MATHEMATICS PROBLEM 152

A cubical cake is sliced ten times, each slice being a plane cut through the cake. Pieces are not removed until all of the cuts are made. What is the maximum number of pieces resulting from ten planar cuts?

The mathematical version: What is the maximum number of regions that three dimensional space is divided into by ten planes?

Each person submitting a correct solution with a correct explanation to Problem 152 by noon Friday 16 December 2005 will be recognized as a solver on the next problem. Submit solutions to Prof. Wardlaw at mathprob@usna.edu (please no attachments!) or by slipping under his office door, Chauvenet 375.

Correct solutions to Math Problem 151 were submitted by Midshipmen Matt Knitt, Kevin K. McCadden, Trevor McLemore, Reeve Hartman Meck, Christopher A. Nigus, Christina Partridge, Daniel Ryan, and Brooks Rogers, by LT Josh Wood, and by Professors Mark Kidwell and Mark Meyerson. My solution to Math Problem 151 is on the back of this page and is posted on the Math Dept bulletin board on the third floor of Chauvenet Hall across from the Mathematics Department Office.

MATHEMATICS PROBLEM 151

Consider the numbers 3, 8, 13, … , 103, 108; that is, all positive integers less than 110 which leave a remainder of 3 when divided by 5. What is the smallest k such that every collection of k of these numbers will always contain a pair which sums to 121?

Solution. Put the numbers in the following 12 "boxes":

[3], [8], [13, 108], [18, 103], [23, 98], [28, 93], [33, 88],

[38, 83], [43, 78], [48, 73], [53, 68], [58, 63]

So that a pair of numbers sum to 121 if and only if they lie in the same "box". 12 or fewer numbers can be chosen without choosing any two numbers from the same box, hence without having a pair adding to 121. But if 13 numbers are chosen, at least two must be chosen from the same box, and so at least that pair must sum to 121. Hence, $k = 13$ is the smallest such that every collection of k of the numbers will always contain a pair which sums to 121.

Remark. This solution is an example of the "pigeonhole principle": If you have more than n pigeons in n pigeon holes, at least one pigeonhole must contain more than one pigeon. This very simple idea can solve a wide array of problems.