5° AOA, App D.

$c_\ell = 0.67$

$c_{m_{c/4}} = -0.025$

(Note: Two sets of lift and moment coefficient data are given for the NACA 1412 airfoil -- with and without flap deflection. Make certain to read the code properly, and use only the unflapped data, as given above. Also, note that the scale for $c_{m_{c/4}}$ is different than that for c_ℓ -- be careful in reading the data.)

With regard to c_d, first check the Reynolds number,

$$\text{Re} = \frac{\rho_\infty V_\infty c}{\mu_\infty} = \frac{(0.002377)(100)(3)}{(3.7373 \text{ x } 10^{-7})}$$

$$\text{Re} = 1.9 \text{ x } 10^6$$

In the airfoil data, the closest Re is 3 x 10^6. Use c_d for this value.

$$c_d = 0.007 \quad (\text{for } c_\ell = 0.67)$$

The dynamic pressure is

$$q_\infty = \frac{1}{2}\rho_\infty V_\infty^{\,2} = \frac{1}{2}(0.002377)(100)^2 = 11.9 \text{ lb/ft}^2$$

The area per unit span is $S = 1(c) = (1)(3) = 3 \text{ ft}^2$

Hence, per unit span,

$$L = q_\infty S c_\ell = (11.9)(3)(0.67) = \boxed{23.9 \text{ lb}}$$

$$D = q_\infty S c_d = (11.9)(3)(0.007) = \boxed{0.25 \text{ lb}}$$

$$M_{c/4} = q_\infty S c c_{m_{c/4}} = (11.9)(3)(3)(-0.025) = \boxed{-2.68 \text{ ft.lb}}$$

5.3 $\rho_\infty = \dfrac{p_\infty}{RT_\infty} = \dfrac{(1.01 \times 10^5)}{(287)(303)} = 1.61 \text{ kg/m}^3$

From Appendix D,

$c_\ell = 0.98$

$c_{m_{c/4}} = -0.012$

Checking the Reynolds number, using the viscosity coefficient from the curve given in Chapter 4,

$\mu_\infty = 1.82 \times 10^{-5}$ kg/m sec at T = 303K,

$$Re = \frac{\rho_\infty V_\infty c}{\mu_\infty} = \frac{(1.157)(42)(0.3)}{1.82 \times 10^{-5}} = 8 \times 10^5$$

This Reynolds number is considerably less than the lowest value of 3×10^6 for which data is given for the NACA 23012 airfoil in Appendix D. Hence, we can use this data only to give an educated guess; use

$c_d \approx 0.01$, which is about 10 percent higher than the value of 0.009 given for $Re = 3 \times 10^6$

The dynamic pressure is

$$q_\infty = \frac{1}{2}\rho_\infty V_\infty^2 = \frac{1}{2}(1.161)(42)^2 = 1024 \text{ N/m}^2$$

The area per unit span is $S = (1)(0.3) = 0.3 \text{ m}^2$. Hence,

$L = q_\infty S c_\ell = (1024)(0.3)(0.98) = \boxed{301\text{N}}$

$D = q_\infty S c_d = (1024)(0.3)(0.01) = \boxed{3.07\text{N}}$

$M_{c/4} = q_\infty S c c_m = (1024)(0.3)(0.3)(-0.012) = \boxed{-1.1\text{Nm}}$

5.4 From the previous problem, $q_\infty = 1020 \text{ N/m}^2$

$L = q_\infty S c_\ell$

Hence,

$$c_\ell = \frac{L}{q_\infty S}$$

The wing area $S = (2)(0.3) = 0.6 \text{ m}^2$

Hence,

$$c_\ell = \frac{200}{(1024)(0.6)} = 0.33$$

From Appendix D, the angle of attack which corresponds to this lift coefficient is

$$\boxed{\alpha = 2^\circ}$$

5.5 From Appendix D, at $\alpha = 4^\circ$,

$c_\ell = 0.4$

Also, $V_\infty = 120 \left(\frac{88}{60}\right) = 176 \text{ ft/sec}$

$$q_\infty = \frac{1}{2}\rho_\infty V_\infty^2 = \frac{1}{2}(0.002377)(176)^2 = 36.8 \text{ lb/ft}^2$$

$L = q_\infty S c_\ell$

$$S = \frac{L}{q_\infty c_\ell} = \frac{29.5}{(36.8)(0.4)} = \boxed{2 \text{ ft}^2}$$

5.6 $L = q_\infty S c_\ell$

$D = q_\infty S c_d$

Hence,

$$\frac{L}{D} = \frac{q_\infty S\ c_\ell}{q_\infty S\ c_d} = \frac{c_\ell}{c_d}$$

We must tabulate the values of c_ℓ/c_d for various angles of attack, and find where the maximum occurs. For example, from Appendix D, at $\alpha = 0°$,

$$c_\ell = 0.25$$

$$c_d = 0.006$$

Hence

$$\frac{L}{D} = \frac{c_\ell}{c_d} = \frac{0.25}{0.006} = 41.7$$

A tabulation follows.

α	0°	1°	2°	3°	4°	5°	6°	7°	8°	9°
c_ℓ	0.25	0.35	0.45	0.55	0.65	0.75	0.85	0.95	1.05	1.15
c_d	0.006	0.006	0.006	0.0065	0.0072	0.0075	0.008	0.0085	0.0095	0 .0105
$\frac{c_\ell}{c_d}$	41.7	58.3	75	84.6	90.3	100	106	112	111	110

From the above tabulation,

$$\left(\frac{L}{D}\right)_{max} \approx \boxed{112}$$

5.7 At sea level

$$\rho_\infty = 1.225 \text{ kg/m}^3$$

$p_\infty = 1.01 \times 10^5\ \mathrm{N/m^2}$

Hence,

$$q_\infty = \frac{1}{2}\rho_\infty V_\infty^2 = \frac{1}{2}(1.225)(50)^2 = 1531\ \mathrm{N/m^2}$$

From the definition of pressure coefficient,

$$C_p = \frac{p - p_\infty}{q_\infty} = \frac{(0.95 - 1.01) \times 10^5}{1531} = \boxed{-3.91}$$

5.8 The speed is low enough that incompressible flow can be assumed. From Bernoulli's equation,

$$p + \frac{1}{2}\rho\, V_\infty^2 = p_\infty + \frac{1}{2}\rho_\infty V_\infty^2 = p_\infty + q_\infty$$

$$C_p = \frac{p - p_\infty}{q_\infty} = \frac{q_\infty - \frac{1}{2}\rho\, V^2}{q_\infty} = 1 - \frac{\frac{1}{2}\rho\, V^2}{\frac{1}{2}\rho_\infty V_\infty^2}$$

Since $\rho = \rho_\infty$ (constant density)

$$C_p = 1 - \left(\frac{V}{V_\infty}\right)^2 = 1 - \left(\frac{62}{55}\right)^2 = 1 - 1.27 = \boxed{-0.27}$$

5.9 The flow is low speed, hence assumed to be incompressible. From problem 5.8,

$$C_p = 1 - \left(\frac{V}{V_\infty}\right)^2 = 1 - \left(\frac{195}{160}\right)^2 = \boxed{-0.485}$$

5.10 The speed of sound is

$$a_\infty = \sqrt{\gamma R T_\infty} = \sqrt{(1.4)(1716)(510)} = 1107 \text{ ft/sec}$$

Hence,

$$M_\infty = \frac{V_\infty}{a_\infty} = \frac{700}{1107} = 0.63$$

In problem 5.9, the pressure coefficient at the given point was calculated as -0.485. However, the conditions of problem 5.9 were low speed, hence we identify

$$C_{p_o} = -0.485$$

At the new, higher free stream velocity, the pressure coefficient must be corrected for compressibility. Using the Prandtl-Glauert Rule, the high speed pressure coefficient is

$$C_p = \frac{C_{p_o}}{\sqrt{1 - M_\infty^{\ 2}}} = \frac{-0.485}{\sqrt{1-(0.63)^2}} = \boxed{-0.625}$$

5.11 The formula derived in problem 5.8, namely

$$C_p = 1 - \left(\frac{V}{V_\infty}\right)^2,$$

utilized Bernoulli's equation in the derivation. Hence, it is <u>not valid</u> for compressible flow. In the present problem, check the Mach number.

$$a_\infty = \sqrt{\gamma R T_\infty} = \sqrt{(1.4)(1716)(505)} = 1101 \text{ ft/sec}$$

$$M_\infty = \frac{780}{1101} = 0.708.$$

The flow is clearly compressible! To obtain the pressure coefficient, first calculate ρ_∞ from the equation of state.

$$\rho_\infty = \frac{p_\infty}{RT_\infty} = \frac{2116}{(1716)(505)} = 0.00244 \text{ slug/ft}^3$$

To find the pressure at the point on the wing where V = 850 ft/sec, first find the temperature from the energy equation

$$c_p T + \frac{V^2}{V} = c_p T_\infty + \frac{V_\infty^{\,2}}{2}$$

$$T = T_\infty + \frac{V_\infty^{\,2} - V^2}{2c_p}$$

The specific heat at constant pressure for air is

$$c_p = \frac{\gamma R}{\gamma - 1} = \frac{(1.4)(1716)}{(1.4-1)} = 6006 \frac{\text{ft lb}}{\text{slug R}}$$

Hence,

$$T = 505 + \frac{780^2 - 850^2}{2(6006)} = 505 - 9.5 = 495.5\text{R}$$

Assuming isentropic flow

$$\frac{p}{p} = \left(\frac{T}{T_\infty}\right)^{\frac{\gamma}{\gamma-1}}$$

$$p = (2116)\left(\frac{495.5}{505}\right)^{3\,5} = 1980 \text{ lb/ft}^2$$

From the definition of C_p

$$C_p = \frac{p - p_\infty}{q_\infty} = \frac{p - p_\infty}{\frac{1}{2}\rho_\infty V_\infty^{\,2}} = \frac{1980 - 2116}{\frac{1}{2}(0.00244)(780)^2}$$

$$C_p = \boxed{-0.183}$$
