EA429 Fall AY2008 **Name(s):**_____

Classroom Exercise 3 **Section:**_____

All work must be checked before you leave the laboratory period. You may work in groups of 2 and you should generate a common solution sheet. Show all your work and write up your solutions in an organized, legible format. It may or may not be collected.

1. A four-engine turboprop aircraft is cruising at 200 KTAS at an altitude where the density ratio is 0.5. The propellers are 3.5m (11.48 ft) in diameter, variable pitch, four-bladed, and turn at a constant speed which is 0.044 of the engine speed of 27,272 RPM. The following table lists data for the "line of maximum efficiency for C_S."

C_S	J	η
0.5	0.22	0.55
1.0	0.62	0.74
1.5	1.00	0.82
2.0	1.39	0.83
2.5	1.78	0.81

   Find the aircraft drag and the weight if $L/D = 15$.

2. Each engine of a twin aircraft weighing 4,000 lb is fitted with a constant-speed, 5.90 ft diameter propeller. Each propeller is direct-driven by its engine which develops a maximum static power of 201 HP at a shaft speed of 2,700 RPM in sea level standard conditions. The relevant propeller chart gives $C_T = 0.12$ at $J = 0$.

   a. Determine the aircraft static T/W ratio.

   b. If each propeller is replaced with a ducted fan (generating the same static thrust), calculate the diameter required.

   c. Compare the tip Mach number of a propeller with that of its replacement ducted fan.

3. Do the attached "Propeller Example Problem."

Chapter 3, Problem 6: Given the following: A four engine turboprop flies at 200 knots at the following conditions using a propeller as specified by figure 1.

Flight speed: $V_1 := 200 \cdot knot \qquad V_1 = 102.889 \frac{m}{s}$

Density ratio: $\sigma := 0.5 \qquad \rho := \sigma \cdot 1.225 \cdot \frac{kg}{m^3}$

Propeller diameter: $D := 3.5 \cdot m$

Engine RPM: $N_e := 27272 \cdot \frac{1}{min}$

Propeller RPM (GR = 0.044): $N_p := 0.044 \cdot N_e \qquad N_p = 1.19997 \times 10^3 \frac{1}{min}$

or you can say: $n := N_p$

Propeller data as provided:

$$C_{s_chart} := \begin{pmatrix} 0.5 & 0.22 & 0.55 \\ 1 & 0.62 & 0.74 \\ 1.5 & 1 & 0.82 \\ 2 & 1.39 & 0.83 \\ 2.5 & 1.78 & 0.81 \end{pmatrix}$$

Split out individual vectors: $C_S := C_{s_chart}^{\langle 0 \rangle} \qquad J_p := C_{s_chart}^{\langle 1 \rangle} \qquad \eta_p := C_{s_chart}^{\langle 2 \rangle}$

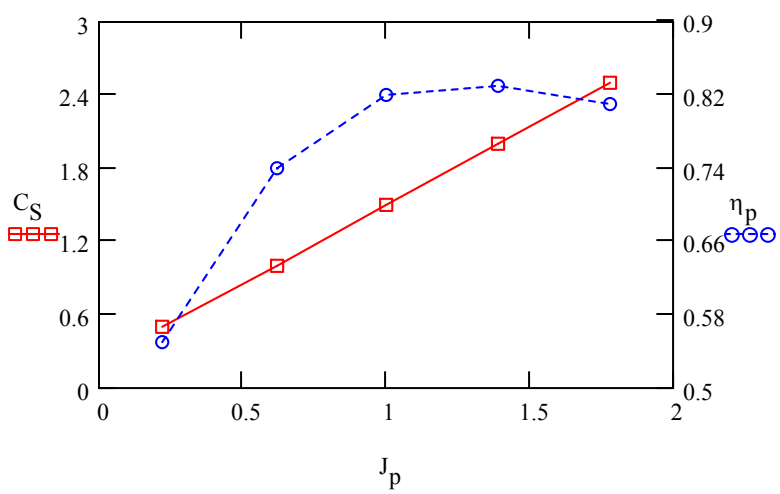

Figure 1. Cs and efficiency versus J

Find the following: (a) Aircraft Drag (T) and (b) Weight for L/D = 15 $\quad LoverD := 15$

a. Us the definition of C_S as:

We have enough information to find J: $J_{inf} := \frac{V_1}{n \cdot D}$ $\quad J_{inf} = 1.47$

Using the data we linear interpolate to get the η and C_S value:

$$\eta_{pJ} := \frac{J_{inf} - J_{p_3}}{J_{p_4} - J_{p_3}} \cdot \left(\eta_{p_4} - \eta_{p_3}\right) + \eta_{p_3} \qquad \eta_{pJ} = 0.826$$

$$C_{SJ} := \frac{J_{inf} - J_{p_3}}{J_{p_4} - J_{p_3}} \cdot \left(C_{S_4} - C_{S_3}\right) + C_{S_3} \qquad C_{SJ} = 2.102$$

With CS you can now find CP, power and thrust:

$$C_p := \left(\frac{J_{inf}}{C_{SJ}}\right)^5 \qquad C_p = 0.167$$

$$P := C_p \cdot \rho \cdot n^3 \cdot D^5 \quad P = 4.29857 \times 10^5\ W \quad P = 576.45\ hp$$

$$T_{ac} := \frac{\eta_{pJ} \cdot P}{V_1} \qquad T_{ac} = 3.45052 \times 10^3\ N$$

$$T_{ac} = 775.71\ lbf$$

b. The weight is found as:

$$W_{ac} := LoverD \cdot T_{ac} \cdot 4$$

$$W_{ac} = 2.070312 \times 10^5\ N \qquad W_{ac} = 46542.47\ lbf$$

Chapter 3, Problem 9. The following informtion is provided for twin engine aircraft with constant speed propellers direct driven by 201 hp engines:

Aircraft Weight: $W_{ac} := 17.78 \cdot 10^3 \cdot N$ $\quad W_{ac} = 3.997 \times 10^3\ lbf$

Prop Diameter and Np: $D := 1.8 \cdot m$ $\quad N_p := 2700 \cdot \frac{1}{min}$ $\quad n := N_p$

Shaft Power: $P_s := 150 \cdot 10^3 \cdot W$ $\quad P_s = 201.153\ hp$

Static Thrust Coefficient: $C_{To} := 0.12$

Air Properties: $\rho_{air} := 1.225 \cdot \frac{kg}{m^3}$ $\quad \gamma := 1.4$ $\quad T_{air} := 288 \cdot K$ $\quad R_{air} := 53.3 \cdot \frac{ft \cdot lbf}{lb \cdot R}$

Speed of sound: $a := \sqrt{\gamma \cdot R_{air} \cdot T_{air}}$ $\quad a = 340.038\ \frac{m}{s}$

Find the following: T/W, Diameter of an equivalent ducted fan, tip Mach number of fan versus propeller.

Part a. Find T and T/W

$$T := C_{To} \cdot \rho_{air} \cdot n^2 \cdot D^4$$

$T = 3124.87\ N$ $\quad T = 702.499\ lbf$

$T_{total} := 2 \cdot T$ $\quad T_{total} = 6.25 \times 10^3\ N$

So the thrust-to-weight ratio is:

$$TtoW := \frac{T_{total}}{W_{ac}} \qquad TtoW = 0.352$$

Part b. For the equivalent fan dimater we take...

$$D_f := \frac{1}{\sqrt{2}} \cdot D \qquad D_f = 1.273\ m$$

Part c. For static conditions we need the linear speed in the plane of the propeller (the $r\omega$ term) and the induced flow at the disk. For static conditions we have the following thrust equations:

$$T_{prop} = \frac{1}{2}\cdot\rho\cdot A_p\cdot v_p^{\,2} \qquad T_{fan} = \rho\cdot A_f\cdot v_f^{\,2}$$

So we need to solve for the propeller and fan wake velocities using the single engine thrust of 3124.87 N from above:

$$v_p := \sqrt{\frac{2\cdot T}{\rho_{air}\cdot\frac{\pi}{4}\cdot D^2}} \qquad v_p = 44.776\ \frac{m}{s}$$

$$v_f := \sqrt{\frac{T}{\rho_{air}\cdot\frac{\pi}{4}\cdot D_f^{\,2}}} \qquad v_f = 44.776\ \frac{m}{s}$$

The $r\omega$ speed is found using the diameter and RPMs:

$$V_{p\omega} := \pi\cdot n\cdot D \qquad V_{p\omega} = 254.469\ \frac{m}{s}$$

$$V_{f\omega} := \pi\cdot n\cdot D_f \qquad V_{f\omega} = 179.937\ \frac{m}{s}$$

The resultant velocity seen by the tip is:

$$V_{pR} := \sqrt{v_p^{\,2} + V_{p\omega}^{\,2}} \qquad V_{pR} = 258.378\ \frac{m}{s}$$

$$V_{fR} := \sqrt{v_f^{\,2} + V_{f\omega}^{\,2}} \qquad V_{fR} = 185.424\ \frac{m}{s}$$

So the tip Mach numbers are:

$$M_p := \frac{V_{pR}}{a} \qquad M_p = 0.76$$

$$M_f := \frac{V_{fR}}{a} \qquad M_f = 0.545$$

Propeller Example Problem

During a flight test of the F-33A Beachcraft Bonanza the following data were recorded:

Altitude: Sea level, standard day
Airspeed: 165 MPH (242 ft/sec)
Engine: IO-520-B providing 178 HP at 2,300 RPM
Propeller: 7 ft diameter two-bladed Clark-Y

1. What is the drag of the aircraft? – in class

2. Using the data and results above, find the optimum propeller diameter. – in class

3. Rework 1 and 2 above using a three-bladed propeller – you try this!

Reference: NACA Report No. 640

1. *Solution*: The procedure is to calculate the advance ratio (J), the power coefficient (C_P) and then use figure 6 of the reference to obtain the thrust coefficient (C_T) and pitch angle (β).

$$J = \frac{V}{n \cdot D} = \frac{242\ ft/\sec}{38.33\sec^{-1} \cdot 7\ ft} = 0.9 \text{ and}$$

$$C_P = \frac{P}{\rho n^3 D^5} = 0.0435$$

From figure 6 of NACA Report No. 640 we obtain for the above J and C_P:

$$\beta \approx 22.5\deg \text{ and}$$

$$C_T \approx 0.04$$

From figure 8 of the reference, we verify C_T using the β above:

$$\text{using } \beta = 22.5\deg \text{ we get } C_T = 0.042$$

Now we can calculate the efficiency as follows:

$$\eta = \frac{C_T}{C_P} J = \frac{0.042}{0.0435} 0.9 = 0.86$$

Assuming steady flight where thrust equals drag:

$$T = D = \eta \frac{P}{V} = \eta \frac{BHP \cdot 550\ ft \cdot lbf / HP \cdot \sec}{V}$$
$$D = 348\ lbf$$

2. <u>*Solution*</u>: For the same data, we will determine the optimum propeller diameter by using figure 7. This curve provides a maximum efficiency curve with the speed coefficient (C_S) as the entering argument.

$$C_S = \left(\frac{\rho}{P \cdot n^2} \right)^{1/5} V = 1.69$$

Using figure 7 we can obtain J_{opt} (= 0.92) allowing for direct calculation of propeller diameter as follows:

$$D = \frac{V}{n \cdot J_{opt}} = 6.89\ ft$$

Thus, a 7 ft propeller seems like an adequate choice.