Lesson 7

- Boundary Conditions
  - Recognize the types and what they each imply
  - Know terminology
    - Bounding Surfaces
      - Rigid surface
      - Free surface
    - Inviscid
    - Viscous

System of Equations for the Atmosphere and Ocean

- 6 equations and 6 unknowns
  - Continuity Equation
  - EOM (3)
  - EOS
  - 1st Law of Thermodynamics
  - P, α (atm), ρ (ocn), T, u, v, w
- In solving this system of equations, we can find the future state of the atmosphere and ocean, i.e., the evolution of the flow

Solutions to the Equations

- In order to calculate the future state of the atmosphere and ocean, we need to know the governing equations as well as:
  - Boundary conditions (BC)
    - Constraints to the problem
    - A set of conditions specified for the behavior of a solution to a set of differential equations at the boundary of a domain
  - Initial conditions (IC)
    - Define initial state
    - A specified value of an unknown function at a given point in the domain

Boundary Conditions

- In order to describe the flow, the EOM need BC and IC
- Since they are PDEs based on x,y,z and t we need to provide a BC *for each variable*
- We also need IC *for each variable* at the initial time to integrate forward in time
- BC and IC have significant control on the character of the solutions!

Bounding Surfaces

- Rigid Surface
  - Confines the fluid
  - Fixed boundary (i.e. it doesn't move regardless of what the fluid does)
  - Examples: closed container; soup can; continents
- Free Surface
  - ***Material*** surfaces of the motion
    - Moves with the fluid
  - Examples: open container; surface of ocean/lake

Rigid Surface - Kinematic

- Since it confines a fluid, the component of the velocity normal to the surface must be zero:

$$\vec{V} \cdot \hat{n} = 0$$

- If F(x,y,z) = rigid boundary surface, then

$$\hat{n} = \nabla F(x, y, z)$$

$$\vec{V} \cdot \nabla F = 0$$

- Kinematic boundary condition for an inviscid flow
  - *Example*: u becomes 0 at the <u>N-S shore</u> and w becomes 0 at the <u>bottom</u>

Free Slip Boundary

- Consider the molecular effects of friction and heat conduction unimportant at the boundary
  - Implies no shear (i.e., no frictional effects) at the boundary

$$\frac{\partial \vec{V}}{\partial \hat{n}} \cdot \vec{T} = 0$$

    - T is any tangent vector
    - n is the normal
  - Useful if only interested in the interior

Non-Slip (No-Slip) Boundary

- The tangential component to a solid boundary must be zero
- Based on observations
  - The consequence of the interactions between the solid and fluid - the friction at the boundary
- Adherence condition
  - When frictional forces are important
  - In addition to $\vec{V} \cdot \hat{n} = 0$, we have $\vec{V} \cdot \vec{T} = 0$ for non-moving surfaces

Free Surface Condition

- The surface is considered a material surface of the flow
- The surface is described by: F(x,y,z,t)=0
- The BC must be: $\frac{DF}{Dt} = 0$
  - i.e. the description of the surface is conserved
  - A particle at the boundary must remain at the boundary