Laboratory

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Airspeed Determination
Bernoulli's Equation and the Pitot-static tube

Introduction

The accurate indication of airspeed is critical for both flight operations and wind tunnel testing of aircraft components. In this laboratory, we will study the nature of Bernoulli's Principle (sometimes referred to as the "Venturi Effect") and apply it to the measurement of the velocity of an air stream. The results will be used to calibrate the speed control on the USNA Flow Bench in Rickover 023, and the implications on aircraft airspeed indicators will be discussed.

Bernoulli's Equation

What is now known as Bernoulli's equation was formulated by mathematician/physicist Daniel Bernoulli (1700-1782). Much of his work involved studies in fluid mechanics and the kinetic motion of gases and their applications to jet propulsion, manometers, and flow in pipes. Although Daniel originally published the concept involving the Bernoulli equation in 1738, it was through the observations of his father, Johann, and the insight of his close friend, Leonhard Euler, that resulted in the present derivation of the Bernoulli Equation.

The Euler Equation originates out of the steady-state momentum equation applied to a streamline when the effects of friction are assumed to be negligible. This assumption is common in fluid dynamics, and characterizes a fluid as *inviscid*. Thus, for an inviscid flow that is invariant with time, the resulting Euler Equation can be written as:

$$dp + \rho V dV + \rho g dh = 0 \tag{4.1}$$

Equation (4.1) holds true as long as we restrict ourselves to the streamline on which the fluid element is traveling. Note that it also includes the term $\rho g dh$, which is known as the hydrostatic term. If we restrict ourselves to the flow of *air* rather than *water*, then this term is negligibly small compared to the others. The result is the form of the Euler Equation as presented in Anderson's *Introduction to Flight* text (equation 4.8 on p.132):

$$dp + \rho V dV = 0 \quad \text{(or } dp = -\rho V dV \text{)} \tag{4.2}$$

This equation is easily integrated over a streamline when we make the additional assumption that the density is constant. This last assumption is valid for low-speed flows, where the velocity is less than about 330 ft/sec (230 mph or 200 knots). When the density is constant throughout the flow, it is said to be in the incompressible flow regime. The resulting indefinite integral is:

$$\int \left(dp + \rho V dV \right) = \int dp + \rho \int V dV = p + \frac{1}{2}\rho V^2 + C_1 \tag{4.3}$$

where C_1 is a constant. This can also be represented as:

$$p + \frac{1}{2}\rho V^2 = \mathrm{C}_2 \tag{4.4}$$

where C_2 is a different constant. Remember that this constant holds everywhere along the streamline, according to our derivation assumptions. Thus, we can relate any two points along this streamline without actually knowing the value of this constant:

$$p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2 \tag{4.5}$$

The constant on the right-hand side of equation (4.4), denoted as the *total* pressure and written as p_T, represents the total energy of the fluid particles along a streamline. This total energy is a constant value when we confine ourselves to the streamline along which we integrated. Thus we can write the total pressure as:

$$p_T = p + \frac{1}{2}\rho V^2 = C_2 = \text{ Constant along a streamline} \tag{4.6}$$

This is known as Bernoulli's Equation. It is obvious that the total pressure consists of two terms that are added. The first term (p) is called the fluid pressure (often called *static* pressure or p_s). The fluid pressure represents the energy associated with the random motion of the fluid particles and is a state property of the fluid. This is analogous to, for instance, the potential energy of a swinging pendulum. The second term ($\frac{1}{2}\rho V^2$) is called the *dynamic* pressure of the moving fluid. The dynamic pressure is denoted by q and is thus defined:

$$q \equiv \frac{1}{2}\rho V^2 \tag{4.7}$$

> ***NOTE:*** *This definition for* q *is* <u>*always true*</u> *and does NOT depend on the assumption of incompressibility, unlike Bernoulli's Equation.*

The dynamic pressure represents the energy associated with the ordered motion of the fluid traveling at velocity V . This is analogous to, for instance, the kinetic energy of a swinging pendulum. In such an analogy, the total energy is the sum of the potential energy and the kinetic energy. Thus, the total pressure is the sum of the static pressure and the dynamic pressure. When consistent units for ρ and V are used, the dynamic pressure will be in units of lbs/ft² or Pa.

With the definition of the dynamic pressure from equation (4.7) and substituting this into equation (4.6), we can thus rewrite Bernoulli's Equation in its simplest form:

$$p_T = p_s + q \tag{4.8}$$

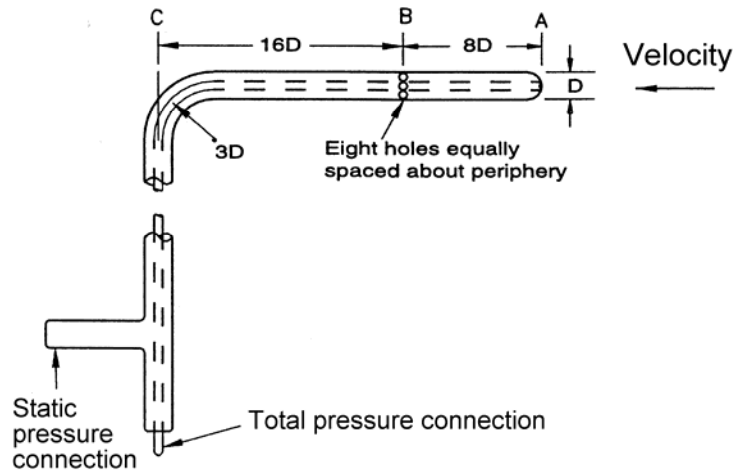
Note that if the dynamic pressure were a known quantity, then the velocity can be calculated by rewriting equation (4.7) as:

$$V = \sqrt{2q/\rho} \tag{4.9}$$

This is the principal approach used to determine the airspeed of a flight vehicle.

Airspeed Measurement

In order to determine airspeed from equation (4.9), we need the value of the dynamic pressure, but it is impossible to directly measure q. However, we can easily measure p_s and p_T using a device called a Pitot-static tube in order to indirectly determine the dynamic pressure. Figure 2 shows the geometry of a standard Pitot-static pressure probe and how it measures both total and static pressures.



• Figure 2. A standard Pitot-static pressure probe

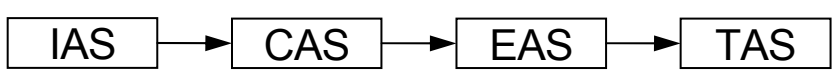
Thus it follows from equations (4.7) and (4.8):

$$V = \sqrt{\frac{2q}{\rho}} = \sqrt{\frac{2\left(p_T - p_s\right)}{\rho}} \tag{4.10}$$

Equation (4.10) is just another formulation of the Bernoulli equation. We see that it's not even necessary to measure both p_T and p_s, but simply the difference between the two. That is why a differential measurement system, like a manometer, a dial-type pressure gauge, or a differential pressure transducer, is often used.

Flight Vehicle Application

Obviously, the calculation of velocity requires an accurate measurement of the dynamic pressure of the air stream and the actual air density of the fluid. However, a conventional cockpit airspeed indicator is a simple dial-type pressure gauge that only measures the pressure difference $\left(p_T - p_s\right)_{measured}$. This pressure difference is also known as the "impact pressure" in the aviation community and is denoted as q_c. Note that the impact pressure is not really an absolute pressure measurement, but a differential measurement. The dial on the airspeed indicator is calibrated to read out in units of knots, but it is labeled "indicated airspeed" and not "true airspeed." Thus, the conventional airspeed indicator calibration involves several assumptions, including standard-day, sea level values for the density (0.002377 slug/ft³) and pressure (2116.2 lb/ft²). The airspeed is thus displayed in units of knots, indicated airspeed (KIAS). The value of the true airspeed must involve several corrections for the probe's position on the aircraft, the vehicle pressure altitude, and the density altitude. The different components of the airspeed are progressively related according to the figure below. The airspeed components are defined as:



Indicated Airspeed (IAS). Indicated airspeed is equal to the Pitot-static airspeed indicator reading as installed in the aircraft. It involves no corrections for system errors, but it does include a correction for sea level compressible flow (which is included in the instrument dial calibration) so that its reading is valid above 200 knots. The assumed air conditions are standard, sea level pressure and density.

Calibrated Airspeed (CAS). CAS is the result of correcting IAS for errors resulting from the positioning or location of the Pitot-static probe installation. The error is provided in a table by the aircraft manufacturer and is designed to be as small as possible, but the static pressure port is generally more susceptible to position and location errors than is the Pitot pressure. At low speeds, CAS will be greater than IAS, but at high

speeds this trend reverses due to compressibility effects. Assuming that the impact pressure has been corrected for these errors, the calibrated airspeed can be calculated:

$$CAS = a_{std}\sqrt{5\left[\left(\frac{q_c}{p_{std}}+1\right)^{2/7}-1\right]} \tag{4.11}$$

The term a_{std} is the standard, sea level value for the speed of sound (1116 ft/sec or 661.5 knots).

Equivalent Airspeed (EAS). EAS is the CAS that has been corrected for altitude compressibility effects. Thus, the equivalent airspeed is literally the "equivalent" flight speed at standard sea level density that would produce the same dynamic pressure at the actual flight speed and altitude. Essentially, it is a correction of CAS to the true pressure altitude, while still assuming a standard, sea level value of the density. At sea level, EAS = CAS. The equation to calculate EAS is given by:

$$EAS = a_{std}\sqrt{5\frac{p}{p_{std}}\left[\left(\frac{q_c}{p}+1\right)^{2/7}-1\right]} \tag{4.12}$$

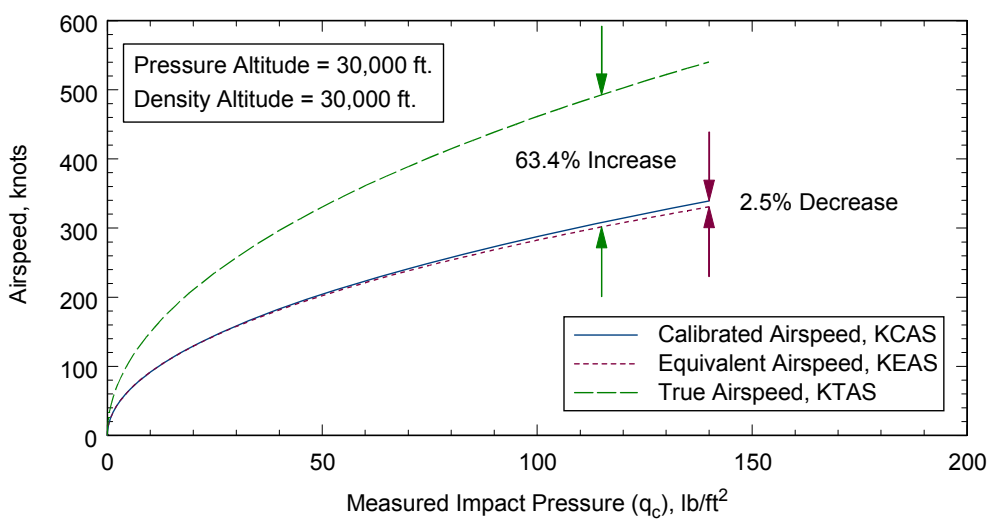
True Airspeed (TAS). The true airspeed results when the EAS is corrected for density altitude. Because the airspeed indicator is calibrated for standard sea level density, it must be corrected to account for the actual density at altitude so that an accurate dynamic pressure determination can be made. Essentially, it is a correction of EAS to the true density altitude. At sea level, TAS = EAS. The equation to calculate TAS is given by:

$$TAS = a_{std}\sqrt{5\frac{p}{p_{std}}\frac{\rho_{std}}{\rho}\left[\left(\frac{q_c}{p}+1\right)^{2/7}-1\right]} \tag{4.13}$$

The common (but not standard) units used for airspeed measurement are knots, or **nautical miles per hour**. Thus, we would notate the airspeed components as "knots indicated airspeed" (KIAS); "knots equivalent airspeed" (KEAS); and "knots true airspeed" (KTAS). For the purpose of this laboratory, we can assume a low-speed, incompressible flow such that the following is true: $IAS \approx CAS = EAS$.

Take note of the conversion factors involving statute miles and nautical miles:

$1\ statute\ mile = 5,280\ ft$ and $1\ nautical\ mile = 6,076\ ft$



• Figure 3. Comparison of airspeed measurements

These different values of airspeed are summarized in Figure 3, which is a plot of airspeed as a function of the impact pressure. These numbers assume the flight conditions of a typical jetliner (standard altitude of 30,000 ft and speeds up to 500 knots). Note that the correction from CAS to EAS is at most a 2.5% decrease. The correction from EAS to TAS is a constant 63.4% increase.

Pressure Conversions and Measurement

There are many different units commonly used for pressure measurement. Care must be taken when working with the different units because fluid equations are usually derived on the basis of standard units. Thus, the standard units of pressure (force per unit area) are N/m² or lb/ft². All other units of pressure measurement are referenced to 1 atmosphere, which is the sea level barometric pressure of the ARDC 1959 Standard Atmosphere (tabulated in Anderson's *Introduction to Flight*, Appendices A and B). To do pressure conversions, just remember that…

1 atmosphere = 101,325 N/m²
101,325 Pa
1013.25 mbar
1.01325 bars
1,013,250 dynes/cm²
760 mm Hg
2116.2 lb/ft²
14.696 lb/in²
29.92 in. Hg
406.78 in. H_2O

Notice how many units of pressure are expressed in the units of the height of a fluid column, such as millimeters of mercury or inches of water. These units originate from the pressure exerted by a column of fluid having a specified height at standard temperature (22 °C). In the laboratory, the fluid column is constrained by U-shaped length of glass tubing known as a manometer, and the difference in height between the two fluid columns represents a difference in pressure. In this experiment, we'll use a manometer filled with green water. When recording data, the height of the fluid column is usually recorded as the symbol h (rather than p), but this is always understood to be a pressure measurement.

Frequently, fluids other than water or mercury are used in the laboratory. On the USNA Fluids Lab Deck, a mixture of ethyl and methyl alcohol is used in most of the manometer tubes. This fluid is dyed pink to increase its visibility. To get the effective pressure in units of inches of water, the height of the alcohol column is multiplied by the specific gravity ($S.G.$) of the fluid. Thus:

$$h\ (in.\ H_2O) = h\ (in.\ alcohol) \times S.G. \tag{4.14}$$

Procedure

This experiment uses the USNA Flow Bench in Rickover 023. To operate the flow bench, a control pendant has a push button that turns the blower on and off and a knob that adjusts the flow velocity. The readout on the pendant gives the Blower Control Voltage (BCV), an analog signal to the motor controller which is proportional to the blower flow rate. The motor controller is capable of a maximum BCV of 10 volts, but the control pendant limits the maximum value to somewhat less than this. The following procedure will be used to calibrate the BCV so it is a reliable indicator of the velocity of the jet.

A Pitot-static probe is mounted 6″ above the nozzle exit plane, and it is connected to a U-tube manometer filled with green-colored water. The manometer has a tape measure glued to the board for height measurements.

Follow these steps to determine the calibration for the flow bench:

1) Record the room temperature and the day's barometric pressure.
2) Measure the diameter of the nozzle exhaust.

3) Ensure that the Pitot-static tube is centered over the nozzle of the jet in the flow bench. Securely connect the total pressure line to the left side of the U-tube manometer. Now connect the static pressure line to the right side so that a differential measurement is possible.

4) Measure both levels in the manometer tube with the blower off. These will be your "Zero" readings, in units of inches of water. Note that this is your first data point!

5) Put on the proper protective equipment.

6) With the BCV set to 0, start the blower. Carefully increase the flow rate until a BCV of about 1 is achieved on the control pendant.

7) After the flow stabilizes, record the height of the two fluid columns in the manometer. These measurements will be your "run" readings, h_1 and h_2, in units of inches of water.

8) Increase the flow rate by BCV = 0.25-0.5 and repeat step 7) until the maximum flow rate is achieved.

9) Turn off the blower and record the final temperature and barometric pressure.

Data Reduction

Your data reduction should involve the following calculations:

1) Calculate the differential height in the manometer, Δh (in. water) for each data point.

2) Convert the differential to the units of psi for each point. This is the dynamic pressure, $p_T - p_S$.

3) From your raw data, use the air temperature and barometric pressure to calculate the air density (slug/ft^3) for each point. If the temperature varied much, assume a linear increase with time, from the initial temperature at the first measurement, to the final temperature at the last measurement.

4) Calculate EAS and TAS in knots, and the percent difference between them for each data point. Do not be tempted to use equations (4.12) and (4.13); these are too involved for this test. Instead, assume that the flow is incompressible (and it is) so that you can use Bernoulli's equation (4.10).

5) Plot the EAS as a function of BCV (i.e., BCV on the x-axis). What type of relationship does this appear to be? Put in a trendline and determine the equation for EAS in terms of BCV. Make sure you include the first data point (0, 0). In other words, your trendline has to go through (0, 0). Make sure your plot follows an appropriate format for technical plots.

6) Plot the dynamic pressure as a function of BCV. What type of relationship does this appear to be? Put in a trendline and determine the equation for q in terms of BCV.

Deliverables

Although the data were gathered in teams, each write-up must be done and submitted on an individual basis. You must hand in the following deliverables to complete this laboratory:

- A formal **EXPERIMENTAL SETUP** write-up for this experiment
- Two plots and a data table of your data
- Sample calculations for one data point that was used to obtain your graphical results
- Two calibration equations, one for the blower velocity and one for the dynamic pressure
- Answers to the following questions

Questions

1) What is the maximum dynamic pressure (in psi), velocity (in KEAS), and volumetric flow rate (ft^3/min or CFM) of the test? What are the maximum capabilities of the USNA Flow Bench (i.e., if we could get BCV = 10 volts)?

2) An aircraft is flying 165 KTAS at an altitude of 5,000 ft. What is the dynamic pressure in psi? What is the static pressure in psi? What is the total pressure in psi?

3) How do we convert from EAS to the TAS for an incompressible flow regime? Derive the relationship and <u>write out the equation</u>. (Note: review the definition of the equivalent airspeed and how it relates to true airspeed).

4) If we are flying a Cessna Citation V at an airspeed of 165 KEAS at an altitude of 5,000 ft, what is the percent difference from our true airspeed? What would be our actual airspeed (i.e., KTAS)?

• Table 3. Raw data table for airspeed experiment

Date & Time: _____

"Zero" Readings (h_0): _____

Barometric Pressures (start/end): _____

Room Temperatures (start/end): _____

Data Point No.	BCV (volts)	h_1 (in. water)	h_2 (in. water)	Δh (in. water)	$q = \Delta p$ (lbs/in²)	Airspeed (KEAS)	Airspeed (KTAS)	Percent Difference
1								
2								
3								
4								
5								
6								
7								
8								
9								
10								
11								
12								
13								
14								
15								