Laboratory

5

Drag on Basic Shapes
Determination of the Drag Coefficient

Introduction

An object's speed and shape greatly affect its aerodynamic characteristics. In this laboratory, we will study the effects of velocity, size, and geometry on the aerodynamic drag of several basic shapes. A comparison of the results will underscore the competing influences between the friction drag and the pressure drag on the overall measured drag.

Dimensional Analysis

The formula describing aerodynamic drag on an object of fixed orientation a the flow stream can be derived using the Buckingham Pi Theorem, which is outlined in section 5.3 of John Anderson's *Introduction to Flight*. This approach uses dimensional analysis and assumes that the drag, D, can be expressed in terms of the fluid density, ρ, the flow velocity, V, an area representing the size of the object in some way, S, the fluid viscosity, μ, and the local speed of sound, a. The equation for the drag on an object can thus be written in these terms:

$$D = Z\rho^b V^c S^d \mu^e a^f \tag{5.1}$$

Equation (5.1) depends on six dimensionless constants: Z, b, c, d, e, and f. Dimensionally, this combination of units must yield a force (say, Newtons). Thus, this equation can be used to compare the combination of units on both sides of the equation. Plugging-in standard SI units:

$$N = \frac{kg\text{ - }m}{sec^2} = \left(\frac{kg}{m^3}\right)^b \left(\frac{m}{sec}\right)^c \left(m^2\right)^d \left(\frac{kg}{m - sec}\right)^e \left(\frac{m}{sec}\right)^f \tag{5.2}$$

Because the units need to work out, this leads directly to equating the value of each exponent on a dimensional basis only:

$1 = b + e$ (mass, in kg),

$-2 = -c - e - f$ (time, in sec), and

$1 = -3b + c + 2d - e + f$ (length, in m) (5.3)

But we have three equations and five unknowns, so some of these constants will remain undetermined. Solving these equations in terms of e and f results in:

$b = 1 - e$,

$c = 2 - e - f$, and

$$d = \frac{(1 + 3b - c + e - f)}{2} = \frac{1 + 3(1 - e) - (2 - e - f) + e - f}{2} = 1 - \frac{e}{2} \tag{5.4}$$

Now these relationships can be substituted into equation (5.1):

$$D = Z \cdot \rho^{1-e} \cdot V^{2-e-f} \cdot S^{1-\frac{e}{2}} \cdot \mu^e \cdot a^f \tag{5.5}$$

Combining and separating some terms, we arrive at:

$$D = Z \cdot \rho \cdot V^2 \cdot S \cdot \frac{\mu^e}{\rho^e V^e S^{e/2}} \cdot \frac{a^f}{V^f} = Z \rho V^2 S \left(\frac{\mu}{\rho V S^{1/2}} \right)^e \left(\frac{a}{V} \right)^f \tag{5.6}$$

It is obvious that the units of $S^{1/2}$ are length. Thus, we can define some representative length, ℓ. Then we see the last two terms become:

$$\left(\frac{\mu}{\rho V S^{1/2}} \right)^e = \left(\frac{\mu}{\rho V \ell} \right)^e = \left(\frac{1}{Re} \right)^e \text{ and}$$

$$\left(\frac{a}{V} \right)^f = \left(\frac{1}{M} \right)^f \tag{5.7}$$

Note that these are the dimensionless Reynolds and Mach numbers, respectively. The former describes the relative interaction between momentum and viscous effects and the latter, compressibility effects on the flowfield.

Drag Formulation

Having determined the functionality of the different terms in the drag equation (5.6), all the dimensionless terms can be collected and renamed $C_D/2$. This now defines C_D as the dimensionless **drag coefficient**. Different values of C_D for several common shapes are listed in Figure 4. Note that these drag coefficients assume a laminar boundary layer development.

Now, with this information, the drag equation can be written in its simplest form:

$$D = \frac{C_D}{2} \rho V^2 S = C_D \left(\frac{1}{2} \rho V^2 \right) S = C_D q S \tag{5.8}$$

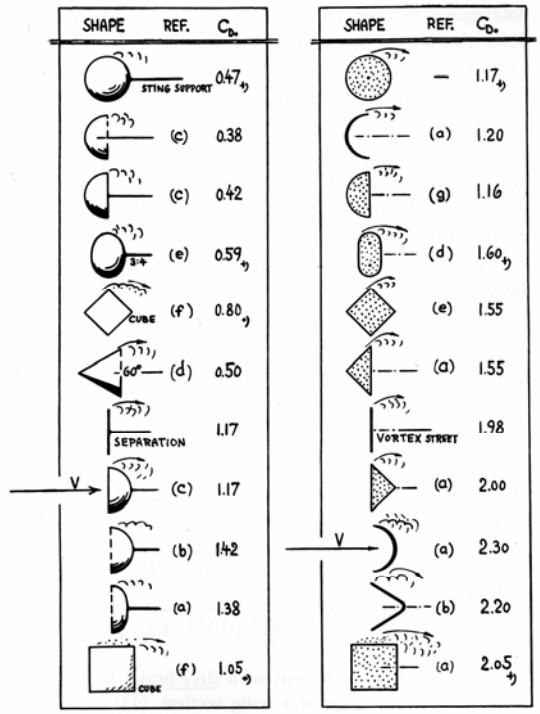
This gives us understanding as to the factors which affect the drag coefficient. Recall that in functional notation, it can be written:

$$C_D = C_D \left(Re, M \right) \tag{5.9}$$

Later on, we will show how C_D also varies with the orientation of the body in the flow (i.e., incidence angle or angle of attack). But for now, it is important to note that the drag coefficient is NOT a function of anything other than the Reynolds and Mach numbers. Thus, if we put two objects having the same <u>shape</u> in a flow, then the drag coefficients will be equal if we match Re and M. This is a necessary and sufficient condition. It doesn't require us to match the velocity, or the density, or even the size in order to obtain the same value of the drag coefficient. By this basic principle, wind tunnel tests are conducted on sub-scale models, and reliable full-scale aerodynamic drag coefficients are obtained. The only condition is that the Reynolds and Mach numbers be matched.

In this lab, we'll measure the area, the velocity, and the drag so that the drag coefficient can be determined using graphical means. Should we be interested in a single-point calculation, we could solve for C_D:

$$C_D = \frac{D}{qS} \tag{5.10}$$



• Figure 4. Drag on basic shapes with laminar flow[9]

Procedure

Follow these steps to determine the variation of drag with dynamic pressure:

1) Apply AC power to the control computer of the Educational Wind Tunnel (EWT). Let the electronics "warm up" for at least 30 minutes before operating the wind tunnel.
2) Record the barometric pressure and the room temperature.
3) Measure the diameter of each object. This will be our representative length, ℓ.
4) Install the flat disk model on the mini-sting force balance in the test section. Do not over-tighten the set screw!
5) Hit the button to "zero out" the balance readouts for velocity, normal force, axial force, and pitching moment.
6) After clearing the area immediately in front of the inlet and behind the exhaust, turn on the VFD (variable frequency drive) of the motor controller. By using the slider control, bring up the tunnel speed manually to 60% speed.
7) Let the flow through the test section stabilize and make a reading of the axial force on the model. Do your best to get an "eyeball average" from the digital display. This will be your drag measurement (in lbs).
8) Repeat step 7) in speed increments of 10%, up to 100% (or maximum speed).
9) Repeat steps 4) through 8) for the cup, the smooth sphere, and the "tripped" sphere models.

[9] 3D shapes are shown on the left half of the figure, and 2D shapes are shown on the right

Data Reduction

Your data reduction should involve the following calculations:

1) Calculate the reference area, S, for the models. Use the cross-sectional (or frontal) area for this area.
2) Calculate the air density in slug/ft³.
3) Tabulate your raw data for each model: the model geometry, the indicated test section velocity, and the measured drag.
4) Tabulate your calculated data for each model: the true test section velocity, the dynamic pressure, and the Reynolds number based on ℓ. You may combine the two data tables.
5) For the disk and cup models, make a separate plot of D as a function of q using the proper units. For the smooth and tripped spheres, put both models on the same plot. Fit an appropriate trendline or curvefit through your data and force it to pass through $(0,0)$ (What is the most appropriate trendline for this relationship, theoretically?). Make sure you print out the value of the slope on your plot. The value of the slope for this curvefit is equal to the product of the area and the averaged drag coefficient for the model, $\bar{C}_D \cdot S$. Using your plots, determine $\bar{C}_D$ for each model. You should review the graphics conventions to ensure your plots are properly formatted.
6) Make a table comparing the averaged drag coefficients for each model. Present the data in two ways: as a value of $\bar{C}_D$ and as a percentage of $\bar{C}_{D_{disk}}$. Compare both of these with the benchmark values presented in Figure 4; calculate the percent deviation from the benchmark where possible.

Deliverables

Although the data were gathered in teams, each write-up must be done and submitted on an individual basis. You must hand in a formal **RESULTS** write-up for this experiment, which must include the following:

- The plots required. You should present the plots within the text of your write-up.
- A final $\bar{C}_D$ data table containing the comparisons. Place this within the text of your write-up.
- Raw and calculated data tables should be appended as an appendix. This may be your hand-written results from the day of the experiment. Note that your data is effectively presented in the plots, so duplicate data tables aren't necessary unless you're going to point out the numbers.
- A sample calculation for one data point, appended as an appendix. This should match your graphical results, and may be hand-written as well.
- In the discussion of your results, you should focus on the following (broken up roughly into paragraphs):
- What measurements were made? Make reference to your data tables (even if they're in the appendices). What is presented? How did you arrive at those numbers?
- Make reference to and discuss the meaning of your plots. What is presented? What trends are observed? Why do the different shapes have different drag coefficients? By the values of the Reynolds numbers, what type of flow might you expect over these objects?
- Which shape has the highest drag coefficient? Why?
- Which shape has the lowest drag coefficient? Why?
- What is the effect of putting a wire loop on the front of the sphere? Why? Is there any significance of the range of Reynolds numbers tested?

- Your discussion of the drag coefficients should involve the relative balance between the two contributions of drag discussed in class.
- Can you estimate the <u>precision</u> of your velocity and drag measurements? In other words, how much were the measurements varying when you did your "eyeball average?" What affect does this have on the final $\bar{C}_D$ numbers?

• Table 4. Raw data table for drag coefficient experiment

Date & Time: _____ Barometric Pressure: _____

Reference Area: _____ Room Temperature: _____

Data Point No.	Model Installed	Motor Power (percent)	Indicated Speed (mph)	Drag (lbs)	True Speed (fps)	q (psf)	Reynolds Number
1	Flat Disk	0	0	0	0	0	0
2							
3							
4							
5							
6							
7							
8							
9							
10							
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