

$$7.1 \quad C_{M_{cg}} = C_{M_{ac}} + C_L (h - h_{ac})$$

$$C_{M_{ac}} = C_{M_{cg}} - C_L (h - h_{ac}) = 0.005 - 0.05 (0.03) = \boxed{-0.01}$$


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$$7.2 \quad q_{\infty} = \frac{1}{2} \rho_{\infty} V_{\infty}^2 = \frac{1}{2} (1.225)(100)^2 = 6125 \text{ N/m}^2$$

At zero lift, the moment coefficient about c.g. is

$$C_{M_{cg}} = \frac{M_{cg}}{q_{\infty} S c} = \frac{-12.4}{(6125)(1.5)(0.45)} = -0.003$$

However, at zero lift, this is also the value of the moment coefficient about the a.c.

$$C_{M_{ac}} = \boxed{-0.003}$$

At the other angle of attack,

$$C_L = \frac{L}{q_{\infty} S} = \frac{3675}{(6125)(1.5)} = 0.4$$

and 
$$C_{M_{cg}} = \frac{M_{cg}}{q_{\infty} S c} = \frac{20.67}{(6125)(1.5)(0.45)} = 0.005$$

Thus, from Eq. (7.9) in the text,

$$C_{M_{cg}} = C_{M_{ac}} + C_L (h - h_{ac})$$

Thus,

$$h - h_{ac} = \frac{C_{M_{cg}} - C_{M_{ac}}}{C_L} = \frac{0.005 - (-0.003)}{0.4}$$

$$h - h_{ac} = \boxed{0.02}$$

The aerodynamic center is two percent of the chord length ahead of the center of gravity.

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7.3 From the results of problem 7.2,

$$q_{\infty} = 6125 \text{ N/m}^2$$

$$C_{M_{ac}} = -0.003$$

$$h - h_{ac} = 0.02$$

In the present problem, the c.g. has been shifted 0.2c rearward. Hence,

$$h - h_{ac} = 0.02 + 0.2 = 0.22$$

$$\text{Also, } C_L = \frac{L}{q_{\infty} S} = \frac{4000}{(6125)(1.5)} = 0.435$$

$$\text{Thus, } C_{M_{cg}} = C_{M_{ac}} + C_L (h - h_{ac})$$

$$C_{M_{cg}} = -0.003 + 0.435 (0.22) = \boxed{0.0927}$$

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$$7.6 \quad h_n = h_{a c_{wb}} + V_H \frac{a_t}{a} \left( 1 - \frac{\partial \varepsilon}{\partial \varepsilon} \right)$$

From Problem 7.2,  $h_n - h_{a c_{wb}} = 0.02$

Hence,

$$h_{a c_{wb}} = h - 0.02 = 0.26 - 0.02 = 0.24$$

$$h_n = 0.24 + 0.593 \left( \frac{0.12}{0.09} \right) (1 - 0.42)$$

$$h_n = \boxed{0.70}$$

By definition,

$$\text{static margin} = h_n - h = 0.70 - 0.26 = \boxed{0.44}$$

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$$7.7 \quad \delta_{\text{trim}} = \frac{C_{M_0} + \left( \frac{\partial C_{M_0}}{\partial \alpha_n} \right) \alpha_n}{V_H \left( \frac{\partial C_{L_t}}{\partial \delta_e} \right)}$$

$$C_{M_0} = 0.139 \text{ (from Problem 7.5)}$$

$$\frac{\partial C_{M_0}}{\partial \alpha_n} = -0.039 \text{ (from Problem 7.5)}$$

$$\alpha_n = 8^\circ$$

$$V_H = 0.593 \text{ (from Problem 7.4)}$$

$$\frac{\partial C_{L_t}}{\partial \delta_e} = 0.04 \text{ (given)}$$

$$\text{Thus, } \delta_{\text{trim}} = \frac{0.139 + (-0.039)(8)}{(0.593)(0.04)} = \boxed{-7.29^\circ}$$

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