

Hilbert's Grand Hotel: No Vacancy?

Mathematician David Hilbert (1862–1943) discussed an imaginary hotel with an infinite number of rooms, numbered 1, 2, 3, Even though each room is occupied, the hotel is able to accommodate new guests as they arrive. Consider the following three scenarios.

- A traveler approaches the hotel one afternoon. The hotel clerk says “Every room is occupied, but we can accommodate you.” The clerk moves the guest in room 1 to room 2; the guest in room 2 to room 3; and in general, the guest in room n goes to room $n + 1$. Room 1 is then available to the newcomer.

$$\underline{1} \quad \underline{2} \quad \underline{3} \quad \underline{4} \quad \underline{5} \quad \underline{6} \quad \underline{7} \quad \underline{8} \quad \underline{9} \quad \dots \implies * \quad \underline{1} \quad \underline{2} \quad \underline{3} \quad \underline{4} \quad \underline{5} \quad \underline{6} \quad \underline{7} \quad \underline{8} \quad \dots$$

- Later that day, a limousine pulls up with infinitely many potential guests, numbered 1, 2, 3, The hotel clerk says “Every room is occupied, but we can accommodate all of you.” He moves the guest in room 1 to room 2; the guest in room 2 to room 4; and in general, the guest in room n goes to room $2n$. This leaves all of the odd-numbered rooms empty and available to the newcomers.

$$\underline{1} \quad \underline{2} \quad \underline{3} \quad \underline{4} \quad \underline{5} \quad \underline{6} \quad \underline{7} \quad \underline{8} \quad \underline{9} \quad \dots \implies * \quad \underline{1} \quad * \quad \underline{2} \quad * \quad \underline{3} \quad * \quad \underline{4} \quad * \quad \dots$$

- That night, infinitely many limousines (numbered 1, 2, 3, . . .) pull up, each with with infinitely many potential guests, numbered 1, 2, 3, The hotel clerk says “Every room is occupied, but we can accommodate all of you.” He again moves the guest in room 1 to room 2; the guest in room 2 to room 4; and in general, the guest in room n goes to room $2n$. This leaves all of the odd-numbered rooms empty and available to the newcomers. The clerk then puts the guests in the first limousine into rooms 3, 3^2 , 3^3 , and so on; the guests in the second limousine go in rooms 5, 5^2 , 5^3 , and so on; in general, the guests in the k -th limousine move into the rooms p_k , p_k^2 , p_k^3 , . . . , where p_k is the k -th odd prime. All the new guests have rooms. In fact, many rooms unoccupied—for instance, 15, 21, and 35.

Mathematics Faculty Profile

Lieutenant Kerry Bosché graduated from the United States Naval Academy in 2005 with a Bowman Scholarship and a degree in Quantitative Economics. Prior to graduation, he spent part of his 1/C summer as an intern analyst at The World Bank in Washington, DC, assisting in the research and production of Global Economic Prospects 2005. Following graduation, he studied Operations Analysis at the Naval Postgraduate School, completing a thesis on the optimization of computer-aided screening methods to pre-identify important factors in a wide variety of scenarios. Upon completion of nuclear power training in Charleston, SC, Lieutenant Bosché reported to the ballistic missile submarine USS Wyoming (SSBN-742)(Gold) as the Reactor Control Assistant, where he received his gold dolphins in January 2009. He later served as the ship's Communications Officer, completing four deterrent patrols and four refit periods. Following detachment from USS Wyoming, he returned to his dream shore tour at USNA teaching math. His mathematical interests, though many, are centered on the mathematical and theoretical concept of infinity. In fact, he can't go a day without talking about it, as many of his students will tell you.