MATHEMATICS PROBLEM 135

The nth Fermat number is $F_n = (2^{(2^n)}) + 1$. Fermat knew that $F_0 = 3$, $F_1 = 5$, $F_2 = 17$, $F_3 = 257$, and $F_4 = 65537$ were prime, and he believed that all of these numbers were prime.

a. Show that if $h \neq k$ then $\gcd(F_h, F_k) = 1$.

b. Use part a to show that there are infinitely many primes.

c. (Extra credit) Find a nontrivial factor of F_5.

Each midshipman submitting a correct solution with a correct explanation to Problem 135 by 1700 Friday 30 January 2004 will be recognized as a solver on the next problem. Submit solutions to Prof. Wardlaw at mathprob@usna.edu (please no attachments!) or via his mailbox in Chauvenet 301.

Midshipman Stephen McMath submitted a correct solution to Mathematics Problem 134. A solution is on the back of this page and posted on the wall.

MATHEMATICS PROBLEM 134

The infinite series

and it converges if and only if $|x| < 1$. To see this we note that when $n = 0$ the series is just the geometric series

which is known to converge if and only if $|x| < 1$.

Now assume that for some nonnegative integer n,

and it converges if and only if $|x| < 1$. Differentiating both sides gives

and hence

converges when $|x| < 1$. The latter series diverges when $|x| \geq 1$ since the terms do not approach 0 then. This justifies the desired result by induction on n.°