Gas Dynamics and Thermodynamics
Review Problems
Assignment 1 (EA429 and EA365)

1. An airplane flies at 600 mph (880 ft/sec) at an altitude where the temperature is 60 F and the pressure is 10 psia. What temperatures will the nose of the aircraft and leading edge of any blunt surfaces approach?

Given the following:

Flight velocity: $Vinf := 600 \cdot \frac{mi}{hr}$ $\quad Vinf = 880 \frac{ft}{sec}$

Atmospheric conditions: $Tinf := (459.67 + 60) \cdot R$ $\quad Tinf = 519.67\ R$

$T_K := Tinf$

$R = 0.556\ K$ $\quad ratio := \frac{1}{R}$ $\quad ratio = 1.8 \frac{1}{K}$

Fluid (air) properties: $cp := 0.24 \cdot \frac{BTU}{lb \cdot R}$ $\quad cp = 1004.832 \frac{J}{kg \cdot K}$ $\quad \gamma := 1.4$

$Rair := 53.3 \cdot \frac{ft \cdot lbf}{lb \cdot R}$ $\quad Rair = 286.771 \frac{N \cdot m}{kg \cdot K}$

Relationships: $h_t = h + \frac{V^2}{2}$ $\quad M = \frac{V}{a}$ $\quad h = c_p \cdot T$

Solution:

Method 1: Static enthalpy is: $h := cp \cdot Tinf$

Total enthalpy is: $ht := h + \frac{Vinf^2}{2}$

Stagnation temperature is: $Tt := \frac{ht}{cp}$ $\quad Tt = 584.108\ R$

$Tt = 324.505\ K$

Method 2: $a := \sqrt{\gamma \cdot Rair \cdot Tinf}$ $\quad a = 1116.977 \frac{ft}{sec}$ $\quad a = 340.455 \frac{m}{s}$

$Minf := \frac{Vinf}{a}$ $\quad Minf = 0.788$

$Tt := Tinf \cdot \left(1 + \frac{\gamma - 1}{2} \cdot Minf^2\right)$ $\quad Tt = 584.181\ R$ $\quad Tt = 324.545\ K$

Method 3: At Minf = 0.79 we have T/Tt ratio of: $T_Tt := 0.889$

So the stagnation temperature is: $Tt := \frac{Tinf}{T_Tt}$

$Tt = 324.753\ K$

2. Show that the maximum velocity of the gas from a chamber with (high) temperature T_o is:

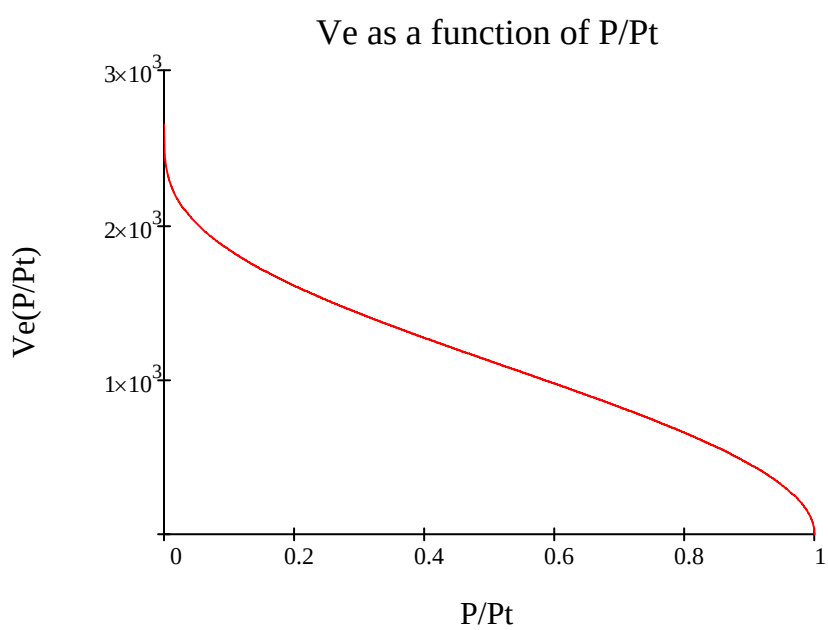
$V = \sqrt{2 \cdot cp \cdot T_t}$

This problem was discussed in class and a velocity equation was derived as a function of total temperature, pressure and exit pressure. This equation is plotted below:

$Pe_to_Pt := 0, 0.001 .. 1$ $Tt := 3500 \cdot K$

$$Ve(Pe_to_Pt) := \sqrt{\frac{2 \cdot \gamma}{\gamma - 1} \cdot Rair \cdot Tt \cdot \left(1 - Pe_to_Pt^{\frac{\gamma - 1}{\gamma}}\right)}$$

$Ve(0) = 2650.64\ \frac{m}{s}$



3. A smooth 3 in diameter hole is punched into the side of a large chamber where Nitrogen is stored at 500 R and 150 psia. Assume frictionless flow.

a. Compute the initial mass flow rate from the chamber if the surrounding pressure is 15 psia
b. What is the flow rate if the pressure in the surroundings is reduced to 0 psia?
c. What is the flow rate if the chamber pressure is raised to 300 psia?

Given the following:

From the problem statement:

Hole diameter: $Dh := 3 \cdot in$

Tank conditions $Tt := 500 \cdot R$ $Pt1 := 150 \cdot psi$ $Pt2 := 300 \cdot psi$

Back pressure: $Patm1 := 15 \cdot psi$ $Patm2 := 0 \cdot psi$

Fluid properties of interest:

$\gamma := 1.4$ $M_wt := 28.02 \cdot \frac{kg}{kmol}$ $R_u := 8314 \cdot \frac{N \cdot m}{kmol \cdot K}$

$R_u = 3406.718 \frac{ft \cdot lbf}{kmol \cdot R}$ $R_{N2} := \frac{R_u}{M_wt}$ $R_{N2} = 55.149 \frac{ft \cdot lbf}{lb \cdot R}$

$R_{N2} = 296.717 \frac{N \cdot m}{kg \cdot K}$

Relationships: Isentropic relations $mass_flow = p \cdot A \cdot M \cdot \sqrt{\frac{\gamma}{R \cdot T}}$

Solution: Assuming the conditions in the tank are stagnation conditions we must first examine P/Pt. For the ratios given we have:

Since: $$\frac{P}{Pt} = \left(1 + \frac{\gamma - 1}{2} \cdot M^2\right)^{\frac{\gamma}{\gamma - 1}}$$

We can solve for M

$$M := \sqrt{\frac{2}{\gamma - 1} \cdot \left[\left(\frac{Pt1}{Patm1}\right)^{\frac{\gamma - 1}{\gamma}} - 1\right]} \qquad M = 2.157$$

In the tables this is tabulated as P/Pt (a ratio 1 or < 1):

$$ratio := \frac{Patm1}{Pt1} \qquad ratio = 0.1$$

From the tables we have the Mach number is 2.15

Since the hole in the side of the tank does not have a diverging section the flow cannot attain supersonic Mach numbers at the plane of the hole exit. Therefore we can assume the "nozzle" is choked. What about the higher tank pressure or the lower atmospheric (back) pressure?

For choked flow the relationship between Total and Static conditions are set for a give set of gas properties (γ, R_{gas} etc.) since M = 1.

$$Pstar_to_Pt := \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma}{\gamma - 1}} \qquad Pstar_to_Pt = 0.528$$

$$Tstar_to_Tt := \left(\frac{2}{\gamma + 1}\right) \qquad Tstar_to_Tt = 0.833$$

Part a. With these relations we can calculate the static conditions at the throat (the * or reference conditions), but why do that at all when we can just substitute P/Pt and T/Tt multiplied by total pressure and temperature respectively into the mass flow equation given above?

$$Astar := \frac{\pi}{4} \cdot Dh^2 \qquad Astar = 0.005\ m^2$$

$$mdot := Pstar_to_Pt \cdot Pt1 \cdot Astar \cdot \sqrt{\frac{\gamma}{R_{N2} \cdot Tstar_to_Tt \cdot Tt}}$$

$$mdot = 24.8\ \frac{lb}{sec} \qquad mdot = 11.249\ \frac{kg}{s}$$

Part b. Changing the back pressure will have no effect on the mass flow for a choked flow.

Part c. How should doubling the tank pressure effect the mass flow?

$$mdot := Pstar_to_Pt \cdot Pt2 \cdot Astar \cdot \sqrt{\frac{\gamma}{R_{N2} \cdot Tstar_to_Tt \cdot Tt}}$$

$$mdot = 49.599 \frac{lb}{sec} \qquad mdot = 22.498 \frac{kg}{s}$$

Note: For english units you must use the conversion factor g_c. This gives consistent units inside the radical.

$$g_c := 32.17 \cdot \frac{lb \cdot ft}{lbf \cdot sec^2}$$

also note that for SI units, g_c is unity (1)

$$g_c = 1$$

4. Oxygen flows into an insulated and frictionless device with the following initial conditions: p_1 = 30 psia, T_1 = 750 R, V_1 = 639 ft/sec and the area changes from A_1 = 6 ft^2 to A_2 = 5 ft^2.

a. Determine M_1, p_{o1}, T_{o1}

b. Is this device a nozzle or a diffuser?

Given the following:

Flight velocity: $V_1 := 639 \cdot \frac{ft}{sec}$ $\quad V_1 = 194.767 \frac{m}{sec}$

Atmospheric conditions: $T_1 := 750 \cdot R$ $\quad P_1 := 30 \cdot psi$

Fluid properties: $cp := 0.218 \cdot \frac{BTU}{lb \cdot R}$ $\quad cp = 912.722 \frac{J}{kg \cdot K}$ $\quad \gamma := 1.4$

$R_{O2} := 48.3 \cdot \frac{ft \cdot lbf}{lb \cdot R}$ $\quad R_{O2} = 259.869 \frac{N \cdot m}{kg \cdot K}$

Relationships: $h_t = h + \frac{V^2}{2}$ $\quad M = \frac{V}{a}$ $\quad h = c_p \cdot T$

Varying area, adiabatic flow equations

Solution: Part a. $a := \sqrt{\gamma \cdot R_{O2} \cdot T_1}$ $\quad a = 1277.383 \frac{ft}{sec}$ $\quad a = 389.346 \frac{m}{s}$

Mach number: $M_1 := \frac{V_1}{a}$ $\quad M_1 = 0.5$

Total temperature: $Tt1 := T_1 \cdot \left(1 + \frac{\gamma - 1}{2} \cdot M_1^{\,2}\right)$ $\quad Tt1 = 787.536\ R$

Total pressure: $Pt1 := P_1 \cdot \left(1 + \frac{\gamma - 1}{2} \cdot M_1^{\,2}\right)^{\frac{\gamma}{\gamma - 1}}$ $\quad Pt1 = 35.592\ psi$

Part b. Since the flow is subsonic and the section is converging, it is a nozzle.

Since the area ratio A_1/A_2 is 1.2, what is M_2? For M_1 = 0.5 we find from the tables A/A* is 1.33984. So now we calculate A_2/A^*_2 to find M_2 and all the appropriate temperature/pressure ratios. You should do this and find:

$M_2 := 0.67$ and $V_2 := 800 \cdot \frac{ft}{sec}$

5. A large chamber contains air at 80 psia and 600 R. Air enters a Covergent-Divergent nozzle with an area ratio of 3.0.

a. What pressure ratio must exist in the receiver (exit) for the nozzle to operate at its first critical point?
b. What should the receiver pressure be for the third critical (design) point operation?
c. If operating at the third critical point what are the density and velocity of the air at the nozzle exit plane?
d. For what range of exit pressures can a shock be found inside the nozzle?

Given the following:

Nozzle/Chamber conditions: $Pt := 80 \cdot psi$ $Tt := 600 \cdot R$ $A_ratio := 3$

Relationships:

Varying area adiabatic flow relations and tables.

Solution:

Part a and b. There are two solutions for an area ratio of 3.0. Using the equation solver in Mathcad and the appropriate guess we can find either solution.

$M_{exit} := 2$

$AoverAstar := 3$

Given

$$AoverAstar = \frac{1}{M_{exit}} \cdot \left(\frac{1 + \frac{\gamma - 1}{2} \cdot M_{exit}^{2}}{\frac{\gamma+1}{2}} \right)^{\frac{\gamma+1}{2 \cdot (\gamma-1)}}$$

$M_{exit} := \mathrm{Find}(M_{exit})$

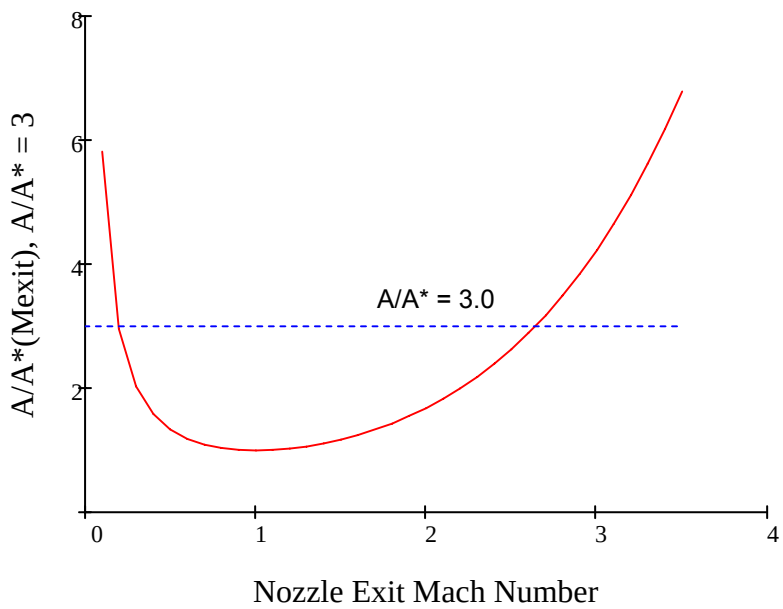
$M_{exit} = 2.637$ also for A/A* = 3, M_{exit} = 0.197

These two Mach numbers represent the conditions at the first and 3rd critical points in terms of nozzle exit conditions.

Another way to examine the problem is to simply plot the equation of a range of Mach numbers as shown below:

$$M_{exit} := 0, 0.1 .. 3.5$$

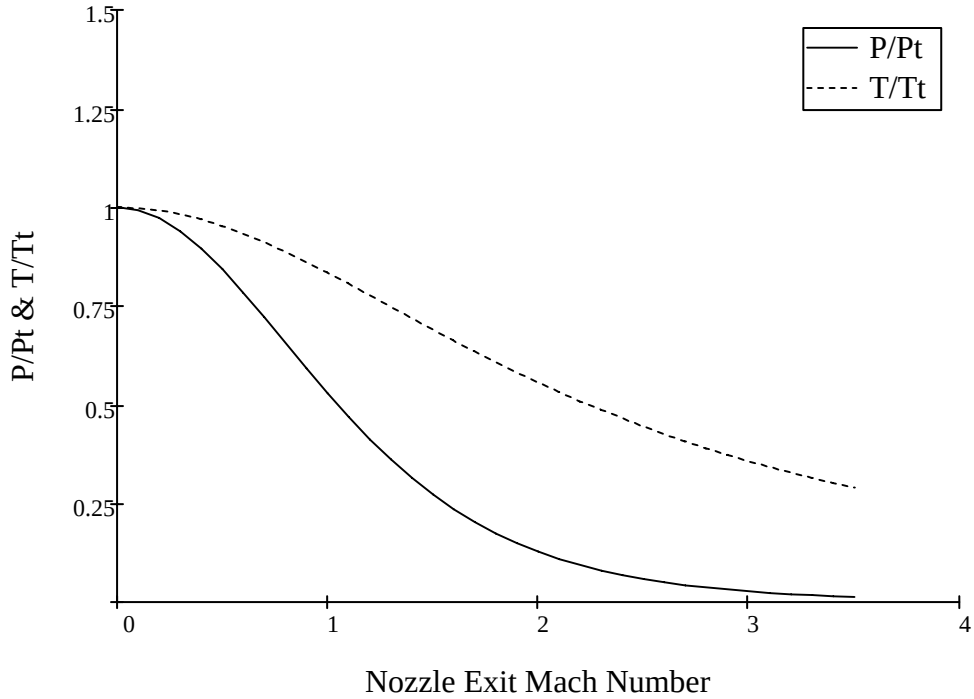
$$Astar_ratio(M_{exit}) := \frac{1}{M_{exit}} \cdot \left(\frac{1 + \frac{\gamma - 1}{2} \cdot M_{exit}^{2}}{\frac{\gamma+1}{2}} \right)^{\frac{\gamma+1}{2\cdot(\gamma-1)}}$$



To get the solution we need to use the isentropic tables to find the static pressures at the exit or perhaps the equations themselves as plotted below:

$$ToverTt(M_{exit}) := \frac{1}{1 + \frac{\gamma - 1}{2} \cdot M_{exit}^{2}}$$

$$PoverPt(M_{exit}) := \left(\frac{1}{1 + \frac{\gamma - 1}{2} \cdot M_{exit}^{2}} \right)^{\frac{\gamma}{\gamma-1}}$$



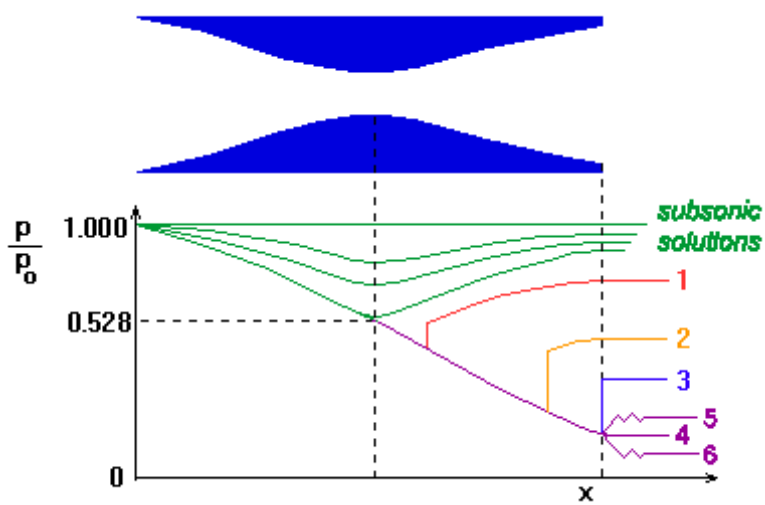


Figure 1. Typical C-D nozzle showing pressure variation with axial position for various regimes

Using either approach we find that the pressure for the first critical (venturi regime) is:

$M_{1st} := 0.197$

$$PoverPt := \left(\frac{1}{1 + \frac{\gamma - 1}{2} \cdot M_{1st}^{2}}\right)^{\frac{\gamma}{\gamma - 1}} \qquad PoverPt = 0.973$$

$P_{exit_1st} := PoverPt \cdot Pt \qquad P_{exit_1st} = 77.864\ psi$

The same calculation for the 3rd critical (design operation) is

$M_{3rd} := 2.64$

$$PoverPt := \left(\frac{1}{1 + \frac{\gamma - 1}{2} \cdot M_{3rd}^{2}}\right)^{\frac{\gamma}{\gamma - 1}} \qquad PoverPt = 0.047$$

$P_{exit_3rd} := PoverPt \cdot Pt \qquad P_{exit_3rd} = 3.769\ psi$

Part b. Now we must determine the density and velocity at the exit for the 3rd critical point.

Find the exit Temperature at the 3rd critical

$$ToverTt := \frac{1}{1 + \frac{\gamma - 1}{2} \cdot M_{3rd}^{2}} \qquad ToverTt = 0.418$$

$T_{exit} := Tt \cdot ToverTt \qquad T_{exit} = 250.635\ R \qquad R_{air} := 53.3 \cdot \frac{ft \cdot lbf}{lb \cdot R}$

Find the speed of sound based on T

$$a := \sqrt{\gamma \cdot R_{air} \cdot T_{exit}} \qquad a = 236.437\ \frac{m}{s}$$

Find velocity using the definition of the speed of sound

$$V_{exit} := M_{3rd} \cdot a \qquad V_{exit} = 2047.883\ \frac{ft}{sec}$$

Find density using the perfect gas law

$$\rho_{exit} := \frac{P_{exit_3rd}}{R_{air} \cdot T_{exit}} \qquad \rho_{exit} = 0.041\ \frac{lb}{ft^{3}}$$

Since we have identified the 1st and 3rd critical points in the nozzle we have also identified the pressures BETWEEN which a shock could be present inside or outside of the nozzle. Next we must identify the 2nd critical point telling us the point in a frictionless adiabatic nozzle when a shock is located exactly at the exit plane.

The way to find second critical point back pressure is to assume design operation UP TO the exit plane and then assume a shock located exactly at the exit. Using the design exit Mach number, the shock tables (or equations) and the associated pressure ratio we can find the the pressure just beyond the shock or the theoretical back pressure for the 2nd crit!

For an M_1 of 2.64 we can find the Mach number behind the shock using the following relationship or from normal shock tables as:

$$M_1 := 2.64$$

$$M_2 := \sqrt{\frac{M_1^{\,2} + \frac{2}{\gamma - 1}}{\left(\frac{2\cdot\gamma}{\gamma - 1}\right)\cdot M_1^{\,2} - 1}} \qquad M_2 = 0.5$$

With this Mach number and the pressure ratio across the shock we can find P_{2nd_crit} by multiplying the pressure of the 3rd critical times the pressure ratio from the equation shown below or the normal shock tables.

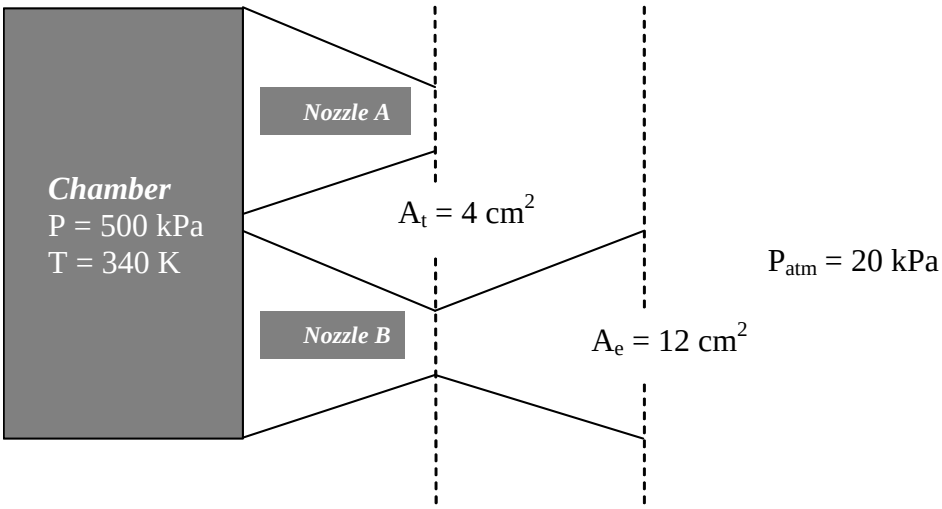
$$P_2 := P_{exit_3rd}\cdot\left[\left(\frac{2\cdot\gamma}{\gamma + 1}\right)\cdot M_1^{\,2} - \left(\frac{\gamma - 1}{\gamma + 1}\right)\right]$$

$$P_2 = 30.017 \text{ psi}$$

So now we know that for back pressures between 78 psi and 30 psi there is a shock inside the nozzle and for back pressures between 30 psi and 3.8 psi there is a shock outside the nozzle.

6. Given two air nozzle configurations A and B (assume frictionless and adiabatic) in **Figure 1** below find:

a. The exit Mach number for each configuration and
b. The nozzle with the largest mass flow and the magnitude of this mass flow.



Given the following:

Chamber conditions: $Pt := 500 \cdot 10^3 \cdot Pa$ $\quad Tt := 340 \cdot K$

Throat area: $A_{throat} := 4 \cdot cm^2$ *And the exit area for nozzle A*

Back pressure: $Patm := 20 \cdot 10^3 \cdot Pa$

Exit area for nozzle B: $A_{exit} := 12 \cdot cm^2$

Assumptions: Assume isentropic flow and use the isentropic table or relations to solve.

Solutions: For nozzle A and the B the ration between Patm and Pt or back pressure and chamber pressure is 0.04. For γ = 1.4 (air) this is WELL BELOW a ratio of 0.528 indicating the conditions at the nozzle throat are choked (M = 1).

Nozzle A: Since nozzle A is a converging only section the exit Mach number must be unity (1) and the exhaust velocity is exactly the speed of sound.

$T_{exit} := 0.833 \cdot Tt$ $\quad T_{exit} = 283.22\ K$

$$a := \sqrt{\gamma \cdot R_{air} \cdot T_{exit}} \qquad a = 337.205 \ \frac{m}{s}$$

Nozzle B: Nozzle B is a converging-diverging (C-D) and therefore will have an exit Mach number based on the area ratio.

$$Area_ratio := \frac{A_{exit}}{A_{throat}} \qquad Area_ratio = 3$$

As discovered in the problem 5 above, this corresponds to M_{exit} = 2.64 and an exit pressure of 23.5 kPa (P/Pt = 0.04711). Is this nozzle over or under-expanded?

The exit velocity can be found using the definition of stagnation enthalpy. From problem 5 above...

$$PoverPt = 0.047$$

$$V_{exit} := \sqrt{\frac{2 \cdot \gamma}{\gamma - 1} \cdot R_{air} \cdot Tt \cdot \left(1 - PoverPt^{\frac{\gamma - 1}{\gamma}}\right)}$$

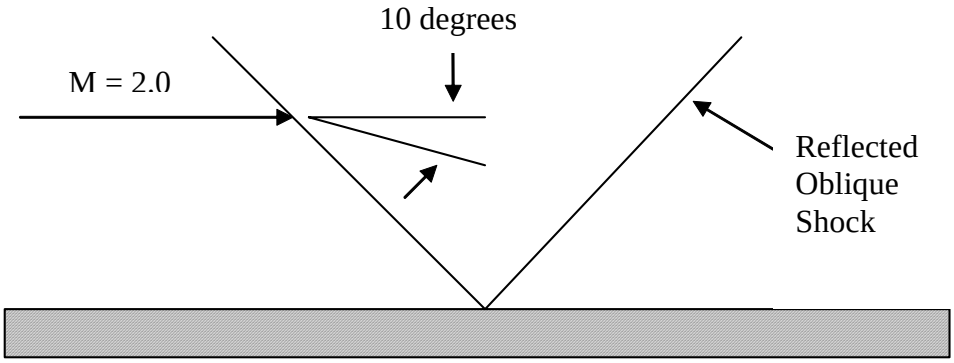
$$V_{exit} = 630.406 \ \frac{m}{s}$$

Mass flow for Nozzle A & B: Since both nozzles are choked, the mass flow must be identical for both configurations

$$mdot_{AB} := 0.685 \cdot \frac{Pt \cdot A_{throat}}{\sqrt{R_{air} \cdot Tt}}$$

$$mdot_{AB} = 0.439 \ \frac{kg}{s} \qquad mdot_{AB} = 0.967 \ \frac{lb}{s}$$

8. A Mach 2 flow passes through an oblique shock as shown and deflects 10 degrees. A second oblique shock reflects from the solid wall. What is the pressure ratio across the two-shock system? Assume that there is no boundary layer near the wall; that is, the flow is uniform in each of the regions bounded by the shocks (See Figure 2).



Given the following: $\delta := 10 \cdot deg$ $M_1 := 2.0$ $\gamma := 1.4$

Relationships: The oblique and normal shock charts and relations.

Assumptions: Here we assume the flow is uniform with a negligible bounday layer. Given the geometry we can assume the flow must conform with the flat plate.

Solution: The basic sequence here is to:

1. Determine the shock angle of the first shock.
2. Use the shock angle to find the Mach number normal to the first shock
3. Use the normal shock relations/tables to determine the outlet Mach number normal to the shock and the pressure (and temperature) ratio(s).
4. Use a similar relation as in 2 above, $\sin(\theta-\delta)$, to find the outlet Mach number
5. Repeat for shock 2.

a. Using the oblique shock chart - or - relationships we get θ_1.

Guess value: $\theta := 40 \cdot deg$

Given

$$\tan(\delta) = 2 \cdot \cot(\theta) \cdot \left[\frac{M_1^2 \cdot \sin(\theta)^2 - 1}{M_1^2 \cdot (\gamma + \cos(2 \cdot \theta)) + 2} \right]$$

$\theta_1 := \mathrm{Find}(\theta)$ $\theta_1 = 39.314\ deg$

From the chart you find θ ~ 40 deg.

b. Now we must find M_{1N}, and from the normal shock tables M_{2N} as well as the pressure ratios

Normal Mach number:

$$M_{1N} := M_1 \cdot \sin(\theta_1) \qquad M_{1N} = 1.267$$

Normal shock relations (tables work too!):

$$M_{2N} := \sqrt{\frac{M_{1N}^2 + \frac{2}{\gamma - 1}}{\left(\frac{2 \cdot \gamma}{\gamma - 1}\right) \cdot M_{1N}^2 - 1}} \qquad M_{2N} = 0.803$$

Pressure ratio across the first shock:

$$P2_to_P1 := \left[\left(\frac{2 \cdot \gamma}{\gamma + 1}\right) \cdot M_{1N}^2 - \left(\frac{\gamma - 1}{\gamma + 1}\right)\right]$$

$$P2_to_P1 = 1.707$$

Now we need the Mach number in the direction of flow (along the wall):

$$M_2 := \frac{M_{2N}}{\sin(\theta_1 - \delta)} \qquad M_2 = 1.641$$

Note the flow is still supersonic

c. Now since the flow must conform to the geometry because of our assumptions, δ must be 10 degrees for shock 2. First lets find the shock angle again as above.

Guess value: $\theta := 40 \cdot deg$

Given

$$\tan(\delta) = 2 \cdot \cot(\theta) \cdot \left[\frac{M_2^{\,2} \cdot \sin(\theta)^2 - 1}{M_2^{\,2} \cdot (\gamma + \cos(2 \cdot \theta)) + 2} \right]$$

$\theta_2 := \mathrm{Find}(\theta)$ $\qquad \theta_2 = 49.384\ deg$

Now we literally repeat the steps above. Tedious but very satisfying because we are all powerful!

d. Now we must find M_{1N}, and from the normal shock tables M_{2N} as well as the pressure ratios

Normal Mach number:

$M_{2N} := M_2 \cdot \sin(\theta_2)$ $\qquad M_{2N} = 1.245$

Normal shock relations (tables work too!):

$$M_{3N} := \sqrt{\frac{M_{2N}^{\,2} + \frac{2}{\gamma - 1}}{\left(\frac{2 \cdot \gamma}{\gamma - 1}\right) \cdot M_{2N}^{\,2} - 1}} \qquad M_{3N} = 0.815$$

Pressure ratio across the first shock:

$$P3_to_P2 := \left[\left(\frac{2 \cdot \gamma}{\gamma + 1}\right) \cdot M_{2N}^{\,2} - \left(\frac{\gamma - 1}{\gamma + 1}\right) \right]$$

$P3_to_P2 = 1.643$

Now we need the Mach number in the direction of flow (along the wall):

$$M_3 := \frac{M_{3N}}{\sin(\theta_2 - \delta)} \qquad M_3 = 1.285$$

Note the flow is still supersonic

e. The final pressure ratio is simply the product of the two pressure ratios from shock 1 and 2:

$$P3_to_P1 := P2_to_P1 \cdot P3_to_P2$$

$$P3_to_P1 = 2.803$$

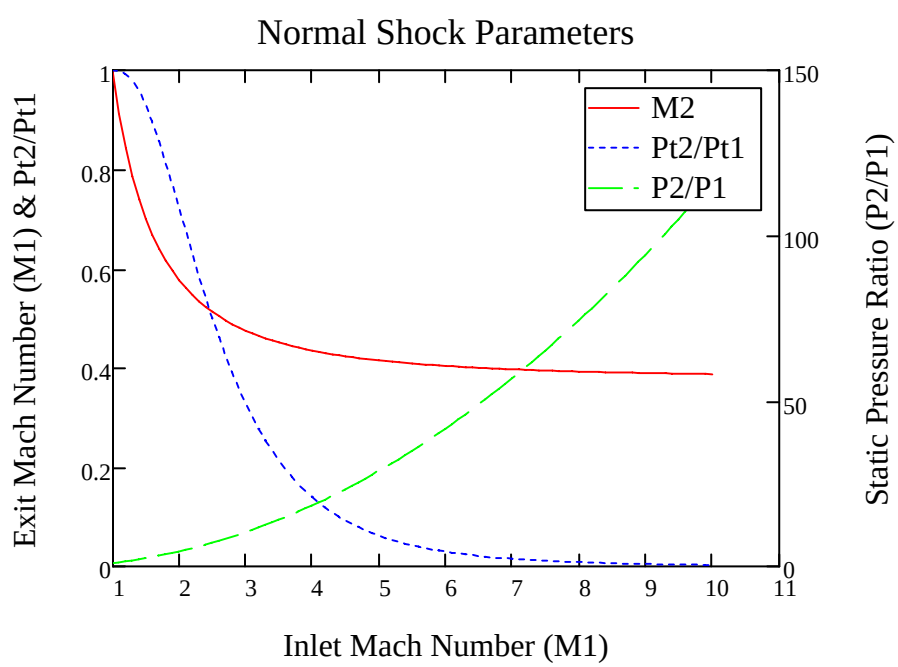
By the way lets look at the normal shock relation as a function of inlet Mach number alongside pressure ratio and total pressure ratio.

$$M_1 := 1, 1.1 .. 10$$

$$M_2(M_1) := \sqrt{\frac{M_1^{\,2} + \frac{2}{\gamma - 1}}{\left(\frac{2 \cdot \gamma}{\gamma - 1}\right) \cdot M_1^{\,2} - 1}}$$

$$P2_to_P1(M_1) := \left[\left(\frac{2 \cdot \gamma}{\gamma + 1}\right) \cdot M_1^{\,2} - \left(\frac{\gamma - 1}{\gamma + 1}\right)\right]$$

$$Pt2_to_Pt1(M_1) := P2_to_P1(M_1) \cdot \left(\frac{1 + \frac{\gamma - 1}{2} \cdot M_2(M_1)^2}{1 + \frac{\gamma - 1}{2} \cdot M_1^{\,2}}\right)^{\frac{\gamma}{\gamma - 1}}$$



9. Steam enters a turbine with an enthalpy of 1600 BTU/lbm and a velocity of 100 ft/sec, at a flow rate of 80,000 lbm/hr. The steam leaves the turbine with an enthalpy of 995 BTU/lbm and a velocity of 150 ft/sec. Compute the power output of the turbine assuming 100% efficiency. Neglect any heat transfer or potential energy changes.

Given the following:

Steam conditions at inlet: $h_1 := 1600 \cdot \frac{BTU}{lb}$ $V_1 := 100 \cdot \frac{ft}{sec}$

$m_{stm} := 80000 \cdot \frac{lb}{hr}$

Steam conditions at exit: $h_2 := 995 \cdot \frac{BTU}{lb}$ $V_2 := 150 \cdot \frac{ft}{sec}$

Assumptions: Assume steady adiabatic flow without losses and no change in potential (z).

Relationships: The steady flow energy equation

$$h_1 + \frac{V_1^2}{2} + z_1 + q = h_2 + \frac{V_2^2}{2} + z_2 + w$$

Solution: Q will be neglected since we assume adiabatic flow so we can solve directly for work in this case

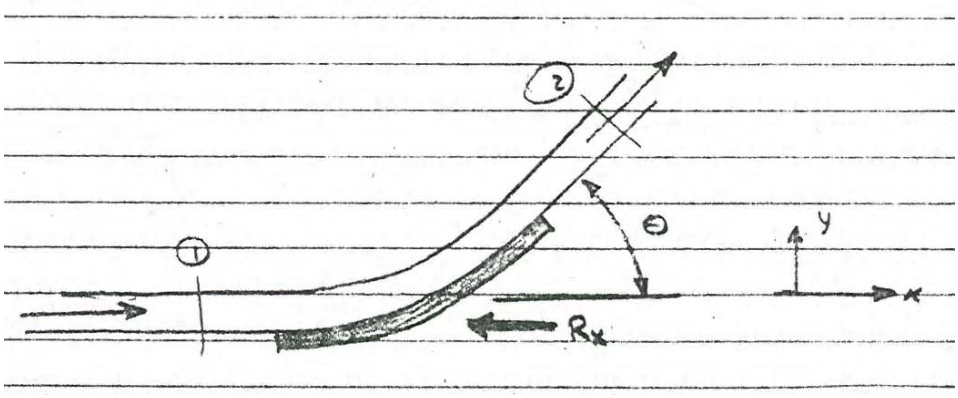
$$w := h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} \qquad w = 604.75 \frac{BTU}{lb} \qquad w = 470598.148 \frac{ft \cdot lbf}{lb}$$

In this context, to obtain power we multiply the work by mass flow rate:

$$P := m_{stm} \cdot w \qquad P = 4.838 \times 10^7 \frac{BTU}{hr}$$

$$P = 19014.067 \text{ hp}$$

10. A fluid jet strikes a vane and is deflected through an angle of θ as shown in figure 2. For a given jet (fluid, area and velocity constant) what deflection angle will cause the greatest axial or x-component of force between the fluid and vane? Assume incompressible flow with no friction along the vane. If the fluid is air at standard conditions, velocity is 300 m/sec and the streamtube diameter is 10 mm, what is the reaction force if the θ is 45 degrees?



Given the following: $\theta := 45 \cdot deg$ $\quad V_{jet} := 300 \cdot \frac{m}{sec}$ $\quad D_{jet} := 10 \cdot mm$

Asumptions: The flow is assumed to be incompressible with no friction. We will further assume the flow is adiabatic with no shaft work. What does this mean in terms of the conditions in the jet? How about applying the incompressible pressure-energy equation?

$$\frac{P_1}{\rho_1} + \frac{V_1^2}{2} + g \cdot z_1 = \frac{P_2}{\rho_2} + \frac{V_2^2}{2} + g \cdot z_2$$

Since the flow is exposed to the atmosphere the pressure must be constant. Since the potential rise in the flow is small we shall assume it is negligent. The only conclusion is that $V_1 = V_2$ and the only force exerted on the vane is from the change in momentum.

We are concerned only with the reaction in the x-direction. So the to obtain the force at (2) we multiply by the cosine.

$$F_x = -R_x = mdot \cdot \left(V_2 \cdot \cos(\theta) - V_1\right) \quad \text{where } V_1 = V_2 = V_{jet}$$

then

$$R_x = \rho \cdot A \cdot V_{jet}^2 \cdot (1 - \cos(\theta))$$

To find the maximum/minimum θ we must take the derivative with respect to θ and set equal to zero.

$$\frac{dR_x}{d\theta} = \rho \cdot A \cdot V_{jet}^{2} \cdot \sin(\theta) = 0$$

The maximum occurs with θ = 180 degrees and the mimumum occurs when θ = 0 degrees.

If we assume the fluid is water with a density of 1.225 kg/m^3...

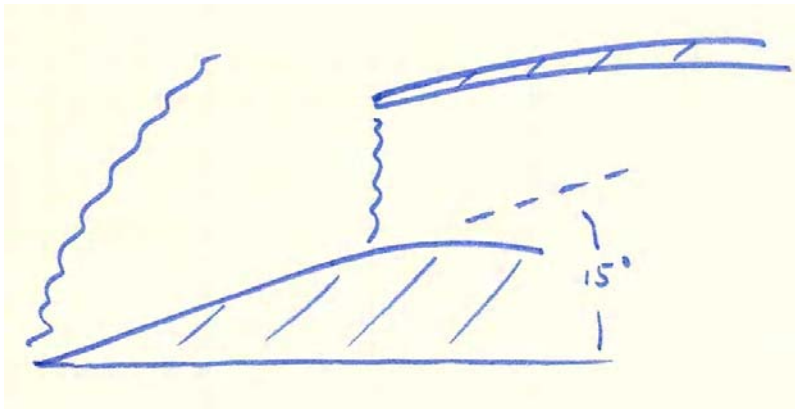
$$A_{jet} := \frac{\pi}{4} \cdot D_{jet}^{2} \qquad A_{jet} = 7.854 \times 10^{-5}\,m^2$$

$$\rho := 1.225 \cdot \frac{kg}{m^3} \qquad \theta := 45 \cdot deg$$

$$R_x := \rho \cdot A_{jet} \cdot V_{jet}^{2} \cdot (1 - \cos(\theta))$$

$$R_x = 2.536\,N \qquad R_x = 0.57\,lbf$$

11. A jet aircraft inlet operates at 50,000 ft at $M_0 = 2.5$. If the two-shock inlet has a wedge angle of 15 degrees such that an oblique shock is established on the leading edge and normal shock occurs at the inlet as shown below find the air pressure and temperature behind the normal shock.



Given the following: $\delta := 15 \cdot deg$ $\quad M_1 := 2.5$ $\quad \gamma := 1.4$

$P_1 := 243 \cdot \frac{lbf}{ft^2}$ $\quad T_1 := 392 \cdot R$

Relationships: The oblique and normal shock charts and relations.

Assumptions: Here we assume the flow is uniform with a negligible bounday layer. Given the geometry we can assume the flow must conform to the wedge shown after the oblique shock and then pass through the normal shock. The procedures is similar to the problem above.

Solution: The basic sequence here is to:

1. Determine the shock angle of the oblique shock.
2. Use the shock angle to find the Mach number normal to the oblique shock
3. Use the normal shock relations/tables to determine the outlet Mach number normal to the shock and the pressure (and temperature) ratio(s).
4. Use a similar relation as in 2 above, $\sin(\theta-\delta)$, to find the outlet Mach number
5. For the second shock we only need the normal shock relations .

a. Using the oblique shock chart - or - relationships we get θ_1.

Guess value: $\theta := 40 \cdot deg$

Given

$$\tan(\delta) = 2 \cdot \cot(\theta) \cdot \left[\frac{M_1^2 \cdot \sin(\theta)^2 - 1}{M_1^2 \cdot (\gamma + \cos(2 \cdot \theta)) + 2} \right]$$

$\theta_1 := \text{Find}(\theta)$ $\qquad \theta_1 = 36.945\ deg$

From the chart you find $\theta \sim 37$ deg.

b. Now we must find M_{1N}, and from the normal shock tables M_{2N} as well as the pressure ratios

Normal Mach number:

$$M_{1N} := M_1 \cdot \sin(\theta_1) \qquad M_{1N} = 1.503$$

Normal shock relations (tables work too!):

$$M_{2N} := \sqrt{\frac{M_{1N}^2 + \frac{2}{\gamma - 1}}{\left(\frac{2 \cdot \gamma}{\gamma - 1}\right) \cdot M_{1N}^2 - 1}} \qquad M_{2N} = 0.7$$

Pressure, Total Pressure and Temperature ratios across the oblique shock:

Static Pressure:

$$P2_to_P1 := \left[\left(\frac{2 \cdot \gamma}{\gamma + 1}\right) \cdot M_{1N}^2 - \left(\frac{\gamma - 1}{\gamma + 1}\right)\right]$$

$$P2_to_P1 = 2.468$$

Total Pressure:

$$Pt2_to_Pt1 := P2_to_P1 \cdot \left(\frac{1 + \frac{\gamma - 1}{2} \cdot M_{2N}^2}{1 + \frac{\gamma - 1}{2} \cdot M_{1N}^2}\right)^{\frac{\gamma}{\gamma - 1}}$$

$$Pt2_to_Pt1 = 0.929$$

Temperature ratio:

$$T2_to_T1 := \frac{1 + \frac{\gamma - 1}{2} \cdot M_{1N}^2}{1 + \frac{\gamma - 1}{2} \cdot M_{2N}^2}$$

$$T2_to_T1 = 1.322$$

Now we need the Mach number in the direction of flow (along the wedge):

$$M_2 := \frac{M_{2N}}{\sin(\theta_1 - \delta)} \qquad M_2 = 1.874$$

Note the flow is still supersonic

Finally, the flow must pass through the normal shock on the way into our imaginary inlet.

$$M_3 := \sqrt{\frac{M_2^{\,2} + \frac{2}{\gamma - 1}}{\left(\frac{2\cdot\gamma}{\gamma - 1}\right)\cdot M_2^{\,2} - 1}} \qquad M_3 = 0.601$$

And once again we need to find the pressure, total pressure and temperature ratios:

Static Pressure:

$$P3_to_P2 := \left[\left(\frac{2\cdot\gamma}{\gamma + 1}\right)\cdot M_2^{\,2} - \left(\frac{\gamma - 1}{\gamma + 1}\right)\right]$$

$$P3_to_P2 = 3.928$$

Total Pressure:

$$Pt3_to_Pt2 := P3_to_P2\cdot\left(\frac{1 + \frac{\gamma - 1}{2}\cdot M_3^{\,2}}{1 + \frac{\gamma - 1}{2}\cdot M_2^{\,2}}\right)^{\frac{\gamma}{\gamma - 1}}$$

$$Pt3_to_Pt2 = 0.78$$

Temperature ratio:

$$T3_to_T2 := \frac{1 + \frac{\gamma - 1}{2}\cdot M_2^{\,2}}{1 + \frac{\gamma - 1}{2}\cdot M_3^{\,2}}$$

$$T3_to_T2 = 1.587$$

So the final static pressureand temperature are:

$T_3 := T_1 \cdot (T3_to_T2 \cdot T2_to_T1)$ $\qquad T_3 = 822.601\ R$

$P_3 := P_1 \cdot (P3_to_P2 \cdot P2_to_P1)$ $\qquad P_3 = 2355.508\ \frac{lbf}{ft^2}$

$P_3 = 16.358\ psi$

Let's also analyze the total pressure losses and examine how this relates to the entropy rise (losses) across the shock two system.

$Pt3_to_Pt1 := Pt3_to_Pt2 \cdot Pt2_to_Pt1$ $\qquad Pt3_to_Pt1 = 0.724$

The total pressure ratio is related to entropy by the following statement:

$$\frac{Pt3}{Pt1} = e^{\frac{-\Delta s}{R_{gas}}}$$

So the change in entropy is:

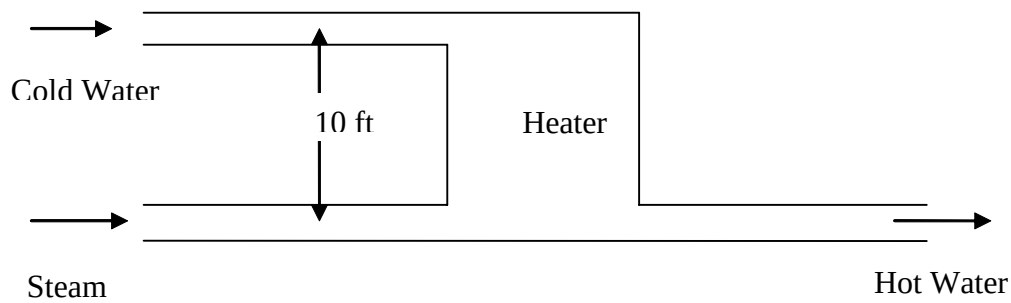
$\Delta s := -R_{air} \cdot \ln(Pt3_to_Pt1)$ $\qquad \Delta s = 0.022\ \frac{BTU}{lb \cdot R}$

The actual total pressure loss should be quantified as well since it is the initial condition in the inlet of the aircraft engine.

$Pt_1 := P_1 \cdot \left(1 + \frac{\gamma - 1}{2} \cdot M_1^2\right)^{\frac{\gamma}{\gamma - 1}}$ $\qquad Pt_1 = 28.833\ psi$

$Pt_3 := Pt_1 \cdot Pt3_to_Pt1$ $\qquad Pt_3 = 20.879\ psi$

12. Cold water with an enthalpy of 8 BTU/lbm enters a heater at a rate of 5 lbm/sec with a velocity of 10 ft/sec and at a potential of 10 feet with respect to the other connections shown. Steam enters at a rate of 1 lbm/sec with a velocity of 50 ft/sec and an enthalpy of 1350 BTU/lbm. These two streams mix in the heater and hot water emerges with an enthalpy of 168 BTU/lbm and a velocity of 12 ft/sec. Determine the heat loss from the apparatus. What is the percentage error if kinetic and potential energy changes are neglected?



Given the following:

Cold water: $h_{cold} := 8 \cdot \frac{BTU}{lb}$ $\quad m_{cold} := 5 \cdot \frac{lb}{sec}$ $\quad V_{cold} := 10 \cdot \frac{ft}{sec}$ $\quad z_{cold} := 10 \cdot ft$

Steam: $h_{stm} := 1350 \cdot \frac{BTU}{lb}$ $\quad m_{stm} := 1 \cdot \frac{lb}{sec}$ $\quad V_{stm} := 50 \cdot \frac{ft}{sec}$ $\quad z_{stm} := 0 \cdot ft$

Hot water: $h_{hot} := 168 \cdot \frac{BTU}{lb}$ $\quad V_{hot} := 12 \cdot \frac{ft}{sec}$ $\quad z_{hot} := 0 \cdot ft$

Relationships: Steady flow continuity equation and energy equation

$$mdot_{in} = mdot_{out}$$

$$mdot_{in} \cdot \left(h_1 + \frac{V_1^2}{2} + z_1 \right) + Q = mdot_{out} \cdot \left(h_2 + \frac{V_2^2}{2} + z_2 \right) + W$$

Solution:

From constinuity the mass flow into the system must be equal to the mass flow out of the system

$m_{hot} := m_{cold} + m_{stm}$ $\qquad m_{hot} = 6 \frac{lb}{sec}$

With mass flow known we can now find the heat exchange in the device

$$Q := m_{hot} \cdot \left(h_{hot} + \frac{V_{hot}^{2}}{2} + z_{hot} \cdot g \right) - m_{cold} \cdot \left(h_{cold} + \frac{V_{cold}^{2}}{2} + z_{cold} \cdot g \right) - m_{stm} \cdot \left(h_{stm} + \frac{V_{stm}^{2}}{2} \right.$$

$$Q = -382.107 \frac{BTU}{sec}$$

If we neglect the changes in kinetic and potential energy in the problem we find:

$$Q_{wo} := m_{hot} \cdot \left(h_{hot} \right) - m_{cold} \cdot \left(h_{cold} \right) - m_{stm} \cdot \left(h_{stm} \right)$$

$$Q_{wo} = -382 \frac{BTU}{sec}$$

$$\%Diff := \frac{Q - Q_{wo}}{Q} \cdot 100 \qquad \%Diff = 0.028$$

13. A fan accelerates quiescent air to 10 m/sec at a rate of 4 m^3/sec. Determine the minimum power required to power the fan using sea level standard day conditions.

$$Vf := 10 \cdot \frac{m}{sec} \qquad AV := 4 \cdot \frac{m^3}{sec} \qquad \rho := 1.18 \cdot \frac{kg}{m^3}$$

$$Wfan := \rho \cdot AV \cdot \left(\frac{Vf^2}{2} \right) \qquad Wfan = 236\ W$$

14. A classroom that normally seats 40 students is to be air-conditioned with window air-conditioning units of 5 kW capacity. There are ten light bulbs in room with a rating of 100 W. The rate of heat transfer to the classroom through the walls is 15,000 kJ/h. Assuming a person at rest (not asleep) dissipates heat at a rate of 360 kJ/h and the room is to be maintained at 21 degrees centigrade, determine the number of window air-conditioning units required.

$$stu := 40 \qquad q_stu := 360 \cdot 10^3 \cdot \frac{J}{hr}$$

$$lights := 10 \qquad q_light := 100 \cdot W$$

$$q_c := 15000 \cdot 10^3 \cdot \frac{J}{hr}$$

$$q_ac := 5000 \cdot W$$

$$Qin := stu \cdot q_stu + lights \cdot q_light + q_c \qquad Qin = 9166.667\ W$$

$$Qout := q_ac \qquad Qout = 5000\ W$$

To handle the load you have at least two (2) air-conditioning units