

## C2M15

### Vectors and the Scalar Product

The scalar, “dot”, or “inner” product of two vectors is a very important concept that involves the size of each and the amount that each points in the direction of the other. This nebulous statement will have more meaning as you become familiar with vectors. In Maple, use `evalm` when evaluating a vector or matrix expression. Also, you will need the package `linalg`. In earlier versions of Maple one used `dotprod` to find the scalar product of two vectors. However, in Release 6 this command seems to be of better use when working with complex scalars. We are using only the real numbers,  $\mathbb{R}$ , so it is best to use the Maple command `innerprod` when finding the scalar product.

The concept of finding the projection of one vector on another is very important. First, the projection is a vector. Second, it is a scalar multiple of the vector being projected on, which must be a non-zero vector. When that scalar multiple is subtracted from the first vector the result is a vector that is orthogonal to the vector being projected on. Let’s find the projection of  $\vec{v}_1 = \langle 2, 1, -3 \rangle$  on  $\vec{v}_2 = \langle 1, -2, 2 \rangle$ . We will find the scalar  $c$  symbolically and then let Maple do the calculations and verify that it works. We seek  $c$  so that  $(\vec{v}_1 - c\vec{v}_2) \cdot \vec{v}_2 = 0$ . The *projection of  $\vec{v}_1$  on  $\vec{v}_2$*  is the vector  $c\vec{v}_2$ . Recall that two vectors are orthogonal (or perpendicular) if their scalar or dot product is 0.

$$(\vec{v}_1 - c\vec{v}_2) \cdot \vec{v}_2 = \rightarrow$$

Recall that  $\vec{a} \cdot \vec{b} = \|\vec{a}\| \|\vec{b}\| \cos \theta$ , where  $\theta$  is the angle between  $\vec{a}$  and  $\vec{b}$ . Obviously, for non-zero vectors  $\vec{a}$  and  $\vec{b}$  we can solve for  $\cos \theta$  and thereby determine  $\theta$ .

$$\cos \theta = \frac{\vec{a} \cdot \vec{b}}{\|\vec{a}\| \|\vec{b}\|}$$

Maple saves us the work with the command “angle”.

```
> theta:=angle(v1,v2);
```

$$\theta := \pi - \arccos\left(\frac{1}{21}\sqrt{14}\sqrt{9}\right)$$

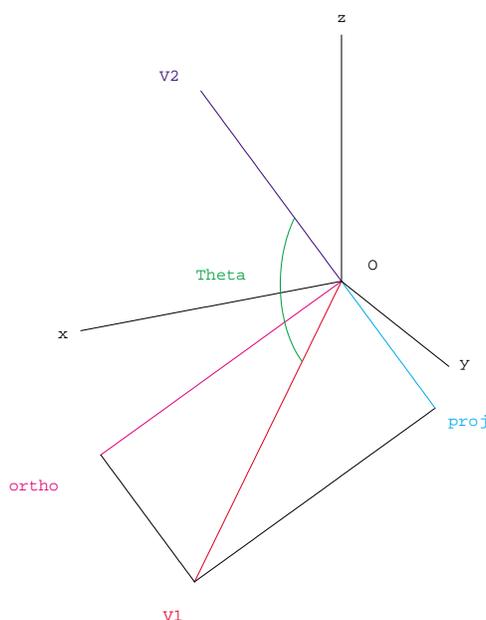
```
> theta:=evalf(theta);
```

$$\theta := 2.134738969$$

```
> theta1:=evalf(theta*180/Pi);
```

$$\theta_1 := 122.3115332$$

This makes sense in the following diagram where we see that  $\theta$  must be an obtuse angle since the projection is opposite to the direction of  $\vec{v}_2$ .



### The projection of $\vec{v}_1$ on $\vec{v}_2$

A line between two points  $P$  and  $Q$  is parameterized by  $\alpha(t) = (1-t)\vec{P} + t\vec{Q}$  for  $0 \leq t \leq 1$ . We will use this in combination with `spacecurve` to plot lines.

**Example:** Plot the coordinate axes and the line segment from  $P(4, -1, 2)$  to  $Q(1, 4, 4)$ .

This task may be accomplished in two distinct ways. The way which is displayed below is direct and uses the basics. Instead of defining the function `alpha`, and plotting it using `spacecurve`, one could use a command in `plottools` named `line`. The syntax would look like

```
> with(plottools):
> LPQ:=line([4,-1,2],[1,4,4],color=red):
```

It suits our purposes here to have you learn how to set up a function like `alpha` because in Calculus III this is a skill which will be essential.

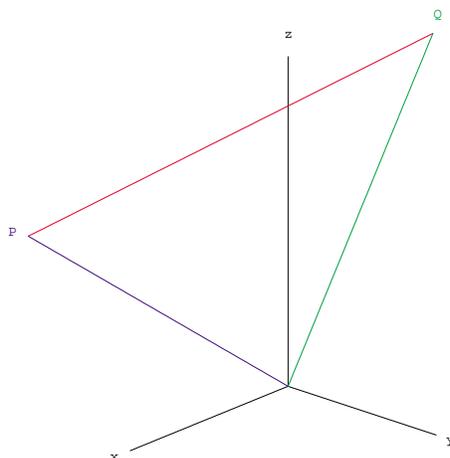
```
> with(plots):
> P:=vector([4,-1,2]): Q:=vector([1,4,4]):
> xaxis:=spacecurve([t,0,0],t=0..3,color=black):
> yaxis:=spacecurve([0,t,0],t=0..3,color=black):
> zaxis:=spacecurve([0,0,t],t=0..3,color=black):
> alpha:=evalm((1-t)*P+t*Q);
```

$$\alpha := [4 - 3t, -1 + 5t, 2 + 2t]$$

```

> LPQ:=spacecurve(alpha,t=0..1,color=red):
> LP:=spacecurve(evalm(t*P),t=0..1,color=blue):
> LQ:=spacecurve(evalm(t*Q),t=0..1,color=green):
> display(xaxis,yaxis,zaxis,LPQ,LP,LQ);

```



### C2M15 Problems:

1. Use Maple to find the projection of  $\vec{v} = \langle 3, -3, 5 \rangle$  on  $\vec{w} = \langle -6, 3, 2 \rangle$ . Find a unit vector in the direction of  $2\vec{v} - \vec{w}$ .
2. Use Maple to find the angle between  $\vec{v} = \langle 2, 2, -1 \rangle$  and  $\vec{w} = \langle 3, -4, 5 \rangle$ . Find a unit vector in the direction of  $-\vec{v}$ .
3. Use Maple to plot the line segment from  $P(4, 1, 4)$  to  $Q(-1, 3, 1)$  and the segments from  $P$  and  $Q$  to the origin. Include the coordinate axes as shown above. Use colors.