric curves.

Relationships between trigonometric functions, and in particular the Pythagorean identities, are used quite often when eliminating the parameter and thereby identifying the curve that has been described parametrically. For your convenience, we provide the Pythagorean identities:

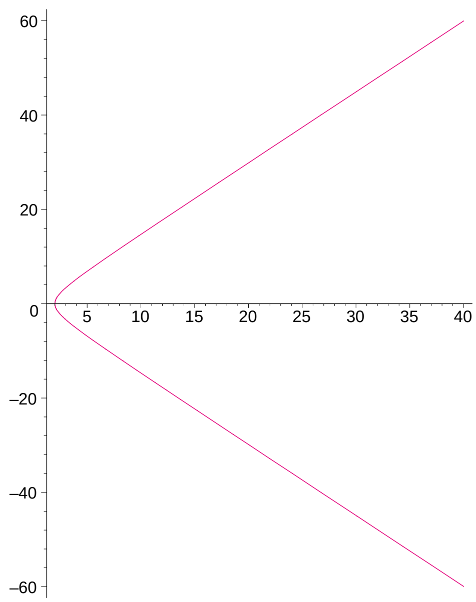
$$\sin^2(\theta) + \cos^2(\theta) = 1 \qquad \tan^2(\theta) + 1 = \sec^2(\theta) \qquad 1 + \cot^2(\theta) = \csc^2(\theta)$$

Maple Example: Plot $x(t) = 2\csc(t)$, $y(t) = 3\cot(t)$ for $0 < t < \pi$ and eliminate the parameter. We solve for the trigonometric functions first and then apply the third identity.

$$\csc(t) = \frac{x}{2}, \quad \cot(t) = \frac{y}{3} \quad \Rightarrow \quad 1 + \frac{y^2}{9} = \frac{x^2}{4} \quad \Rightarrow \quad \frac{x^2}{4} - \frac{y^2}{9} = 1$$

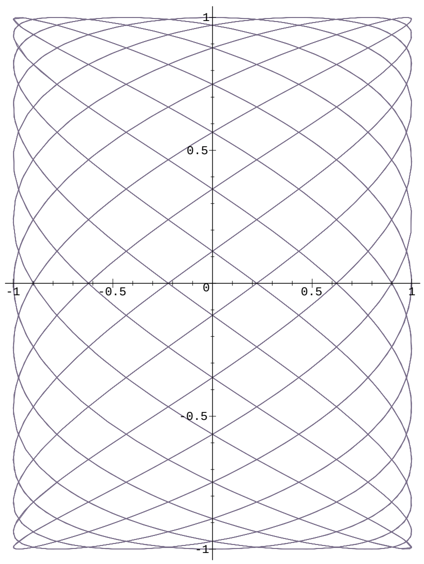
We see that our curve is a hyperbola and we check out the domain and find that not only must x always be positive, it must always be greater than 2. Because we must avoid the endpoints of the domain, note how this is done in our plot. Is it easy to see from the plot that the hyperbola is asymptotic to the straight lines $y = \pm\frac{3}{2}x$?

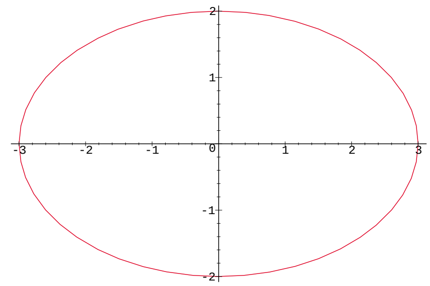
```
> plot([2*csc(t),3*cot(t),t=.05..Pi-.05],color=magenta);
```



Maple Example: Plot $x(t) = \sin(13t)$, $y(t) = \cos(7t)$ for $0 \le t \le 6\pi$ which produces a *lissajou.* The plot follows on the left. As you can see, the scaling is a little off because the "square" is two units on each side. For a little fun, increase the coefficients to say 43 and 37 and see what happens.

```
> plot([sin(13*t),cos(7*t),t=0..6*Pi],color=navy);
```





Maple Example: Ellipses are easy this way. Plot $\dfrac{x^2}{3^2} + \dfrac{y^2}{2^2} = 1$. The Maple output is above on the right.
When you have $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ you may plot this by using $x(t) = a\ \cos(t)$ and $y(t) = b\ \sin(t)$ for $0 \le t \le 2\pi$.

```
> plot([3*cos(t),2*sin(t),t=0..2*Pi]);
```

C1M2 Problems: Use Maple to display the parametric graphs of the given functions. Reminder:
♠ The exponential function, e^x, is accessed by `exp(x)` in Maple.

1. $x = e^t,\ y = e^{2t},\ -1 \le t \le 2$
2. $x = 2\sec t,\ y = \tan t,\ -\pi/2 < t < \pi/2$
3. $x = t - \sin t,\ y = 1 - \cos t,\ 0 \le t \le 4\pi$
4. $x = \cos^3 t,\ y = \sin^3 t,\ 0 \le t \le 2\pi$