Assignment 2 Thrust and Efficiencies

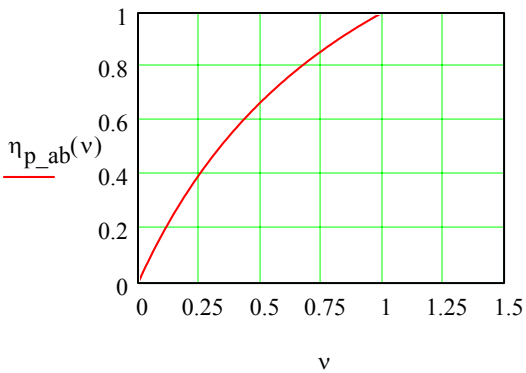
1. Plot the propulsive efficiency, dimensionless thrus and power for the rocket and air breather as a function of the speed ratio betwee zero and one. Compare the two types of propulsion systems on the same graphs.

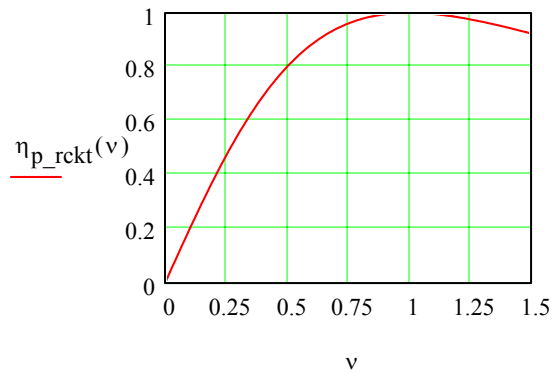
AIR BREATHER **ROCKET**

$\nu := 0, 0.01 .. 1.5$ (ν is defined as the ratio of V1/V4)

$$\eta_{p_ab}(\nu) := \frac{2\cdot\nu}{1+\nu}$$

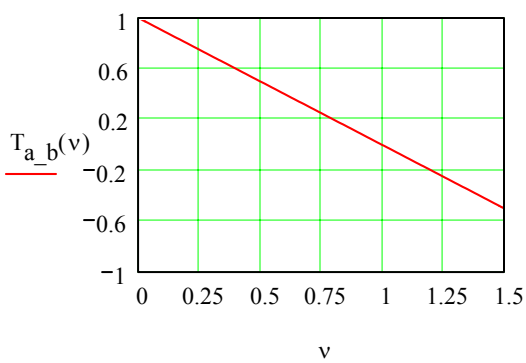
$$\eta_{p_rckt}(\nu) := \frac{2\cdot\nu}{1+\nu^2}$$





$$T_{a_b}(\nu) := (1-\nu)$$

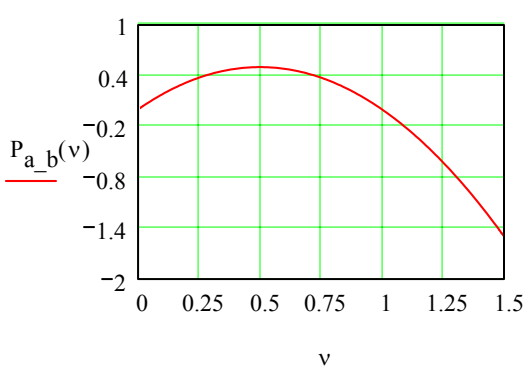
$$T_{rckt}(\nu) := 1$$

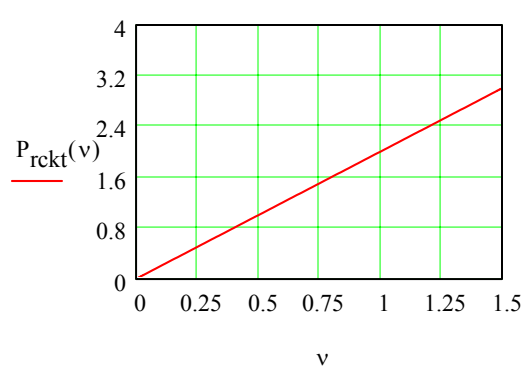


Not a function of ν

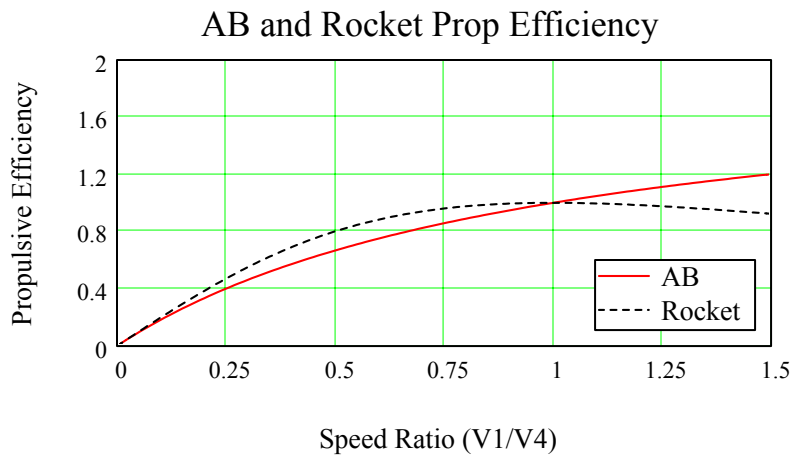
$$P_{a_b}(\nu) := 2\cdot\nu\cdot(1-\nu)$$

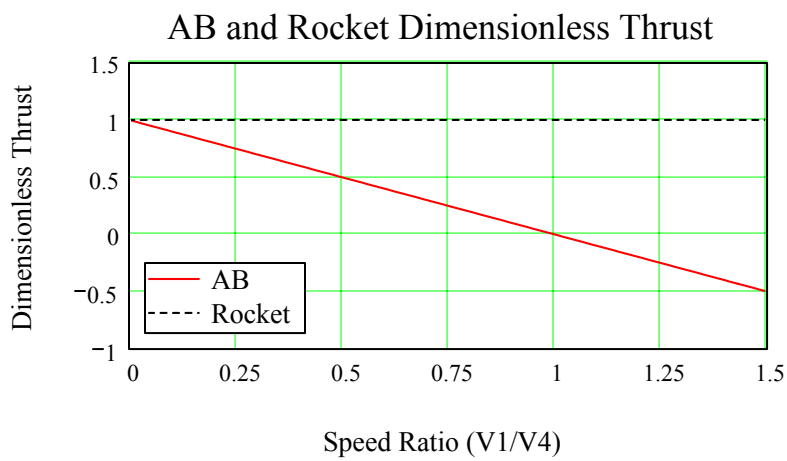
$$P_{rckt}(\nu) := 2\cdot\nu$$

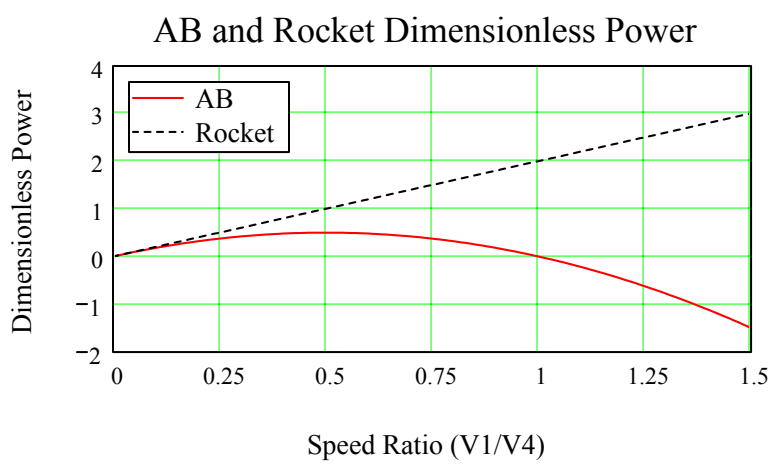




Comparison of AB and Rocket Propulsive efficiency, Dimensionless Thrust and Power







2. Find propulsive efficiency for the following two engines (from Cumpsty):
   a. An RB211 (Rolls-Royce Turbofan) at 31,000 ft, flight Mach number is 0.85 and approximate jet velocity is 390 m/sec.
   b. An Olympus 593 (Concorde) at 51,000 ft (p_{inf} = 11.0 kPa, T_{inf} = 216.7 K), flight Mach number 2.0 and an approximate jet velocity of 1009 m/sec.

For part a we are given: $M_{inf} := 0.85 \quad \gamma := 1.4 \quad V_e := 390 \cdot \frac{m}{sec}$

$R_{gas} := 287 \cdot \frac{N \cdot m}{kg \cdot K} \quad T_h := 408.3 \cdot R \quad T_h = 226.8333\ K$

The speed of sound is $a := \sqrt{\gamma \cdot R_{gas} \cdot T_h} \quad a = 301.8967\ \frac{m}{s}$

The required velocities are: $V_{inf} := M_{inf} \cdot a \quad V_{inf} = 256.6122\ \frac{m}{s} \quad \nu := \frac{V_{inf}}{V_e} \quad \nu = 0.6$

and ηp is calculated: $\eta_{P_RB211} := \frac{2 \cdot \nu}{1 + \nu} \quad \eta_{P_RB211} = 0.7937$

For part b we are given: $M_{inf} := 2 \quad \gamma := 1.4 \quad V_e := 1009 \cdot \frac{m}{sec}$

$R_{gas} := 287 \cdot \frac{N \cdot m}{kg \cdot K} \quad T_h := 216.7 \cdot K \quad T_h = 390.06\ R$

The speed of sound is $a := \sqrt{\gamma \cdot R_{gas} \cdot T_h} \quad a = 295.0764\ \frac{m}{s}$

The required velocities are: $V_{inf} := M_{inf} \cdot a \quad V_{inf} = 590.1527\ \frac{m}{s} \quad \nu := \frac{V_{inf}}{V_e} \quad \nu = 0.584$

and ηp is calculated: $\eta_{P_Concorde} := \frac{2 \cdot \nu}{1 + \nu} \quad \eta_{P_Concorde} = 0.7381$

3. In class handout problems 2,3,4 and 9

Problem 2:

Given: The following information about a Turbojet

$$u := 700 \cdot \frac{ft}{sec} \qquad pa := 6.5 \cdot psi \qquad Ae := 1.65 \cdot ft^2$$

$$ue := 1500 \cdot \frac{ft}{sec} \qquad pe := 10 \cdot psi \qquad mdot := 175 \cdot \frac{lb}{sec}$$

Find: Net thrust and effective exhaust velocity

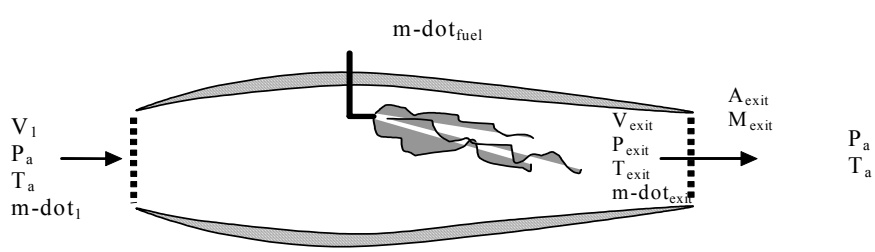
$$Tj := mdot \cdot (ue - u) + (pe - pa) \cdot Ae \qquad Tj = 2.3055 \times 10^4\ N$$

$$Tj = 5182.933\ lbf$$

$$Ce := ue + \frac{(pe - pa) \cdot Ae}{mdot} \qquad Ce = 1652.891\ \frac{ft}{sec}$$

Problem 3 (and Quiz #1). An aircraft operates at a Mach number of 0.8 at 40,000 ft where the ambient pressure is 2.6 psia and temperature is 390 R. The engine mass flow rate of air is 150 lb/sec with a fuel-to-air ratio of 0.012. If the nozzle has an exit Mach number of one (1.0), an area of 2 ft^2 and an exit temperature and pressure of 1375 R and 5 psia respectively find the following:

a. Net thrust
b. Gross thrust
c. Inlet area
d. Propulsive efficiency
e. TSFC



Given: Inlet conditions:

$$M_1 := 0.8 \quad P_1 := 2.6 \cdot \frac{lbf}{in^2} \quad T_1 := 390 \cdot R \quad mdot_1 := 150 \frac{lb}{sec}$$

Combustor conditions:

$$f := 0.012$$

Exit conditions:

$$M_3 := 1 \quad P_3 := 5 \cdot \frac{lbf}{in^2} \quad T_3 := 1375 \cdot R \quad A_3 := 2 \cdot ft^2$$

Gas conditions:

$$R_{gas} := 53.3 \cdot \frac{ft \cdot lbf}{lb \cdot R} \quad \gamma := 1.4$$

Solution:

Part a.

$$V_1 := M_1 \cdot \sqrt{\gamma \cdot R_{gas} \cdot T_1} \quad V_3 := M_3 \cdot \sqrt{\gamma \cdot R_{gas} \cdot T_3}$$

$$V_1 = 774.1102 \frac{ft}{sec} \quad V_3 = 1816.903 \frac{ft}{sec}$$

$$T_{net} := mdot_1 \cdot \left[(1 + f) \cdot V_3 - V_1\right] + \left(P_3 - P_1\right) \cdot A_3$$

$$T_{net} = 5654.498\, lbf$$

Part b.

$$V_4 := V_3 + \frac{\left(P_3 - P_1\right) \cdot A_3}{mdot_1 \cdot (1 + f)} \quad V_4 = 1963.4034 \frac{ft}{sec}$$

$$T_{gross} := mdot_1 \cdot (1 + f) \cdot V_4$$

$$T_{gross} = 9263.51\, lbf$$

Part c. $mdot_1 = \frac{P_1}{R_{gas} \cdot T_1} \cdot A_1 \cdot V_1$

$$A_1 := \frac{mdot_1 \cdot R_{gas} \cdot T_1}{P_1 \cdot V_1}$$

$$A_1 = 10.7583\ ft^2$$

Part d. $\nu := \frac{V_1}{V_4}$ $\eta_p := \frac{2 \cdot \nu}{1 + \nu}$

$$\eta_p = 0.5656$$

Part e. $TSFC := \frac{f \cdot mdot_1}{T_{net}}$ $TSFC = 1.146 \frac{lb}{lbf \cdot hr}$

Problem 9:

Given: The following data for a rocket

$Isp := 300 \cdot sec$ $mdot := 43.8 \cdot \frac{kg}{sec}$ $Ae := 0.0929 \cdot m^2$ $Ae = 1\ ft^2$

$pa := 1.0135 \cdot 10^5 \cdot Pa$ $pa = 14.6996\ psi$

Find: Thrust, effective exhaust velocity, thrust at sea level with the following:

a. The effective exhaust velocity:

$Ce := Isp \cdot g$ $Ce = 2941.995 \frac{m}{s}$ $Ce = 9652.215 \frac{ft}{sec}$

b Rocket thrust:

$Tr := mdot \cdot Ce$ $Tr = 128859.38\ N$ $Tr = 28968.74\ lbf$ $u_e := 1 \cdot \frac{m}{sec}$

c. Rocket thrust with Pe = 1/2 Pa:

$$T := mdot \cdot u_e + (pe - pa) \cdot Ae$$

$$Ce_n := Ce + \frac{(-0.5) \cdot pa \cdot Ae}{mdot} \qquad Ce_n = 2834.5131 \ \frac{m}{s}$$

$$Tsl := mdot \cdot Ce_n \qquad Tsl = 1.2415 \times 10^5 \ N$$

or we can write:

$$Tsl := Tr + pa \cdot (-0.5) \cdot Ae \qquad Tsl = 1.2415 \times 10^5 \ N \quad Tsl = 2.791 \times 10^4 \ lbf$$

5. (H&P #5.2) Given a commercial airliner with a range of 6000 miles using 200 tons of JP4 fuel, estimate the range of the aircraft burning either gaseous or liquid hydrogen and*

a. The same mass of hydrogen as JP4,
b. The same volume of hydrogen as JP4

The dry mass of the aircraft is 800 tons. The fuel properties are as follows:

Fuel	Density (kg/m3)	Heating Value (kJ/kg)
JP4	800.0	45,000
H_2 (gaseous, S.T.P.)	0.0824	120,900
H_2 (liquid at 1 atm)	70.8	120,900

Problem #4 (Hill and Peterson problem 5.2)

Range: $S_{JP4} := 6000 \cdot 5280 \cdot ft$ Burning JP4 fuel

A/C mass: $m_{JP4} := 200 \cdot 1000 kg$

$$m_f := 800 \cdot 1000 \cdot kg \qquad m_f = 8 \times 10^5 \ kg$$

$$m_i := m_f + m_{JP4} \qquad m_i = 1 \times 10^6 \ kg$$

Fuel densities and heating values:

$$\rho_{JP4} := 800 \cdot \frac{kg}{m^3} \qquad HV_{JP4} := 45000 \cdot 10^3 \cdot \frac{J}{kg} \qquad \rho_{H2l} := 0.088 \cdot \frac{gm}{cm^3}$$

$$\rho_{H2g} := 0.0824 \cdot \frac{kg}{m^3} \qquad HV_{H2g} := 120900 \cdot 10^3 \cdot \frac{J}{kg} \qquad \rho_{H2l} = 88 \frac{kg}{m^3}$$

$$\rho_{H2l} := 70.8 \cdot \frac{kg}{m^3} \qquad HV_{H2l} := 120900 \cdot 10^3 \cdot \frac{J}{kg}$$

Note: The ration of heating values: $HVratio := \frac{HV_{H2g}}{HV_{JP4}}$

$$HVratio = 2.6867$$

1. What is the volume of the fuel tanks aboard this aircraft?

$$V_{JP4} := \frac{m_{JP4}}{\rho_{JP4}} \qquad V_{JP4} = 2.5 \times 10^5 \ L$$

2. Now how much gaseous and liquid H2 can be carried in the same volume?

$$m_{H2g} := \rho_{H2g} \cdot V_{JP4} \qquad m_{H2g} = 20.6 \ kg$$

$$m_{H2l} := \rho_{H2l} \cdot V_{JP4} \qquad m_{H2l} = 1.77 \times 10^4 \ kg$$

3. Assuming the overall efficiency (ηo) and L/D remain the same, then we can calculate a ratio of new range to old based on original aircraft data:

a. Tank volume the same:

$$Sratio_H2g := \frac{\ln\left(\frac{m_f + m_{H2g}}{m_f}\right) \cdot HV_{H2g}}{\ln\left(\frac{m_f + m_{JP4}}{m_f}\right) \cdot HV_{JP4}}$$

$$Sratio_H2g = 3.1003 \times 10^{-4}$$

$$S_{H2g} := Sratio_H2g \cdot S_{JP4} \qquad S_{H2g} = 1.8602\,mi$$

$$Sratio_H2l := \frac{\ln\left(\frac{m_f + m_{H2l}}{m_f}\right) \cdot HV_{H2l}}{\ln\left(\frac{m_f + m_{JP4}}{m_f}\right) \cdot HV_{JP4}}$$

$$Sratio_H2l = 0.2635$$

$$S_{H2l} := Sratio_H2l \cdot S_{JP4} \qquad S_{H2l} = 1580.896\,mi$$

Alternatively you could find the quantity ηo times L/D:

$$\eta oL_D := \frac{S_{JP4}}{\ln\left(\frac{m_f + m_{JP4}}{m_f}\right) \cdot \frac{HV_{JP4}}{g}}$$

$$\eta oL_D = 9.4303$$

$$SH2g_{alt} := \eta oL_D \cdot \left(\ln\left(\frac{m_f + m_{H2g}}{m_f}\right) \cdot \frac{HV_{H2g}}{g}\right)$$

$$SH2g_{alt} = 1.86\,mi$$

b. Now calculate range for a mass of H2 same as JP4:

$$Sratio_H2g := \frac{\ln\left(\frac{m_f + m_{JP4}}{m_f}\right) \cdot HV_{H2g}}{\ln\left(\frac{m_f + m_{JP4}}{m_f}\right) \cdot HV_{JP4}}$$

$Sratio_H2g = 2.6867$

$S_{H2g} := Sratio_H2g \cdot S_{JP4}$ $\quad S_{H2g} = 16120\,mi$

4. What would the tank volume required be for the two cases?

$V_{H2g} := \frac{m_{JP4}}{\rho_{H2g}}$ $\quad V_{H2g} = 2.4272 \times 10^{9}\ L$

$V_{H2l} := \frac{m_{JP4}}{\rho_{H2l}}$ $\quad V_{H2l} = 2.8249 \times 10^{6}\ L$

$Vratio_{H2g} := \frac{V_{H2g}}{V_{JP4}}$ $\quad Vratio_{H2g} = 9708.7379$

$Vratio_{H2l} := \frac{V_{H2l}}{V_{JP4}}$ $\quad Vratio_{H2l} = 11.2994$

5. Another way to look at this metric is Joules/m^3?

$Energy_Desnity_{JP4} := \rho_{JP4} \cdot HV_{JP4}$ $\quad Energy_Desnity_{JP4} = 3.6 \times 10^{10}\ \frac{J}{m^3}$

$Energy_Desnity_{H2l} := \rho_{H2l} \cdot HV_{H2l}$ $\quad Energy_Desnity_{H2l} = 8.5597 \times 10^{9}\ \frac{J}{m^3}$

$Energy_Desnity_{H2g} := \rho_{H2g} \cdot HV_{H2g}$ $\quad Energy_Desnity_{H2g} = 9.9622 \times 10^{6}\ \frac{J}{m^3}$

6. (H&P 5.3) The idling engines of a landing turbojet produce forward thrust when operating in a normal manner, but they can produce reverse thrust if the jet is properly deflected. Suppose that while the aircraft rolls down the runway at 150 km/hour the idling engine consumes 50 kg/sec and produces exhaust velocity of 150 m/sec*.

a. What is the forward thrust of the engines?
b. What are the magnitude and direction of the (i.e. forward or reverse) if the exhaust is deflected 90 degrees without affecting the mass flow?
c. What are the magnitude and direction of the thrust as above after the plane has come to a stop, with 90 degree exhaust deflection and airflow of 40 kg/sec?

Problem 5. Hill and Peterson 5.3:

$V_{roll} := 150 \cdot \frac{km}{hr}$ $\quad V_{roll} = 41.6667 \frac{m}{s}$

$mdot := 50 \cdot \frac{kg}{sec}$

$Ve := 150 \cdot \frac{m}{sec}$

$F_a := mdot \cdot (Ve - V_{roll})$ $\quad F_a = 5416.6667\ N$

If Ve is deflected orthogonal to the flight path of thrust line then:

$F_b := mdot \cdot (0 - V_{roll})$ $\quad F_b = -2083.3333\ N$

If Ve is still deflected, but forward speed is now zero then use the original flight conditions to find inlet area and density:

$\rho A_1 := \frac{mdot}{V_{roll}}$ $\quad \rho A_1 = 1.2 \frac{kg}{m}$

Then calculate the new V1 for the stationary condition: $mdot_s := 40 \cdot \frac{kg}{sec}$

$V_1 := \frac{mdot_s}{\rho A_1}$ $\quad V_1 = 33.3333 \frac{m}{s}$ lote: the official answer is zero thrust. 'his is what would really happen since your induce a velocity of V1.

$F_c := mdot_s \cdot (0 - V_1)$ $\quad F_c = -1333.3333\ N$

7. A rocket has a thrust of 8896 N and propellant consumption of 3.867 kg/sec. The vehicle flies at a velocity of 400 m/sec and the propellant specific energy content (heat of combustion) is 16.9 megajoule/kg (from Sutton). Find the following:

a. Effective exhaust velocity (2300 m/sec)
b. Kinetic energy of the jet for 1 kg of fuel (2.64 MJ)
c. Internal efficiency (38.3%)
d. Propulsive efficiency (33.7%)
e. Overall efficiency (13.3%)
f. Specific impulse (234.7 sec)
g. Specific propellant consumption (0.00426 sec^{-1})

Given the following from the problem statement:

$T := 8896 \cdot N \qquad mdot := 3.867 \cdot \frac{kg}{sec} \qquad u := 400 \cdot \frac{m}{sec} \qquad Q_R := 16.9 \cdot 10^6 \cdot \frac{J}{kg}$

Find a through g from above:

a. Effective exhaust velocity: $Ce := \frac{T}{mdot} \qquad Ce = 2300.491 \frac{m}{s}$

b. Specific kinetic energy of the jet: $KE_{jet} := 0.5 \cdot Ce^2 \qquad KE_{jet} = 2.6461 \times 10^6 \frac{J}{kg}$

c. Internal efficiency $\eta_{int} := \frac{KE_{jet}}{Q_R} \qquad \eta_{int} = 0.1566$

d. Propulsive efficiency

The speed ratio: $\nu := \frac{u}{Ce} \qquad \nu = 0.1739$

$\eta_p := \frac{2 \cdot \nu}{1 + \nu^2} \qquad \eta_p = 0.3375$

e. Overall efficiency:

$\eta_o := \frac{T \cdot u}{mdot \cdot Q_R} \qquad \eta_o = 0.054$

OR - $\eta_o := \eta_p \cdot \eta_{int} \qquad \eta_o = 0.0529$

f. Specific impulse: $I_{sp} := \frac{Ce}{g} \qquad I_{sp} = 234.5848\ s$

g. Specific propellant consumption: $TSFC_w := \frac{1}{I_{sp}} \qquad TSFC_w = 0.0042629 \frac{1}{s}$

8. A rocket has an effective exhaust velocity of 7000 ft/sec, consumes 280 lbm/sec of propellant, which liberate 2400 BTU/lbm. The rocket operates for 65 seconds. If the flight velocity is 5000 ft/sec, calculate the following (from Sutton):

a. Internal efficiency
b. Propulsive efficiency (what is the overall efficiency? – how would you calculate it?)
c. Specific impulse (217.5 sec)
d. Total impulse (3,960,000 lbf/sec)
e. Weight of propellants required (18,200 lbf)
f. Volume propellants occupy if the average specific gravity is 0.925 (315 ft^3)

Given the following information from the problem statement:

$$Ce := 7000 \cdot \frac{ft}{sec} \qquad mdot := 280 \cdot \frac{lb}{sec} \qquad Q_R := 2400 \cdot \frac{BTU}{lb}$$

$$t_{burn} := 65 \cdot sec \qquad u := 5000 \cdot \frac{ft}{sec}$$

Find the following from above:

a. Internal efficiency:

$$KE_{jet} := 0.5 \cdot Ce^2 \qquad KE_{jet} = 2.2761 \times 10^6 \frac{J}{kg}$$

$$\eta_{int} := \frac{KE_{jet}}{Q_R} \qquad \eta_{int} = 0.4077$$

b. Propulsive efficiency:

$$\eta_p := \frac{Ce \cdot u}{Ce \cdot u + 0.5 \cdot (Ce - u)^2} \qquad \eta_p = 0.9459$$

or you can...

$$\nu := \frac{u}{Ce} \qquad \nu = 0.7143$$

$$\eta_p := \frac{2 \cdot \nu}{1 + \nu^2} \qquad \eta_p = 0.9459$$

c. Specific impulse:

$$I_{sp} := \frac{Ce}{g} \qquad I_{sp} = 217.5667\ s$$

d. Total impulse:

$$I_t := mdot \cdot Ce \cdot t_{burn} \qquad I_t = 3.95971 \times 10^6\ lbf \cdot sec$$

$$I_t := I_{sp} \cdot mdot \cdot t_{burn} \cdot g \qquad I_t = 3.9597 \times 10^6\ lbf \cdot sec$$

e. Weight of propellants: $w_p := mdot \cdot g \cdot t_{burn}$ $w_p = 18200\ lbf$

f. Volume of propellants: $SG := 0.925$ $\rho_{h2o} := 62.4 \cdot \frac{lb}{ft^3}$

$\rho_{prop} := SG \cdot \rho_{h2o}$ $\rho_{prop} = 57.72\ \frac{lb}{ft^3}$

$V_{prop} := \frac{mdot \cdot t_{burn}}{\rho_{prop}}$ $V_{prop} = 315.3153\ ft^3$