MATHEMATICS PROBLEM 140

a. What is the largest number of equilateral triangles with sides of unit length that can be formed from 12 matchsticks, each of unit length?

b. What is the largest number of squares with sides of unit length that can be formed from 12 matchsticks, each of unit length?

Each midshipman submitting a correct solution with a correct explanation to Problem 140 by 1700 Friday 10 September 2004 will be recognized as a solver on the next problem. Submit solutions to Prof. Wardlaw at mathprob@usna.edu (please no attachments!) or via his Math Dept mailbox in Preble Hall, second deck.

A solution to Math Problem 139 is on the back of this page and is posted on the Math Dept bulletin board on the second floor of Preble Hall across from the stairs. (Please let me know if you submitted a solution to this problem; my records were messed up in the move from Chauvenet.) Solvers will be recognized on the next problem.

MATHEMATICS PROBLEM 139

a. What is the volume of a regular tetrahedron with side length 1 ?

b. What is the volume of a regular octahedron with side length 1 ?

Solution. A *generalized cone* is the obtained from a region B, called the *base,* in a plane and a point A, called the *apex*, which is not in the plane, as follows: Each point P in the base B is joined to the apex A with a straight line segment AP. The generalized cone is the collection of all of the points which lie on any of these line segments AP. The *height* h of the generalized cone is the perpendicular distance from the plane of the base to the apex A. A simple integral of cross sectional areas shows that the volume of the generalized cone is

$$V = |B|h/3,$$

where $|B|$ is the area of the base.

a. The base of the tetrahedron is an equilateral triangle B with altitude $a = (\sqrt{3})/2$ and area $|B| = (\sqrt{3})/4$. The height of the tetrahedron is found by the Pythagorean theorem to be $h = \sqrt{(1 - ((2a/3)^2)} = \sqrt{(2/3)}$. Therefore, the volume of the tetrahedron with side length 1 is $V = |B|h/3 = \mathbf{(\sqrt{2})/12}$ **.**

b. A regular octahedron with side length 1 is made of two pyramids (generalized cones with square bases). The area of the base of each pyramid is $|B| = 1$ and the height of each pyramid is $h = (\sqrt{2})/2$, and so the volume of the regular octahedron with side 1 is $v = 2|B|h/3 = \mathbf{(\sqrt{2})/3}$ **.**