At 33,000 ft: $\rho = 7.9656 \times 10^{-4}$ slug/ft^3

At 33,500 ft: $\rho = 7.8165 \times 10^{-4}$ slug/ft^3

Hence, the density altitude is

$$33{,}000 + 500\left(\frac{7.9656 - 7.919}{7.9656 - 7.8165}\right) = \boxed{33{,}156 \text{ ft}}$$

4.1 $A_1V_1 = A_2V_2$

Let points 1 and 2 denote the inlet and exit conditions respectively. Then,

$$V_2 = V_1\left(\frac{A_1}{A_2}\right) = (5)\left(\frac{1}{4}\right) = \boxed{1.25 \text{ ft/sec}}$$

4.2 From Bernoulli's equation,

$$p_1 + \rho\,\frac{V_1^2}{2} = p_2 + \rho\,\frac{V_2^2}{2}$$

$$p_2 - p_1 = \frac{\rho}{2}\,(V_1^2 - V_2^2)$$

In consistent units,

$$\rho = \frac{62.4}{32.2} = 1.94 \text{ slug/ft}^3$$

Hence,

$$p_2 - p_1 = \frac{1.94}{2}\,[(5)^2 - (1.25)^2]$$

$$p_2 - p_1 = 0.97\,(23.4) = \boxed{22.7 \text{ lb/ft}^2}$$

4.3 From Appendix A; at 3000m altitude,

$p_1 = 7.01 \times 10^4 \text{ N/m}^2$

$\rho = 0.909 \text{ kg/m}^3$

From Bernoulli's equation,

$$p_2 = p_1 + \frac{\rho}{2}(V_1^2 - V_2^2)$$

$$p_2 = 7.01 \times 10^4 + \frac{0.909}{2}[60^2 - 70^2]$$

$$p_2 = 7.01 \times 10^4 - 0.059 \times 10^4 = \boxed{6.95 \times 10^4 \text{ N/m}^2}$$

4.4 From Bernoulli's equation,

$$p_1 + \frac{\rho}{2}V_1^2 = p_2 + \frac{\rho}{2}V_2^2$$

Also from the incompressible continuity equation

$$V_2 = V_1 (A_1/A_2)$$

Combining,

$$p_1 + \frac{\rho}{2}V_1^2 = p_2 + \frac{\rho}{2}(A_1/A_2)^2$$

$$V_1 = \sqrt{\frac{2(p_1 - p_2)}{\rho\left[(A_1/A_2)^2 - 1\right]}}$$

At standard sea level, $\rho = 0.002377 \text{ slug/ft}^3$. Hence,

$$V_1 = \sqrt{\frac{2(80)}{(.002377)[(4)^2 - 1]}} = \boxed{67 \text{ ft/sec}}$$

Note that also $V_1 = 67 \left(\frac{60}{80}\right) = 46$ mi/h. (This is approximately the landing speed of World War I vintage aircraft).

4.5 $$p_1 + \frac{1}{2}\rho V^2 = p_3 + \frac{1}{2}\rho V^2$$

$$V_1^2 = \frac{2(p_3 - p_1)}{\rho} + V_3^2 \qquad (1)$$

$$A_1 V_1 = A_3 V_3, \text{ or } V_3 = \frac{A_1}{A_3} V_1 \qquad (2)$$

Substitute (2) into (1)

$$V_1^2 = \frac{2(p_3 - p_1)}{\rho} + \left(\frac{A_1}{A_3}\right)^2 V_1^2$$

or, $$V_1 = \sqrt{\frac{2(p_3 - p_1)}{\rho\left[1 - \left(\frac{A_1}{A_3}\right)^2\right]}} \qquad (3)$$

Also,

$$A_1 V_1 = A_2 V_2$$

or, $$V_2 = \left(\frac{A_1}{A_2}\right) V_1 \qquad (4)$$

Substitute (3) into (4)

$$V_1 = \frac{A_1}{A_2} \sqrt{\frac{2(p_3 - p_1)}{\rho\left[1 - \left(\frac{A_1}{A_3}\right)^2\right]}}$$

$$V_2 = \frac{3}{1.5}\sqrt{\frac{2(1.00 - 1.02)\ \text{x}\ 10^5}{(1.225)\left[1 - \left(\frac{3}{2}\right)^2\right]}}$$

$$V_2 = \boxed{102.22\ \text{m/sec}}$$

Note: It takes a pressure difference of <u>only</u> 0.02 atm to produce such a high velocity.