MATHEMATICS PROBLEM 166

How many 10 symbol passwords can be formed by rearranging the symbols in the password below?

AABBB12345

Each person submitting a correct solution to Mathematics Problem 166 by 1200 Thursday 17May 2007 will be recognized as a solver when the next problem is announced. Submit solutions to Prof. Wardlaw at wpw@usna.edu.

Correct solutions to Mathematics Problem 165 were submitted by Professors Russell Jackson and Mark Meyerson. My solution to Problem 165 is on the back of this page and on the Mathematics Department bulletin board on the third floor of Chauvenet Hall across from the Mathematics Department Office.

MATHEMATICS PROBLEM 165

Show that any 3 x 3 matrix A over the integers modulo 2 which satisfies the equation

$$A^3 + A + I = 0,$$

where I is the 3 x 3 identity matrix and 0 is the 3 x 3 zero matrix, must have order ord(A) = 7. That is, the seventh power of A is I, but no lower positive power is.

Solution. Remember that we are working over the integers modulo 2, so + and - are the same. Thus, $A^3 + A + I = 0$ implies $A^3 = A + I$ implies

$$A^7 = A(A^3)^2 = A(A + I)^2 = A(A^2 + 2A + I)$$

$$= A(A^2 + I) = (A^3 + A) = A + I + A = I.$$

Thus $A^7 = I$ and 7 is the lowest positive power of A which is I, since $A^k = I$ for $k < 7$ implies that k divides 7 and hence $k = 1$. But $A^1 = I$ is impossible since it gives the contradiction $A^3 + A + I = I + I + I = I$ rather than $A^3 + A + I = 0$.