MATHEMATICS PROBLEM 165

Show that any 3 x 3 matrix A over the integers modulo 2 which satisfies the equation

$$A^3 + A + I = 0,$$

where I is the 3 x 3 identity matrix and 0 is the 3 x 3 zero matrix, must have order ord(A) = 7. That is, the seventh power of A is I, but no lower positive power is.

Each person submitting a correct solution to Mathematics Problem 165 by 1200 Thursday 26 April 2007 will be recognized as a solver when the next problem is announced. Submit solutions to Prof. Wardlaw at wpw@usna.edu.

Correct solutions to Mathematics Problem 164 were submitted by Midshipmen Reeve Meck, and Professors Russell Jackson and Mark Meyerson. Problem 164 was jointly proposed by LT Josh Wood and myself, and it appears as problem 1763 in the February 2007 issue of *Mathematics Magazine*. Our solution is on the back of this page and on the Mathematics Department bulletin board on the third floor of Chauvenet Hall across from the Mathematics Department Office.

MATHEMATICS PROBLEM 164

Two lines are fixed in three space at a distance d apart and have an angle θ between them. Place a line segment of length a on one of these lines and another line segment of length b on the other line. Find the volume of the convex hull of these two line segments.

Solution. The volume is (1/6)abd sin(θ). For if we assign directions to the line segments to obtain vectors $\mathbf{a}$ and $\mathbf{b}$ of lengths a and b, respectively, and let $\mathbf{c}$ be the vector from the initial point of $\mathbf{a}$ to the initial point of $\mathbf{b}$, then the convex hull of the two line segments is the tetrahedron with three concurrent edges given by the vectors $\mathbf{a}$, $\mathbf{c}$, and $\mathbf{b}+\mathbf{c}$. Hence the volume is $(1/6)|(\mathbf{a} \times \mathbf{c})\cdot(\mathbf{b} + \mathbf{c})| = (1/6)|(\mathbf{a} \times \mathbf{b})\cdot\mathbf{c}| = (1/6)|ab \sin(\theta)\, \mathbf{n}\cdot\mathbf{c}| = (1/6)abd \sin(\theta)$

because **n** is a unit vector perpendicular to the two lines and **c** is a vector joining points on the two lines, so $|\mathbf{n}\cdot\mathbf{c}| = d$ is the distance between the lines.