EA429 Fall AY2008 **Name(s):**_____

Classroom Exercise 6 **Section:**_____

All work must be checked before you leave the laboratory period. You may work in groups of 2 and you should generate a common solution sheet. Show all your work and write up your solutions in an organized, legible format. It may or may not be collected.

1. A rocket motor is designed to have a propellant gas with γ = 1.25 and $\mathcal{M}$ = 10 kg/kg-mol. The design combustion temperature ($T_{T,2}$) is 3000K. Test stand measurements yield a combustion chamber pressure ($p_{T,2}$) of 10.13 MPa and $\dot{m}$ = 450 kg/sec. The throat area is 0.1 m².
   a. Calculate the nominal characteristic velocity (theoretical design).
   b. Calculate the expansion characteristic velocity (from the experiment).
   c. What is c^*_{exp} / c^*_{nom} ?

2. A solid rocket has a combustion temperature of 5,000 deg R and a chamber pressure of 68 atm. The nozzle design exit pressure is 2116 psf. Assume γ = 1.3 and $\mathcal{M}$ = 23 slug/slug-mol.
   (**NOTE**: $\mathcal{R}$ = 49,751 ft-lb/slug-mol/°R).

   a. What is the design altitude for this rocket motor?
   b. Calculate the maximum and actual exit velocities ($V_{e,max}$, V_e) at the design altitude.
   c. Calculate the specific impulse at the design altitude.
   d. Calculate the ideal design thrust coefficient ($C_{T,d}$).
   e. Calculate the thrust coefficient at an altitude of 50,000 ft altitude (p_a = 1.682 psia).
   f. If this engine is to produce 100,000 lbs of thrust at sea level, determine the massflow rate (lbm/sec) and the nozzle throat area (ft²).
   g. Determine the total impulse of this engine if it has enough propellant to fire for a maximum of 65 sec.

3. A rocket operates at an altitude of 10 km where p_a = 26.5 kPa. The chamber pressure is 7MPa, the chamber temperature is 3,000 K, and the nozzle is designed for $p_{T,2} / p_e$ = 70. The throat radius is 20 cm. The exhaust gas has γ = 1.2 and $\mathcal{M}$ = 25 slug/slug-mol.
   a. Calculate the design thrust coefficient ($C_{T,d}$).
   b. Calculate the actual thrust coefficient (C_T).
   c. Determine the thrust and massflow, and I_{sp} .

$\boxed{1}$ $R = \mathcal{R}/\mathcal{M} = 8314/10 = 831.4 \ \frac{J}{kg\,K}$

$C_p = \frac{\gamma R}{\gamma - 1} = 4157 \ \frac{J}{kg\,K}$

$\Gamma(1.25) = 0.6581$

$\Gamma_T(1.25) = 2.081$

A great way to start every rocket problem!

a) $C^*_{nom} = \frac{\sqrt{RT_{T2}}}{\Gamma} = \frac{\sqrt{831.4(3000)}}{0.6581} = \boxed{2{,}400 \text{ m/s}}$

b) $C^*_{exp} = \frac{P_{T2} A^*}{\dot{m}} = \frac{(10.13\times10^6)(0.1)}{450} = \boxed{2{,}251 \text{ m/s}}$

c) $C^*_{exp}/C^*_{nom} = \boxed{0.938}$

$\boxed{2}$ a) The design altitude is the point of optimum thrust ... where $P_a = P_e$

Since $P_e = 2116$ psf, $\boxed{h_{design} = \text{Sea Level}}$

b) $R = \mathcal{R}/\mathcal{M} = 49751/23 = 2163 \ \frac{ft\,lb}{slug\,°R}$

$C_p = \frac{\gamma R}{\gamma - 1} = 9{,}373 \ \frac{ft\,lb}{slug\,°R}$

$E = \sqrt{1 - \left(\frac{P_e}{P_{T2}}\right)^{\frac{\gamma-1}{\gamma}}} = \sqrt{1 - \left(\frac{1}{68}\right)^{\frac{0.3}{1.3}}} = 0.789$

$\Gamma(1.3) = 0.6673$

$\Gamma_T(1.3) = 1.964$

$V_{e\,max} = \sqrt{2 C_p T_{T2}} = \sqrt{2(9373)(5000)} = \boxed{9{,}681 \text{ fps}}$

$$V_e = E\,V_{e_{max}} = 0.789(9681) = \boxed{7{,}639 \text{ fps}}$$

c) $I_{sp} = \dfrac{T}{\dot{m}g}$

At design altitude, $T = \dot{m}V_e$

so $I_{sp} = \dfrac{V_e}{g} = \dfrac{7639}{32.2} = \boxed{237 \text{ sec}}$

d) $C_T = \Gamma E + \cancel{\dfrac{P_e - P_a}{P_{T_2}} \cdot \dfrac{A_e}{A^*}}$ → 0 at design alt.

so $C_{T,d} = (1.964)(0.789) = \boxed{1.550}$

e) $C_T = C_{T,d} + \dfrac{P_e - P_a}{P_{T_2}} \cdot \dfrac{A_e}{A^*}$

$$\frac{A_e}{A^*} = \frac{\Gamma}{\sqrt{\frac{2\gamma}{\gamma-1}\left(\frac{P_e}{P_{T_2}}\right)^{\frac{1}{\gamma}} E}} = \frac{0.6673}{(2.944)(0.0389)(0.789)}$$

$$= 7.378$$

$$C_T = 1.550 + \frac{2116 - 1.682 \times 144}{68(2116)}(7.378)$$

$$= 1.550 + 0.0961$$

$$= \boxed{1.646}$$

f) $T = C_T P_{T_2} A^* = C_T \dot{m} c^*$

$$c^* = \frac{\sqrt{RT_{T_2}}}{\Gamma} = \frac{\sqrt{2163(5000)}}{0.6673} = 4{,}928 \text{ fps}$$

(Also... $C_T c^* = c$)

$$\dot{m} = \frac{T}{C_T c^*} = \frac{100{,}000}{(1.550)(4928)} = 13.09 \text{ slug/sec}$$

$$= \boxed{422 \frac{\text{lbm}}{\text{sec}}}$$

$$A^* = \frac{T}{C_T P_{T_2}} = \frac{100{,}000}{(1.550)(68 \times 2116)} = \boxed{0.448 \text{ ft}^2}$$

g) $I_{tot} = \int T(t)\,dt$ TOTAL IMPULSE

But $I_{sp} = \frac{T}{\dot{m}g}$ so $T = \dot{m} g I_{sp}$

$$I_{tot} = \int \dot{m} g I_{sp}\,dt = I_{sp} g \int \dot{m}\,dt$$

$$= I_{sp} g \int \frac{dm}{dt} \cdot dt = I_{sp} g \int dm$$

$$= I_{sp} g \Delta m = I_{sp} \Delta W$$

$$\Delta W = \dot{m} g \Delta t = \left(422 \frac{\text{lbm}}{\text{sec}}\right)(65 \text{ sec})$$

$$= 27{,}401 \text{ lbs}$$

The change in weight is the propellant burned!

$$I_{tot} = (237 \text{ sec})(27{,}401 \text{ lb})$$

$$= \boxed{6.49 \times 10^6 \text{ lb-sec}}$$

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$\boxed{3}$ $R = \mathcal{R}/\mathcal{M} = 8314/25 = 332.6 \frac{J}{kg\,K}$

$C_p = \frac{\gamma R}{\gamma - 1} = 1995 \frac{J}{kg\,K}$

$\Gamma(1.2) = 0.6485$

$\Gamma_T(1.2) = 2.247$

$E = \sqrt{1 - \left(\frac{1}{70}\right)^{\frac{0.2}{1.2}}} = 0.712$

a) $C_{T,d} = \Gamma_T E = 2.247(0.712) = \boxed{1.601}$

b) $\frac{A_e}{A^*} = \frac{\Gamma}{\sqrt{\frac{2\gamma}{\gamma-1}\left(\frac{P_e}{P_{t_2}}\right)^{\frac{1}{\gamma}}} E} = \frac{0.6485}{(3.464)(0.029)(0.712)}$

$= 9.063$

$C_T = C_{T,d} + \frac{P_e - P_a}{P_{t_2}} \cdot \frac{A_e}{A^*}$

$= 1.601 + \frac{100{,}000 - 26{,}500}{7 \times 10^6}(9.063)$

$= 1.601 + 0.0952 = \boxed{1.696}$

c) $A^* = \pi r^2 = 0.1257\,m^2$

$T = C_T P_{t_2} A^* = (1.696)(7 \times 10^6)(0.1257)$

$= \boxed{1.49\ MN}$

$$T = C_T P_{T_2} A^* = C_T \dot{m} C^*$$

$$C^* = \frac{\sqrt{R T_{T_2}}}{\Gamma} = \frac{\sqrt{332.6(3000)}}{0.6485} = 1540 \text{ m/s}$$

$$\dot{m} = \frac{I}{C_T C^*} = \frac{1.49 \text{ MN}}{(1.696)(1540)} = \boxed{571 \text{ kg/sec}}$$

$$I_{sp} = \frac{I}{\dot{m} g} = \frac{1.49 \text{ MN}}{(571)(9.81)} = \boxed{266 \text{ sec}}$$

Also ... $$\frac{I}{\dot{m} g} = \frac{C_T C^*}{g} = \frac{(1.696)(1540)}{9.81}$$

$$= \boxed{266 \text{ sec}}$$

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