

# MATHEMATICS PROBLEM #120

Midshipman Joe asks Midshipman Bob “Is the following expression correct?” and writes

$$f \circ (x^3) = 3x^2$$

on the blackboard. Bob replies, “Well it could be, but I don’t think that’s what you mean.”

What do you think Joe meant? Find a function which makes what Joe wrote correct.

Each midshipman submitting a correct solution with a correct explanation to Problem 120 by 1700 on Friday 30 November 2001 will win a cookie. Submit solutions to Prof. Wardlaw at [mathprob@usna.edu](mailto:mathprob@usna.edu) (please no attachments!) or via his mailbox in Chauvenet 301.

No correct solutions to Mathematics Problem #119 were submitted by midshipmen. Professors Jodi Lockhart, Mark Kidwell, and Mark Meyerson submitted solutions to Problem #119. My solution to Problem #119 is posted on the board and is on the reverse side of this sheet.

## MATHEMATICS PROBLEM #119

A spider puts on 8 identical socks and 8 identical shoes. In how many orders can the spider do this, given that on each foot, the sock has to go on before the shoe?

**Solution.** There are  $(16!)/(2^8) = 81,729,648,000$  different orders that the spider can put on the shoes and socks. This is because there are  $16!$  orders to accomplish the

16 tasks, but this number must be divided by  $2^8$  to account for the sock having to go on before the shoe on each foot.

Perhaps this can be made clearer as follows: The spider needs to decide when, among the 16 time slots available, to put the sock on each foot and when to put the shoe on each foot. This amounts to choosing two different numbers between 1 and 16 for each foot, the smaller to designate when the sock goes on that foot and the larger to designate when the shoe goes on that foot. We can count the number of ways of doing this by counting the number of ways that 16 pool balls, numbered 1, 2, 3, ..., 15, 16 can be put into eight numbered bags, with two balls going into each bag. Filling the bags in order, there are 16 choices for the first ball to go into the first bag, and 15 choices for the second ball, but we divide by 2 because we would get the same result if we reversed the order in which these balls were picked. Thus there are  $16 \times 15 / 2$  different ways to fill the first bag. Now there are 14 balls left over, so there are  $14 \times 13 / 2$  ways to fill the second bag. Similarly, there are  $12 \times 11 / 2$  ways to fill the third bag,  $10 \times 9 / 2$  ways to fill the fourth bag, etc., so there are

$$\begin{aligned} & (16 \times 15 / 2)(14 \times 13 / 2)(12 \times 11 / 2) \dots (4 \times 3 / 2)(2 \times 1 / 2) \\ &= (16 \times 15 \times 14 \times 13 \times 12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1) / (2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2) \\ &= (16!) / (2^8) = 81,729,648,000 \end{aligned}$$

different orders in which the socks and shoes can be put on.