

MATHEMATICS PROBLEM #121

The Maryland lottery “Big Game” is played by selecting five of the numbers 1, 2, 3, ... , 50 and then selecting one of the numbers 1, 2, 3, ... , 36. The first selection must consist of five different numbers with no repeats, and the order in which they are picked does not matter. The second selection may or may not repeat one of the first five numbers.

How many ways can this be done? Put differently, how many plays would you have to purchase (for one dollar each) in order to be sure that you have the winning play? (Be sure to include a careful explanation of your reasoning and calculations in your solution!)

Each midshipman submitting a correct solution with a correct explanation to Problem 121 by 1700 on Thursday 31 January 2002 will win a cookie. Submit solutions to Prof. Wardlaw at mathprob@usna.edu (please no attachments!).

A correct solution to Mathematics Problem #120 was submitted by Midshipman Alan M. Montara. Professors Gary Fowler, Mark Kidwell, and Mark Meyerson also submitted solutions to Problem #120. My solution to Problem #120 is posted on the board and is on the reverse side of this sheet.

MATHEMATICS PROBLEM #120

Midshipman Joe asks Midshipman Bob “Is the following expression correct?” and writes

$$f \circ (x^3) = 3x^2$$

on the blackboard. Bob replies, "Well it could be, but I don't think that's what you mean."

What do you think Joe meant? Find a function which makes what Joe wrote correct.

Solution. Joe probably thought that " f° " meant "take the derivative of what follows", or $f^\circ = d/dx$.

To find the function f which makes Joe's statement $f^\circ(x^3) = 3x^2$ correct, we let $x^3 = t$, so that $t = x^{1/3}$ and substitute to get $f^\circ(t) = f^\circ(x^3) = 3x^2 = 3t^{2/3}$. It follows that $f^\circ(x) = 3x^{2/3}$; integration shows that $f(x) = (9/5)x^{5/3} + c$ is a function which makes Joe's statement correct.