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Faculty Profile: Asst. Prof. Max Wakefield

Prof. Max Douglas Wakefield was born on July 15, 1977 in Kodiak, Alaska. His father was a crab fisherman and his mother a marine biologist. After attending K-12 in Seattle, WA, Prof. Wakefield began working at Boeing as an aerospace machinist while simultaneously earning a B.S. degree in mathematics from Seattle University. Then in 2001 Prof. Wakefield left Seattle and Boeing to enroll in graduate school at the University of Oregon.



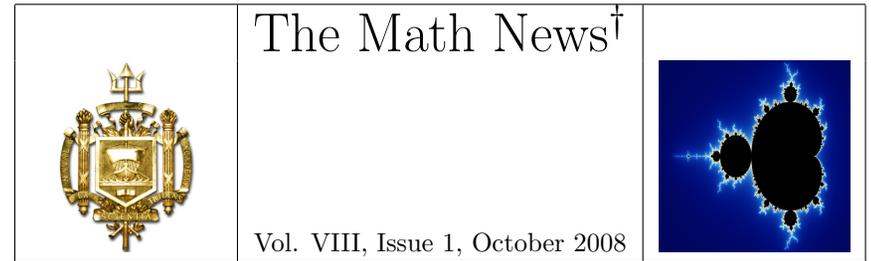
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A house has 3 lights in the basement, and 3 matching light switches in the attic. The owner has forgotten which switch corresponds to which light. He goes to the attic once, flips some of the switches, then goes to the basement once to observe the results. He is able to match the switches with the lights. How? Mids who submit a correct solution to the editor via email will be entered in a random prize drawing. *

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Knots, Links, and Surfaces

by Prof. Mark E. Kidwell (mek@usna.edu)

One of the main theorems in SM221 (Calculus III with Vector Fields) is Stokes's Theorem:

$$\int_C \vec{F} \cdot d\vec{r} = \iint_S \text{curl}(\vec{F}) \cdot d\vec{S}$$

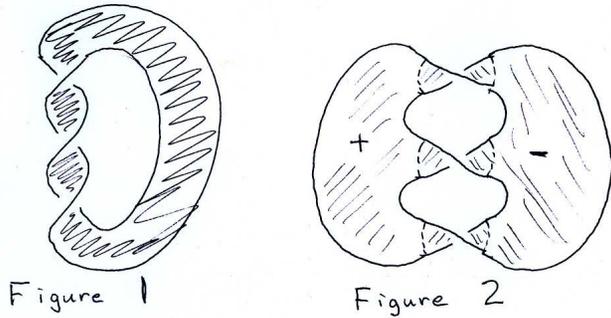
The theorem relates a line integral around a simple closed curve C in 3-space to a surface integral over an orientable surface S whose boundary is the given closed curve. A common choice for C is the unit circle in the xy -plane. Then two possible choices for S are the unit disk in the xy -plane and the upper unit hemisphere, since both have C as their boundary.

You might be able to embarrass your Calc III instructor with the question, "What if the simple closed curve is tied in a knot (with its ends joined); does such a curve bound any surface at all?" You can model the question experimentally by bending a wire into a knot and immersing it in a soap solution. Can a bubble form on the wire without passing through itself? From now on, we will call all simple closed curves "circles", no matter how loopy or knotted they are.

We need to discuss whether our surfaces have a front side and a back side. The example that is invariably given of a non-orientable (one-sided) surface is the Möbius band. One constructs it by taking a long, narrow, rectangular sheet of paper and gluing together the short sides with a half twist. In addition to being one-sided, the Möbius band has only one boundary edge. If the Möbius band is given three half twists instead of one, its boundary

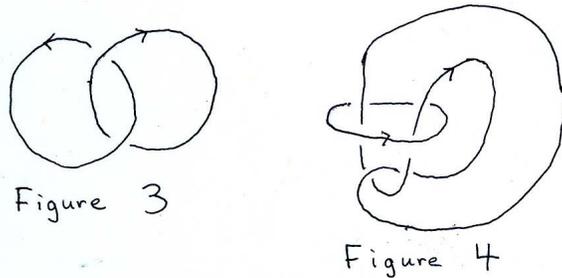
†On the cover: The USNA seal (left) and the Mandelbrot set (right).

edge becomes a knotted circle known as a trefoil (Figure 1).



For many purposes, it is preferable to find an orientable (two-sided) surface whose boundary is a given knot. In the early 1930's, the German mathematician Herbert Seifert found an elegant procedure for doing so. For the knot in Figure 2, another diagram of the trefoil, the surface consists of two disks joined by three "bow ties".

A link consists of two (or more, but all our examples will have two) disjoint circles in 3-space. Figure 3 shows the well known Hopf link, named for Seifert's contemporary Heinz Hopf. It is useful to assign an orientation to each circle, as one often does in an electrical circuit to indicate which way the electrons flow. A "right-hand rule" reminiscent of the cross product then assigns a "back-to-front" direction to an orientable surface whose boundary is the given circle.



With every pair of circles in a link, we can define an integer called the linking number. Carl Friedrich Gauss, one of the greatest mathematicians of all time and an early investigator of electromagnetism, discovered a surface integral which takes these integer values. One can also define the linking number by spanning a Seifert surface (as in Figure 2) in one circle and

counting (number of piercings of the surface by the other circle from back to front) - (number of piercings of the surface by the other circle from front to back). By this definition, the Hopf link of Figure 3 has linking number 1.

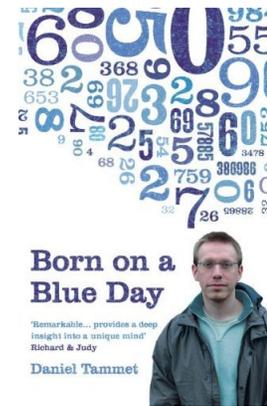
One would think that two circles with linking number 0 could be pulled apart, but the link in Figure 4, named after English mathematician J. H. C. Whitehead, makes this appear unlikely. Mathematicians have been working for nearly one hundred years to find more sophisticated ways to distinguish knots and links; for example, to distinguish the Whitehead link from the "unlink" consisting of two completely separated circles. Highlights of this effort include the Alexander polynomial of the 1920's and the Jones polynomial of the 1980's.

References

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- Rolfsen, D., *Knots and Links*, Berkeley, Publish or Perish, 1976.

Daniel Tammet: Born on a Blue Day

For Daniel Tammet, numbers are not just strings of digits. Numbers have shapes, colors, and even textures.



This neurologically-based phenomenon, in which stimulation of one cognitive pathway leads to involuntary experiences in a second sensory pathway is called *synesthesia*. As one might imagine, Daniel's multi-faceted perception of numbers makes them much easier for him to remember. Indeed, on March 14 (International Pi Day), 2004, he set a European record by reciting 22,514 digits of π without error in a little over 5 hours. Complex calculations are also a trivialeity—each of the numbers in the calculation has a visual representation. The pictures combine automatically in his mind to form a new image, which corresponds to the answer. Daniel is, in fact, an *autistic savant*, not only

capable of remarkable mathematical feats, but also of assimilating new languages virtually instantaneously (he learned Icelandic in a week). In stark contrast with most autistic savants, Daniel is high-functioning, capable of living a fully independent life. In 2007 he published his memoir, *Born on a Blue Day*, in which he explains in detail

how his unique mind processes the world around it. The title refers to the fact that Tammet perceives his birth date, January 31, 1979, as blue, which tells him that it was a Wednesday. The book was a popular success (New York Times Bestseller), and should be especially interesting to the mathematically inclined.

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