Assignment 3: Propellers

1. An aircraft weighing 1000 lb has a propeller of 7 ft diameter and L/D = 5. The aircraft is flying at 2000 ft altitude in a 10 mph head wind. If the actual propeller efficiency is 80% (0.8) and the aircraft's relative ground speed is 90 mph, find the brake shaft horsepower (bhp) and estimate the slipstream velocity in mph.

Given the following:

$W_{ac} := 1000 \cdot lbf$ $\quad LoverD := 5$ $\quad V_{hw} := 10 \cdot mph$ $\quad V_{hw} = 14.667 \frac{ft}{sec}$

$h := 2000 \cdot ft$ $\quad \rho := 2.241 \cdot 10^{-3} \cdot \frac{slug}{ft^3}$ $\quad V_g := 90 \cdot mph$ $\quad V_g = 132 \frac{ft}{sec}$

$\eta_p := 0.80$ $\quad D_p := 7 \cdot ft$

For steady flight, L = W and T = D. This means we can find required thrust:

$$T_{ac} := W_{ac} \cdot \frac{1}{LoverD} \qquad T_{ac} = 200\, lbf$$

$$P_{req} := T_{ac} \cdot \left(V_g + V_{hw}\right) \qquad P_{req} = 53.333\, hp$$

The shaft horsepower is the power require divided by the propeller efficiency:

$$P_S := \frac{P_{req}}{\eta_p} \qquad P_S = 66.667\, hp$$

The slipstream velocity can be estimated using the momentum theory:

$$b := -1 + \sqrt{1 + \frac{8 \cdot T_{ac}}{\rho \cdot \pi \cdot D_p^{\,2} \cdot \left(V_g + V_{hw}\right)^2}}$$

$$b = 0.103$$

After solving for b we can now find the slipstream increment and V_3.

$$v_b := b \cdot \left(V_g + V_{hw}\right) \qquad v_b = 15.04 \frac{ft}{sec}$$

$$V_3 := \left(V_g + V_{hw}\right) + v_b \qquad V_3 = 161.707 \frac{ft}{sec} \qquad V_3 = 110.255\ mph$$

$$V_3 := \left(V_g + V_{hw}\right) \cdot (1 + b) \qquad V_3 = 161.707 \frac{ft}{sec}$$

2. A small sport helicopter weighs 500 lbs with a single crew member weighing 185 lbs. The rotor is 15 ft in diameter with an efficiency of 0.84. The efficiency of all mechanical linkages and shafting is 0.9. If the helicopter is in steady flight hovering at sea level answer the following:

For a helicopter in hover: $T_R = 2\rho A_{disk} v_a^2$ and $P_R = T_R v_a$ (v_a is analagous to $v_a = aV$)

Note: This is basically the momentum theory of propellers applied to helicopter rotors!

a. Find the velocity induced (v_a) by the rotor in ft/sec
b. Find the indicated power of the engine in hp
c. What is the pressure rise across the rotor in psi?
d. The velocity far downstream ($v_{infinity}$) of the rotor has the same relationship to v_a as the slipstream factor does to the inflow factor in our momentum theory of props. What is the downstream velocity?

a. The induced through the rotor can be found by equating the weight to the thrust required. We also need the area of the rotor from the diameter:

Rotor Area: $D_{rotor} := 15 \cdot ft$ $\quad A_{disk} := \frac{\pi}{4} \cdot D_{rotor}^2$

Density: $\rho := 0.002377 \cdot \frac{slug}{ft^3}$

Thrust: $T_R := 500 \cdot lbf + 185 \cdot lbf$ where T_R is as above

Efficiencies: $\eta_p := 0.84$ $\quad \eta_{mech} := 0.9$

Rearranging the thrust to get v_a

$$v_a := \sqrt{\frac{T_R}{2 \cdot \rho \cdot A_{disk}}} \qquad v_a = 28.555 \frac{ft}{sec}$$

b. The thrust power or required power is

$$P_R := T_R \cdot v_a \qquad P_R = 35.564\, hp$$

To get the shaft power we divide the above quantity by η_p. To get the indicated power we divide again by the mechanical efficiency:

$$IP := \frac{P_R}{\eta_p \cdot \eta_{mech}} \qquad IP = 47.042\ hp$$

c. The pressure change across the disk is obtained using T = ΔP x A_{disk}.

$$\Delta P := \frac{T_R}{A_{disk}} \qquad \Delta P = 3.876\ \frac{lbf}{ft^2}$$

d. Finally, the downwash velocity is simply twice the velocity at the disk:

$$v_b := 2 \cdot v_a \qquad v_b = 57.11\ \frac{ft}{sec}$$

3. A propeller with diameter 15 ft. produces a thrust of 8000 lbf while moving at a speed of 400 mph at an altitude where density is 0.002 slug/ft^3. Using the momentum theory determine:

a. Induce flow through the disk (18.7 ft/sec)
b. The final wake velocity (624.4 ft/sec)

a. To find the flow induced through the disk we use the momentum theory thrust equation and solve for the slipstream increment b and thereby get a since a = b/2.

Given the following data:

Prop diameter: $D_p := 15 \cdot ft$

Flight speed: $V_1 := 400 \cdot mph$

Thrust: $T_R := 8000 \cdot lbf$

Density: $\rho := 0.002 \cdot \frac{slug}{ft^3}$

Solving for b we get:

$$b := -1 + \sqrt{1 + \frac{8 \cdot T_R}{\pi \cdot \rho \cdot V_1^2 \cdot D_p^2}} \qquad b = 0.064$$

$$a := \frac{b}{2} \qquad a = 0.032$$

Therefore the induced flow at the disk is:

$$v_a := a \cdot V_1 \qquad v_a = 18.696 \frac{ft}{sec}$$

b. The final wake velocity is simply $V_1 + V_1 * b$

$$V_3 := V_1 \cdot (1 + b) \qquad V_3 = 624.058 \frac{ft}{sec}$$

Chapter 3, Problem 4. After touchdown a 10 ft diameter propeller is being used to provide reverse thrust at sea level conditions. Assume that at a speed of 89 knots on the runway the air flow through the propeller disk is reduced to zero speed in the wake of the propeller. Using the Froude theory. predict the deceleration force.

Given the following: $D_p := 10 \cdot ft$ $\quad \rho := 1.225 \cdot \frac{kg}{m^3}$ $\quad \rho = 2.377 \times 10^{-3} \frac{slug}{ft^3}$

$V_1 := 89 \cdot knot$ $\quad V_1 = 150.215 \frac{ft}{sec}$

$V_3 := 0 \cdot \frac{ft}{sec}$

Assume the Froude theory applies.

Solution:

$$A_{disk} := \frac{\pi}{4} \cdot D_p^{\,2} \qquad A_{disk} = 78.54\ ft^2$$

If the wake velocity must be zero, then the slipstream velocity increment must in 100% of the freestream velocity - or - b = 1.0.

$b := 1$ or one can say: $\quad v_b := V_1 \quad v_a := \frac{1}{2} \cdot v_b$

$$T_{rev} := \rho \cdot A_{disk} \cdot V_1^{\,2} \cdot \left(1 - \frac{b}{2}\right) \cdot b \qquad T_{rev} = 2.106 \times 10^3\ lbf$$

$$T_{rev} := \frac{1}{2} \cdot \rho \cdot A_{disk} \cdot V_1^{\,2} \cdot \left(2 \cdot b - b^2\right) \qquad T_{rev} = 2.106 \times 10^3\ lbf$$

$$T_{rev} := \rho \cdot A_{disk} \cdot \left(V_1 - v_a\right) \cdot v_b \qquad T_{rev} = 2.106 \times 10^3\ lbf$$

Chapter 3, Problem 5: A piston driven aircraft with 200 hp has a direct drive6 ft propeller running at 2700 RPM's. The static thrust coefficient is 0.125. Find the following:

a. Static thrust
b. Propeller tip speed
c. Propeller torque
d. Propeller slipstream speed
e. Diameter of ducted fan with the same static thrust

Given the following:

Prop diameter: $D_p := 6 \cdot ft$ $\rho := 1.225 \cdot \frac{kg}{m^3}$

Shaft power: $P_S := 200 \cdot hp$

RPM's: $N_P := 2700 \cdot \frac{1}{min}$ $n := N_P$

Static thrust coefficient: $C_{To} := 0.125$

Part a: Using the definition of the thrust coefficient:

$$T_R := \rho \cdot n^2 \cdot D_p^{\,4} \cdot C_{To} \qquad T_R = 779.74\ lbf$$

Part b: Find the tip speed using the $r\omega$ or $2\pi nr$:

$$V_{tip} := 2 \cdot \pi \cdot n \cdot \frac{D_p}{2} \qquad V_{tip} = 848.23\ \frac{ft}{sec}$$

Part c. To find the torque, we can use the definition of the power coefficient ($C_P = 2\pi C_Q$):

$P = 2 \cdot \pi \cdot n \cdot Q$ so Q is...

$$Q := \frac{P_S}{2 \cdot \pi \cdot n} \qquad Q = 389.045\ ft \cdot lbf$$

Part d. To find the slipstream velocity we need the static thrust version of the momentum theory equation discussed in class wrt the hovering helicopter:

$$T_R = \frac{1}{2} \cdot \rho \cdot A_{disk} \cdot v_b^{\,2}$$

$$v_b := \sqrt{\frac{8 \cdot T_R}{\rho \cdot \pi \cdot D_p^{\,2}}}$$

$$v_b = 152.331 \ \frac{ft}{sec}$$

Part e. The relationship (via the momentum theory) of the ducted fan diameter (in static thrust) to the unducted propeller diameter is Df ~ 1.4(Dp).

$$D_{fan} := \frac{1}{\sqrt{2}} \cdot D_p \qquad\qquad D_{fan} = 4.243 \ ft$$

Chapter 3, Problem 6: Given the following: A four engine turboprop flies at 200 knots at the following conditions using a propeller as specified by figure 1.

Flight speed: $V_1 := 200 \cdot knot \qquad V_1 = 102.889 \frac{m}{s}$

Density ratio: $\sigma := 0.5 \qquad \rho := \sigma \cdot 1.225 \cdot \frac{kg}{m^3}$

Propeller diameter: $D := 3.5 \cdot m$

Engine RPM: $N_e := 27272 \cdot \frac{1}{min}$

Propeller RPM (GR = 0.044): $N_p := 0.044 \cdot N_e \qquad N_p = 1.19997 \times 10^3 \frac{1}{min}$

or you can say: $n := N_p$

Propeller data as provided:

$$C_{s_chart} := \begin{pmatrix} 0.5 & 0.22 & 0.55 \\ 1 & 0.62 & 0.74 \\ 1.5 & 1 & 0.82 \\ 2 & 1.39 & 0.83 \\ 2.5 & 1.78 & 0.81 \end{pmatrix}$$

Split out individual vectors: $C_S := C_{s_chart}^{\langle 0 \rangle} \qquad J_p := C_{s_chart}^{\langle 1 \rangle} \qquad \eta_p := C_{s_chart}^{\langle 2 \rangle}$

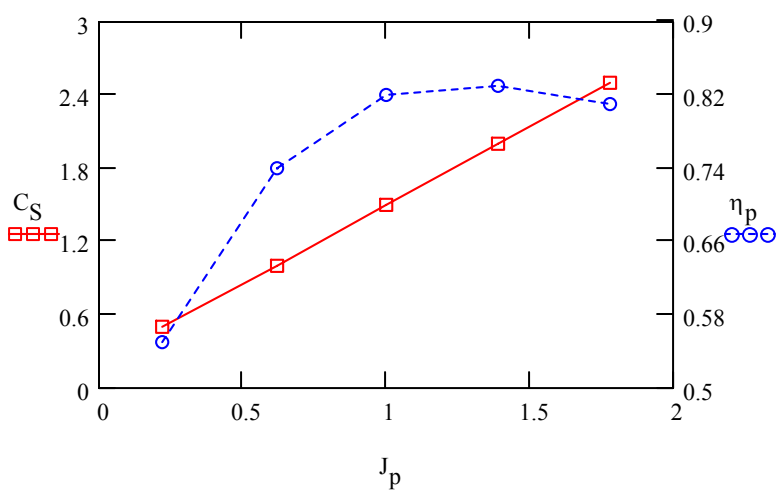

Figure 1. Cs and efficiency versus J

Find the following: (a) Aircraft Drag (T) and (b) Weight for L/D = 15 $\quad LoverD := 15$

a. Us the definition of C_S as:

We have enough information to find J: $J_{inf} := \frac{V_1}{n \cdot D}$ $\quad J_{inf} = 1.47$

Using the data we linear interpolate to get the η and C_S value:

$$\eta_{pJ} := \frac{J_{inf} - J_{p_3}}{J_{p_4} - J_{p_3}} \cdot \left(\eta_{p_4} - \eta_{p_3}\right) + \eta_{p_3} \qquad \eta_{pJ} = 0.826$$

$$C_{SJ} := \frac{J_{inf} - J_{p_3}}{J_{p_4} - J_{p_3}} \cdot \left(C_{S_4} - C_{S_3}\right) + C_{S_3} \qquad C_{SJ} = 2.102$$

With CS you can now find CP, power and thrust:

$$C_p := \left(\frac{J_{inf}}{C_{SJ}}\right)^5 \qquad C_p = 0.167$$

$$P := C_p \cdot \rho \cdot n^3 \cdot D^5 \quad P = 4.29857 \times 10^5\ W \quad P = 576.45\ hp$$

$$T_{ac} := \frac{\eta_{pJ} \cdot P}{V_1} \qquad T_{ac} = 3.45052 \times 10^3\ N$$

$$T_{ac} = 775.71\ lbf$$

b. The weight is found as:

$$W_{ac} := LoverD \cdot T_{ac} \cdot 4$$

$$W_{ac} = 2.070312 \times 10^5\ N \qquad W_{ac} = 46542.47\ lbf$$

Chapter 3, Problem 7 The following information is provided:

A three-bladed prop using NACA Report 640

Propeller design point: $C_S := 2$ $J_p := 1.21$ $\beta := 29 \cdot deg$ $\eta := 0.87$

Engine shaft power: $P_{abs} := 315 \cdot 10^3 \cdot W$ $P_{abs} = 422.422\ hp$

Engine RPM: $N_e := 2200 \cdot \frac{1}{min}$

Pressure/Temperature altitude: $\delta := 0.69$ $t := 0.93$

V_{tip} must be less than 220 m/sec $V_{tip} := 220 \cdot \frac{m}{sec}$ $V_{tip} = 721.785\ \frac{ft}{sec}$

Find the following: V_1 N_P, G.R., D, T, TV_1, Q, M_{tip}

Flight conditions:

Reference conditions and gas properties: $P_o := 14.7 \cdot psi$ $T_o := 518.67 \cdot R$

$\gamma := 1.4$ $R_{gas} := 53.3 \cdot \frac{ft \cdot lbf}{lb \cdot R}$

Pressure at altitude: $P_h := P_o \cdot \delta$ $P_h = 10.143\ psi$

Temperature at altitude: $T_h := t \cdot T_o$ $T_h = 482.363\ R$

Density from perfect gas law: $\rho_h := \frac{P_h}{R_{gas} \cdot T_h}$ $\rho_h = 1.766 \times 10^{-3}\ \frac{slug}{ft^3}$

$\rho_h = 0.057\ \frac{lb}{ft^3}$

Speed if sound and tip Mach number: $a := \sqrt{\gamma \cdot R_{gas} \cdot T_h}$ $a = 1.076 \times 10^3\ \frac{ft}{sec}$

$M_{tip} := \frac{V_{tip}}{a}$ $M_{tip} = 0.671$

a. Start by making use of the tip speed defined by: $V_R^2 = V_1^2 + (\pi \cdot n \cdot D)^2$

The quantity nD is: $nD := \frac{V_{tip}}{\pi}$ $\quad nD = 229.751 \frac{ft}{sec}$

Using the advance ratio: $V_1 := nD \cdot J_p$ $\quad V_1 = 277.999 \frac{ft}{sec}$ $\quad V_1 = 189.545\ mph$

The wheel speed (n) and gear ratio (GR) can be found using C_S:

$$C_S = \left(\frac{\rho V^5}{P n^2}\right)^{0.2}$$ so solving for n:

$$n_p := \sqrt{\frac{\rho_h \cdot V_1^5}{C_S^5 \cdot P_{abs}}} \qquad n_p = 1.191 \times 10^3 \frac{1}{min} \qquad n_p = 19.858 \frac{1}{s}$$

$$N_p := n_p$$

$$GR := \frac{N_e}{N_p} \qquad GR = 1.846$$

Since we know the quantity nD we can get D:

$$D_p := \frac{nD}{n_p} \qquad D_p = 3.526\ m \qquad D_p = 11.57\ ft$$

Using the definition of efficiency we can find T and TV: $\eta = \frac{P_A}{P_S}$ and $P_A = T_A \cdot V$

$$P_A := \eta \cdot P_{abs}$$

$$P_A = 2.74 \times 10^5\ W \qquad P_A = 367.507\ hp$$

$$T_A := \frac{P_A}{V_1} \qquad T_A = 727.085\ lbf$$

Get C_P from C_S and determine C_Q for finding Q:

$$C_S = \frac{J_p}{C_p^{0.2}}$$

$$C_p := \left(\frac{J_p}{C_S}\right)^5 \qquad C_p = 0.081$$

Torque is: $$Q := \frac{C_p}{2\cdot\pi}\cdot\rho\cdot n_p^{\,2}\cdot D_p^{\,5} \qquad Q = 1253.278\ \mathrm{ft\cdot lbf}$$

Chapter 3, Problem 9. The following informtion is provided for twin engine aircraft with constant speed propellers direct driven by 201 hp engines:

Aircraft Weight: $W_{ac} := 17.78 \cdot 10^3 \cdot N$ $\quad W_{ac} = 3.997 \times 10^3\ lbf$

Prop Diameter and Np: $D := 1.8 \cdot m$ $\quad N_p := 2700 \cdot \frac{1}{min}$ $\quad n := N_p$

Shaft Power: $P_s := 150 \cdot 10^3 \cdot W$ $\quad P_s = 201.153\ hp$

Static Thrust Coefficient: $C_{To} := 0.12$

Air Properties: $\rho_{air} := 1.225 \cdot \frac{kg}{m^3}$ $\quad \gamma := 1.4$ $\quad T_{air} := 288 \cdot K$ $\quad R_{air} := 53.3 \cdot \frac{ft \cdot lbf}{lb \cdot R}$

Speed of sound: $a := \sqrt{\gamma \cdot R_{air} \cdot T_{air}}$ $\quad a = 340.038\ \frac{m}{s}$

Find the following: T/W, Diameter of an equivalent ducted fan, tip Mach number of fan versus propeller.

Part a. Find T and T/W

$$T := C_{To} \cdot \rho_{air} \cdot n^2 \cdot D^4$$

$T = 3124.87\ N$ $\quad T = 702.499\ lbf$

$T_{total} := 2 \cdot T$ $\quad T_{total} = 6.25 \times 10^3\ N$

So the thrust-to-weight ratio is:

$$TtoW := \frac{T_{total}}{W_{ac}} \qquad TtoW = 0.352$$

Part b. For the equivalent fan dimater we take...

$$D_f := \frac{1}{\sqrt{2}} \cdot D \qquad D_f = 1.273\ m$$

Part c. For static conditions we need the linear speed in the plane of the propeller (the $r\omega$ term) and the induced flow at the disk. For static conditions we have the following thrust equations:

$$T_{prop} = \frac{1}{2}\cdot\rho\cdot A_p\cdot v_p^{\,2} \qquad T_{fan} = \rho\cdot A_f\cdot v_f^{\,2}$$

So we need to solve for the propeller and fan wake velocities using the single engine thrust of 3124.87 N from above:

$$v_p := \sqrt{\frac{2\cdot T}{\rho_{air}\cdot\frac{\pi}{4}\cdot D^2}} \qquad v_p = 44.776\ \frac{m}{s}$$

$$v_f := \sqrt{\frac{T}{\rho_{air}\cdot\frac{\pi}{4}\cdot D_f^{\,2}}} \qquad v_f = 44.776\ \frac{m}{s}$$

The $r\omega$ speed is found using the diameter and RPMs:

$$V_{p\omega} := \pi\cdot n\cdot D \qquad V_{p\omega} = 254.469\ \frac{m}{s}$$

$$V_{f\omega} := \pi\cdot n\cdot D_f \qquad V_{f\omega} = 179.937\ \frac{m}{s}$$

The resultant velocity seen by the tip is:

$$V_{pR} := \sqrt{v_p^{\,2} + V_{p\omega}^{\,2}} \qquad V_{pR} = 258.378\ \frac{m}{s}$$

$$V_{fR} := \sqrt{v_f^{\,2} + V_{f\omega}^{\,2}} \qquad V_{fR} = 185.424\ \frac{m}{s}$$

So the tip Mach numbers are:

$$M_p := \frac{V_{pR}}{a} \qquad M_p = 0.76$$

$$M_f := \frac{V_{fR}}{a} \qquad M_f = 0.545$$