

MATHEMATICS PROBLEM 148

Suppose p and q are each monic polynomials of degree 4 with real coefficients and the intersection of their graphs is $\{(1, 3), (5, 21)\}$. If $p(3) - q(3) = 20$, what is the area enclosed by their graphs?

Each midshipman submitting a correct solution with a correct explanation to Problem 148 by noon 22 April 2005 will be recognized as a solver on the next problem. Submit solutions to Prof. Wardlaw at mathprob@usna.edu (please no attachments!) or via his Math Dept mailbox in Preble Hall, second deck.

No correct solutions to Math Problem 147 were submitted. Professor Amy Ksir submitted a correct solution to Math Problem 146 which was not acknowledged on MP 147.

A solution to Math Problem 147 is on the back of this page and is posted on the Math Dept bulletin board on the second floor of Preble Hall across from the stairs.

MATHEMATICS PROBLEM 147

A spherical shaped experimental submarine has a spherical interior of radius 6 feet. The lower half of this sphere serves as a ballast tank. This tank has a flat top of pressure withstanding steel plate welded horizontally to the inside center of the sphere. On top of this plate is the "operating room" which is occupied by the sub's operator, air tanks, and equipment. The vessel is designed to float when the ballast tank is less than half full. (That is, when less than half of its volume is filled with water.) You are resting on the bottom of the Atlantic shelf at a depth of 200 feet when you discover that your ballast tank is full of water and your ballast blowing air tank is empty. How much work does it take to pump half of the sea water out of the ballast tank, assuming that sea water weighs 65 pounds per cubic foot and atmospheric pressure at the surface is 14.7 pounds per square inch?

Solution. (In order to be e-mail compatible we use an integral notation like that used in Maple.) First we must determine the level in the hemispherical tank when it is half full of water. If we let u be the distance measured vertically downward from the center of the sphere (radius $a = 6$) to the surface of the water in the tank, we see that the surface of the water is a circle of radius $r = \sqrt{a^2 - u^2}$ and the volume above this surface is the integral $\int_0^u \pi (a^2 - t^2) dt, t = 0..u] = \pi[(a^2 t - t^3 / 3), t = 0..u] = \pi[a^2 u - u^3 / 3]$. Setting this equal to half of the hemispherical volume, $\pi a^3 / 3$ and canceling π gives the polynomial equation $2 a^3 + 3 a^2 u - u^3 = 0$. Substituting $a = 6$ gives $216 + 108u - u^3 = 0$