MATHEMATICS PROBLEM 134

Evaluate the infinite series

$$\sum_{k=0}^{\infty} \binom{n+k}{k} x^k$$

and state for which x it converges. Justify your answers. (Here $\binom{n+k}{k}$ is the binomial coefficient $\frac{(n+k)!}{(n!)(k!)}$.)

Each midshipman submitting a correct solution with a correct explanation to Problem 134 by 1700 Friday 5 December 2003 will be recognized as a solver on the next problem. Submit solutions to Prof. Wardlaw at mathprob@usna.edu (please no attachments!) or via his mailbox in Chauvenet 301.

Midshipman Stephen McMath submitted a correct solution to Mathematics Problem 132. A solution is on the back of this page and posted on the wall.

MATHEMATICS PROBLEM 133

The *Gaussian integers* are complex numbers $a + bi$, where a and b are both integers and $i^2 = -1$. These numbers add and multiply like ordinary complex numbers. If A and B are Gaussian integers, define $A\ B = A + B + AB$. Find all ordered pairs (A, B) of Gaussian integers such that $A\ B = 0$.

Solution. $A\ B = A + B + AB = 0$ if and only if $AB + A + B + 1 = 1$ if and only if $(A + 1)(B + 1) = 1$ if and only if $(A + 1, B + 1)$ is $(1, 1)$, $(i, -i)$, $(-1, -1)$, or $(-i, i)$ (since $1, i, -1,$ and $-i$ are the only Gaussian integers with inverses) if and only if (A, B) is $(0, 0)$, $(-1 + i, -1 - i)$, $(-2, -2)$, or $(-1 - i, -1 + i)$.