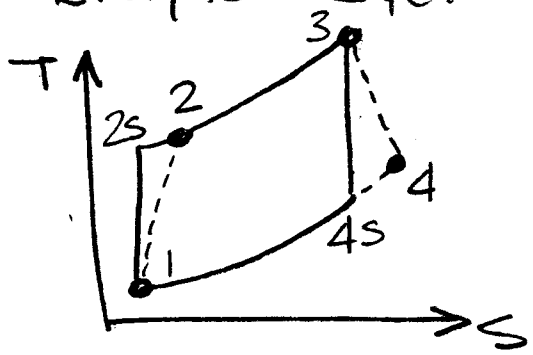
Zucker

$\boxed{12.3}$

Brayton Cycle



$T_1 = 0°C = 273K$

$P_1 = 1$ bar

$T_3 = 1250$ K

$$P_r = \frac{P_2}{P_1} = 10 = \left(\frac{T_{2s}}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$

$$T_{2s} = T_1 P_r^{\frac{\gamma-1}{\gamma}} = 273 \cdot 10^{0.4/1.4}$$

$$= \underline{527K}$$

Also $\frac{T_3}{T_{4s}} = \frac{T_{2s}}{T_1}$

$$T_{4s} = \frac{T_3 T_1}{T_{2s}} = \frac{(1250)(273)}{527} = \underline{647\ K}$$

$$W_c = \frac{C_p(T_{2s} - T_1)}{\eta_c} = \frac{1004\,(527-273)}{0.90}$$

$$= 2.834 \times 10^5\ J/kg$$

$$W_T = C_p \eta_T (T_3 - T_{4s}) = 1004(0.9)(1250-647)$$

$$= 5.445 \times 10^5\ J/kg$$

$$W_{net} = W_T - W_c = \boxed{2.610 \times 10^5\ J/kg}$$

$$\dot{W}_{net} = w_{net}\,\dot{m} = 1\times10^7\ W$$

$$\dot{m} = \frac{1\times10^7\ W}{2.610\times10^5\ J/kg} = \boxed{38.3\ kg/sec}$$

$$q_{in} = c_p(T_3 - T_2)$$

$$= c_p(T_3 - T_1) - c_p(T_2 - T_1)$$

$$= c_p(T_3 - T_1) - \frac{c_p(T_{2s} - T_1)}{\eta_c}$$

$$= c_p(T_3 - T_1) - w_c$$

$$= 1004(1250 - 273) - 2.834\times10^5\ J/kg$$

$$= 9.809\times10^5 - 2.834\times10^5$$

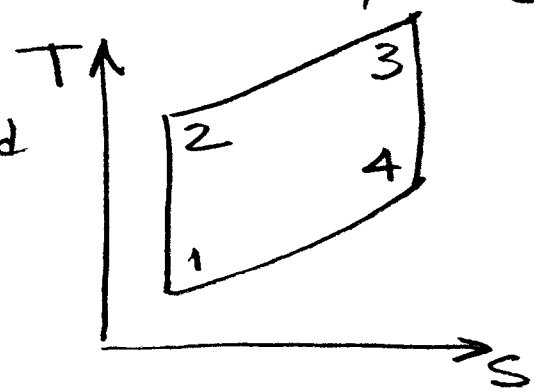
$$= \underline{6.975\times10^5}\ J/kg$$

$$\eta = \frac{w_{net}}{q_{in}} = \frac{2.610\times10^5}{6.975\times10^5} = \boxed{0.374}$$

Note: All these temperatures are really <u>TOTAL</u> temps.

$\boxed{12.5}$

Ideal Brayton Cycle



Let $\alpha = \dfrac{T_{T3}}{T_{T1}}$ and

$\theta = \left(\dfrac{P_{T2}}{P_{T1}}\right)^{\frac{\gamma-1}{\gamma}}$

$$W_{net} = W_t - W_c$$

$$= c_p(T_{T3} - T_{T4}) - c_p(T_{T2} - T_{T1})$$

$$= c_p T_{T1}\left[\left(\frac{T_{T3}}{T_{T1}} - \frac{T_{T4}}{T_{T1}}\right) - \left(\frac{T_{T2}}{T_{T1}} - 1\right)\right]$$

$$= c_p T_{T1}\left[\left(\frac{T_{T3}}{T_{T1}} - \frac{T_{T4}}{T_{T3}} \cdot \frac{T_{T3}}{T_{T1}}\right) - \left(\left(\frac{P_{T2}}{P_{T1}}\right)^{\frac{\gamma-1}{\gamma}} - 1\right)\right]$$

But $\dfrac{T_{T4}}{T_{T3}} = \dfrac{T_{T1}}{T_{T2}} = \left(\dfrac{P_{T2}}{P_{T1}}\right)^{-\frac{\gamma-1}{\gamma}} = \theta^{-1}$

So... $W_{net} = c_p T_{T1}\left[\left(\alpha - \dfrac{\alpha}{\theta}\right) - (\theta - 1)\right]$

$$= c_p T_{T1}\left[\alpha\left(1 - \frac{1}{\theta}\right) - (\theta - 1)\right]$$

$$= c_p T_{T1}\left[\alpha\frac{(\theta - 1)}{\theta} - (\theta - 1)\right]$$

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Thus... $w_{net} = C_p T_{T_1} (\theta - 1) \left[\frac{\alpha}{\theta} - 1 \right]$

$$= C_p T_{T_1} \frac{(\theta - 1)}{\theta} \left[\alpha - \theta \right]$$

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Saarlas

$\boxed{4.5}$ Simple turbojet

$$P_r = 6 = \frac{P_{T2}}{P_{T1}}$$

$$T_1 = 40°F = 500°R$$

$$T_3 = 1240°F = 1700°R$$

IDEAL CYCLE

$$P_r = \frac{P_{T2}}{P_{T1}} = \left(\frac{T_{T2s}}{T_{T1}}\right)^{\frac{\gamma}{\gamma-1}}$$

$$T_{T2s} = T_{T1}\, P_r^{\frac{\gamma-1}{\gamma}} = 500 \cdot 6^{\frac{\gamma-1}{\gamma}} = \underline{834\,°R}$$

$$\frac{T_{T3}}{T_{T4s}} = \frac{T_{T2s}}{T_{T1}}$$

$$T_{T4s} = \frac{T_{T3}\,T_{T1}}{T_{T2s}} = \frac{(1700)(500)}{834} = \underline{1019\,°R}$$

a) $$W_{Cs} = C_p(T_{T2s} - T_{T1}) = 6006(834-500)$$

$$= 2.008 \times 10^6 \ \frac{ft\text{-}lb}{slug}$$

$$W_{Ts} = C_p(T_{T3} - T_{T4s}) = 6006(1700-1019)$$

$$= 4.091 \times 10^6 \ \frac{ft\text{-}lb}{slug}$$

$$W_{net_s} = W_{Ts} - W_{Cs} = \boxed{2.083 \times 10^6 \ \frac{ft\text{-}lb}{slug}}$$

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$$q_{ins} = c_p(T_{T3} - T_{T2s})$$
$$= 6006(1700 - 834) = 5.200 \times 10^6 \ \frac{ft\text{-}lb}{slug}$$

$$\eta_{ideal} = \frac{w_{net,s}}{q_{ins}} = \frac{2.083 \times 10^6}{5.200 \times 10^6} = \boxed{0.401}$$

b) To maximize w_{net},

$$T_c^* = \sqrt{\bar{T}} = \sqrt{\frac{1700}{500}} = 1.844$$

$$P_r^* = T_c^{*\,\gamma/\gamma-1} = \boxed{8.51} \text{ ideal}$$

c) $$\bar{w}_{net} = \frac{w_{net}}{c_p T_{T1}} = \left(\frac{T_c - 1}{T_c}\right)(\bar{T} - T_c)$$

$$= \left(\frac{1.844 - 1}{1.844}\right)(3.400 - 1.844)$$

$$= \boxed{0.7122}$$

$$w_{net,max} = 6006(500)(0.7122)$$

$$= \boxed{2.139 \times 10^6 \ \frac{ft\text{-}lb}{slug}}$$

max. ideal work

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d) $\eta_c = 0.86$, $\eta_t = 0.90$

for $P_r = 6$, $T_c = P_r^{\frac{\gamma-1}{\gamma}} = 6^{\frac{0.4}{1.4}} = 1.669$

$$\eta = \frac{\left[1 - \frac{1}{T_c}\right]\left[\eta_c \eta_T \bar{T} - T_c\right]}{1 + \eta_c(\bar{T}-1) - T_c}$$

$$= \frac{\left(1 - \frac{1}{1.669}\right)(0.86 \cdot 0.90 \cdot 3.400 - 1.669)}{1 + 0.9(3.4-1) - 1.669}$$

$$= \boxed{0.277}$$

e) To maximize W_{net},

$$T_c^* = \sqrt{\eta_c \eta_T \bar{T}} = \sqrt{0.86(0.90)(3.4)}$$

$$= 1.622$$

$$P_r^* = T_c^{*\,\gamma/\gamma-1} = \boxed{5.44}$$ real cycle

f) $$\bar{W}_{net} = \eta_T \bar{T}\left(1 - \frac{1}{T_c}\right) - \frac{(T_c - 1)}{\eta_c}$$

$$= 0.90(3.4)\left(1 - \frac{1}{1.622}\right) - \frac{(1.622-1)}{0.86}$$

$$= \boxed{0.4502}$$

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$$W_{net,max} = 6006(500)(0.4502)$$

$$= \boxed{1.352 \times 10^6 \ \frac{ft\ lb}{slug}}$$

max. real work

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$\boxed{4.7}$

$T_{T_1} = 4°C = 277\,K = 499\,°R$

$P_1 = 14.7$ psia

$T_{T2} = 656\,°F = 1116\,°R$

$P_2 = 161.7$ psia

$\eta_c = 0.80$

$T_{T3} = 2{,}300\,°R$

$$W_{net} = \frac{150\ HP}{lbm/sec}\left(\frac{550\ \frac{ft\,lb}{sec}}{HP}\right)\left(\frac{32.2\ lbm}{slug}\right)$$

$$= \underbrace{2.657 \times 10^6}\ \frac{ft\,lb}{slug} = W_T - W_C$$

$$W_C = C_p(T_{T2} - T_{T1}) = 6006(1116 - 499) = 3.706 \times 10^6\ \frac{ft\,lb}{slug}$$

$$W_T = W_{net} + W_c = 2.657 \times 10^6 + 3.706 \times 10^6$$

$$= 6.362 \times 10^6\ ft\,lb/slug$$

$$\frac{T_{T3}}{T_{T4s}} = \frac{T_{T2}}{T_{T1}} = P_r^{\frac{\gamma-1}{\gamma}} = \left(\frac{161.7}{14.7}\right)^{\frac{\gamma-1}{\gamma}} = 1.984$$

$$T_{T4s} = 2300\,°R / 1.984 = 1159\,°R$$

$$6.362 \times 10^6 = (6006)\eta_T(2300 - 1159)$$

Solve for $\boxed{\eta_T = 0.929}$

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$\boxed{4.12}$

$$T_{T_1} = 300\,K$$
$$T_{T_3} = 1800\,K$$
$$\eta_C = \eta_T = 0.85$$
$$P_r = 30$$

$$\bar{T} = \frac{T_{T3}}{T_{T1}} = \frac{1800}{300} = 6$$

$$\frac{T_{T2S}}{T_{T1}} = \left(\frac{P_{T2}}{P_{T1}}\right)^{\frac{\gamma-1}{\gamma}} = P_r^{\frac{\gamma-1}{\gamma}}$$

$$T_{T2S} = 300(30)^{\frac{\gamma-1}{\gamma}} = 793\,K$$

$$w_C = \frac{C_p(T_{T2S} - T_{T1})}{\eta_C}$$

$$= \frac{1004(793-300)}{0.85} = 5.821 \times 10^5 \ \frac{J}{kg}$$

$$\frac{T_{T3}}{T_{T4S}} = \frac{T_{T2S}}{T_{T1}} = P_r^{\frac{\gamma-1}{\gamma}}$$

$$T_{T4S} = T_{T3}\, P_r^{\frac{1-\gamma}{\gamma}} = 1800(30)^{\frac{1-\gamma}{\gamma}}$$
$$= 681\,K$$

$$w_T = C_p \eta_T (T_{T3} - T_{T4S}) = 1004(0.85)(1800-681)$$
$$= 9.548 \times 10^5 \ J/kg$$

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$$w_{net} = w_t - w_c = 9.548\times10^5 - 5.821\times10^5$$

$$= \underline{3.728\times10^5}\ J/kg$$

$$P_{out} = \dot{W}_{net} = w_{net}\dot{m} = (3.728\times10^5)(100)$$

$$= \boxed{37.3\ MW}$$

37,300 kW
or
50,000 HP

$$BSFC = \frac{\dot{m}_f}{P_{out}} = \frac{\dot{m}_f}{\dot{W}_{net}}$$ But we don't know $\dot{m}_f$!

→ We could assume a value of f, or
→ We could calculate q_{in}.

$$w_c = C_p(T_{T2} - T_{T1})$$

$$T_{T2} = T_{T1} + \frac{w_c}{C_p} = 300 + \frac{5.821\times10^5}{1004} = 880\ K$$

$$q_{in} = C_p(T_{T3} - T_{T2}) = 1004(1800 - 880)$$

$$= \underline{9.239\times10^5}\ \frac{J}{kg}$$

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Thermal efficiency...

$$\eta_{th} = \frac{W_{net}}{Q_{in}} = \frac{3.728\times10^5}{9.239\times10^5} = \boxed{0.404}$$

And $\eta_{th} = \dfrac{\dot{W}_{net}}{\dot{Q}_{in}} = \dfrac{\dot{W}_{net}}{\dot{m}_f H_f}$

so $\dot{m}_f = \dfrac{\dot{W}_{net}}{\eta_{th} H_f}$ Assume $H_f = 43{,}000 \frac{kJ}{kg}$

$$\dot{m}_f = \frac{3.728\times10^7}{(0.404)(4.3\times10^7)} = 2.15 \text{ kg/sec}$$

Note: $f = \frac{2.15}{100} = 0.0215$ ✓ this is OK

$$BSFC = \frac{\dot{m}_f}{\dot{W}_{net}} = 5.756\times10^{-8} \frac{kg/sec}{W}$$

$$= 5.756\times10^{-8} \frac{kg}{sec} \cdot \frac{1}{W} \left| \frac{1000\ W}{kW} \right| \frac{3600\ sec}{hr} \Bigg|$$

$$= \boxed{0.207 \frac{kg}{kW\text{-}hr}}$$

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$\boxed{4.9}$ Simple turbojet (Extra credit)

$T_{T1} = 300\,K$

$T_{T3} = 1500\,K$

$\eta_c = \eta_T = 0.89$

$P_r = 25$

$$P_r = \frac{P_{T2}}{P_{T1}} = \left(\frac{T_{T2s}}{T_{T1}}\right)^{\frac{\gamma}{\gamma-1}}$$

$$T_{T2s} = T_{T1}\, P_r^{\frac{\gamma-1}{\gamma}} = 300(25)^{\frac{\gamma-1}{\gamma}}$$

$$= \underline{753\,K}$$

$$w_c = \frac{c_p(T_{T2s} - T_{T1})}{\eta_c} = \frac{1004\,(753-300)}{0.89}$$

$$= \boxed{5.105 \times 10^5\ J/kg}$$

And $w_c = c_p(T_{T2} - T_{T1})$

$$T_{T2} = T_{T1} + \frac{w_c}{c_p} = 300 + \frac{5.105\times10^5}{1004}$$

$$= \boxed{808\,K}$$

$$\frac{T_{T3}}{T_{T4s}} = \frac{T_{T2s}}{T_{T1}} = (P_r)^{\frac{\gamma-1}{\gamma}}$$

$$T_{T4s} = 1500(25)^{\frac{1-\gamma}{\gamma}} = \underline{598\,K}$$

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$$W_T = C_p \eta_T (T_{T3} - T_{T4s}) = 1004(0.89)(1500-598)$$

$$= \boxed{8.060 \times 10^5 \ \frac{J}{kg}}$$

And $W_T = C_p (T_{T3} - T_{T4})$

$$T_{T4} = T_{T3} - \frac{W_T}{C_p} = 1500 - \frac{8.060 \times 10^5}{1004}$$

$$= \boxed{697 K}$$

$$W_{net} = W_T - W_C = 8.060 \times 10^5 - 5.105 \times 10^5$$

$$= \boxed{2.955 \times 10^5 \ J/kg}$$

$$q_{in} = C_p (T_{T3} - T_{T2}) = 1004(1500-808)$$

$$= \boxed{6.943 \times 10^5 \ J/kg}$$

thus, $\eta_{th} = \frac{W_{net}}{q_{in}} = \frac{2.955 \times 10^5}{6.943 \times 10^5} = \boxed{0.4256}$

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$\boxed{4.14}$ Ideal ramjet

$M_0 = 2$

$P_0 = 0.2(14.7\,\text{psia}) = 2.94\,\text{psia}$

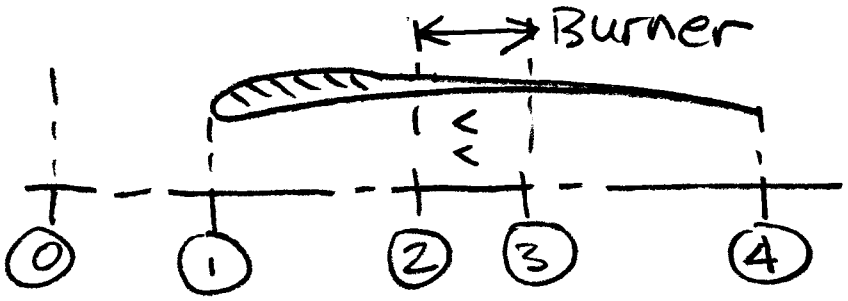
$T_0 = 0.752(519\,°R) = 390\,°R$

$$T_{T_0} = T_0\left(1 + \frac{\gamma-1}{2}M_0^2\right)$$

$$= 390\left(1 + \frac{2.0^2}{5}\right) = \underline{703\,°R}$$

$$a_0 = \sqrt{\gamma R T_0} = \sqrt{1.4(1716)(390)} = 968\,\text{fps}$$

So... $$\frac{T_{net}}{\dot{m}a_0} = M_0\left(\sqrt{\frac{T_{T_4}}{T_{T_0}}} - 1\right)$$



RAMJET

But $T_{T_4} = T_{T_3}$ (and $T_{T_2} = T_{T_0}$)

$$\frac{T_{T_4}}{T_{T_0}} = \frac{T_{T_3}}{T_{T_2}} = \frac{2807\,°R}{703\,°R} = 3.996$$

$$\frac{T_{net}}{\dot{m}\, a_0} = 2.0\left(\sqrt{3.996} - 1\right) = 1.998$$

$$\frac{T_{net}}{\dot{m}} = (1.998)(968\ \text{fps}) = 1{,}935\ \text{fps}$$

$\dot{m} = \rho_0 V_0 A_c$ — by def'n of capture area

$$\rho_0 = \frac{P_0}{RT_0} = \frac{2.94 \times 144}{1716(390)} = 6.321 \times 10^{-4}\ \frac{\text{slug}}{\text{ft}^3}$$

$$V_0 = M_0 a_0 = 2(968) = 1937\ \text{fps}$$

$$\dot{m} = (6.321 \times 10^{-4})(1937)(1.0\ \text{ft}^2) = \boxed{1.224\ \frac{\text{slug}}{\text{sec}}}$$

$$T_{net} = \left(1.224\ \frac{\text{slug}}{\text{sec}}\right)(1{,}935\ \text{fps}) = \boxed{2{,}368\ \text{lbs}}$$

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