Assignment 6: Gas Turbines II

1. Several F100 derivatives are being developed for multiple uses. The design throughput of the basic gas generator (i.e., turbojet) is $\dot{m}$ = 248 lbm/sec and the overall pressure ratio is 23. Calculate (by hand) the net thrust, specific thrust, TSFC, and engine efficiencies (η_p, η_{th}, η_o) for sea level, static conditions. The following component parameters apply:

Diffuser:	$\eta_d = 1$	$\pi_d = 1$	
Compressor:	$\eta_c = 0.87$	$\pi_c = 23$	
Burner:	$\eta_b = 0.97$	$\pi_b = 1$	H_f = 18,500 BTU/lbm
Turbine:	$\eta_T = 0.92$		T_{T4} = 2,460 deg R
	$\eta_{mech,T} = 0.98$		
Nozzle:	$\eta_N = 1$	$\pi_N = 1$	

Assume c_p = constant = 6006 ft-lb/slug/deg R and γ = constant = 1.4. **You must show all your work!!**

2. Now add an afterburner to the base engine of problem 1 and repeat your calculations. The afterburner efficiency is 90%, the afterburner pressure ratio is 0.94, and the maximum allowable temperature is 3,200 deg R. Determine the percent change in total fuel flow rate, T, and $TSFC$.

3. Now add a power turbine to the base engine of problem 1 to make it a turboshaft mated to an electric generator. Repeat your calculations. The power turbine efficiency is 89%. Assume the power turbine has the same mechanical efficiency as the turbine. If the efficiency of the electric generator is 75%, determine the maximum available electric power in kW (HINT: Consider what V_7 should be for maximum power extraction, and then assume that the power turbine expands the flow to an exit pressure of 5 psig, just enough to push the exhaust gases out of the exhaust stack).

4. Now add fan to the base engine of problem 1 to make an unmixed-flow, low-bypass ratio, two-spool turbofan. Repeat your calculations for the same overall pressure ratio of 23. The bypass ratio is β = 0.4, the fan pressure ratio is 3.8, the fan efficiency is 85%, and the fan turbine efficiency is 89%. Assume the fan turbine has the same mechanical efficiency as the turbine. Determine the percent change in T and $TSFC$.

1. F100-PW-229 derivative turbojet

Static test: $M_0 = 0$

$P_{T_0} = 2116 \text{ psf} = P_0$

$T_{T_0} = 519\ °R = T_0$

$\dot{m} = 248 \text{ lb/sec} = 7.702 \text{ slug/sec}$

$H_f = 18{,}500 \text{ BTU/lbm} = 4.636 \times 10^8 \ \frac{\text{ft-lb}}{\text{slug}}$

$P_{T_2} = P_{T_0} \cdot \pi_d = 2116(1) = 2116 \text{ psf}$

$P_{T_3} = P_{T_2} \cdot \pi_c = 2116(23) = 48{,}673 \text{ psf}$

$P_{T_4} = P_{T_3} \cdot \pi_b = 48{,}673(1) = 48{,}673 \text{ psf}$

$$T_{T_2} = T_0\left(1 - \frac{1}{\eta_d}\right) + \frac{T_{T_0}\, \pi_d^{\frac{\gamma-1}{\gamma}}}{\eta_d}$$

$$= 519(1-1) + \frac{519(1)}{(1)} = 519\ °R$$

$$T_{T_3} = T_{T_2}\left(1 + \frac{\pi_c^{\frac{\gamma-1}{\gamma}} - 1}{\eta_c}\right)$$

$$= 519\left(1 + \frac{23^{\frac{\gamma-1}{\gamma}} - 1}{0.87}\right) = 1{,}383\ °R$$

$T_{T_4} = 2460\ °R$

$$f = \frac{C_p T_{T_4} - C_p T_{T_3}}{\eta_b H_f - C_p T_{T_4}} = \frac{T_{T_4} - T_{T_3}}{\frac{\eta_b H_f}{C_p} - T_{T_4}}$$

$$f = \frac{2{,}460 - 1{,}383}{\frac{(0.97)(4.636\times10^{8})}{6006} - 2{,}460} = 0.0149$$

$$\pi_T = \left[1 - \frac{c_p T_{T2}\left(\pi_c^{\frac{\gamma-1}{\gamma}} - 1\right)}{(1+f) c_p \eta_c \eta_T \eta_{mech} T_{T4}}\right]^{\frac{\gamma}{\gamma-1}}$$

$$= (1-0.3839)^{3.5} = 0.6161^{3.5} = 0.1836$$

$$P_{T5} = P_{T4} \cdot \pi_T = 48{,}673(0.1836) = 8{,}934 \text{ psf}$$

$$T_{T5} = T_{T4}\left[1 - \eta_T\left(1 - \pi_T^{\frac{\gamma-1}{\gamma}}\right)\right]$$

$$= 2{,}460\left[1 - 0.92(1 - 0.6161)\right] = 1{,}591\ °R$$

$$P_{T7} = P_{T6} = P_{T5} = 8{,}934 \text{ psf}$$

$$T_{T7} = T_{T6} = T_{T5} = 1{,}591\ °R$$

$$\frac{P_0}{P_{T7}} = \frac{2116}{8934} = 0.2369 < 0.5283$$

It's choked at the exit.

$$P_7 = 0.5283 \cdot P_{T7} = 0.5283(8934) = 4{,}720 \text{ psf}$$

$$T_7 = 0.8333 \cdot T_{T7} = 0.8333(1591) = 1{,}326\ °R$$

$$\rho_7 = \frac{4{,}720}{1716(1326)} = 0.00207 \text{ slug/ft}^3$$

$$\dot m_7 = (1+f)\dot m = 1.0149(7.702) = 7.816 \text{ slug/sec}$$

$$V_7 = \sqrt{2 c_p \eta_N T_{T7}\left[1-\left(\frac{P_7}{P_{T7}}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

$$= \sqrt{2(6006)(1)(1591)\left[1-0.8333\right]} = 1{,}785 \text{ fps}$$

$$A_7 = \frac{\dot{m}_7}{\rho_7 V_7} = \frac{7.816}{(0.00207)(1785)} = 2.111 \text{ ft}^2$$

$$T_{net} = \dot{m}_7 V_7 - \dot{m} V_0 + (P_7 - P_0) A_7$$

$$= (7.816)(1785) - (7.702)(0) + (4720-2116)\cdot(2.111)$$

$$= 13{,}951 - 0 + 5{,}497 = \boxed{19{,}448 \text{ lbs}}$$

$$T/\dot{m} = \frac{19448}{7.702} = \boxed{2{,}525 \text{ ft/sec}}$$

$$TSFC = \frac{f}{T/\dot{m}} = \frac{0.0149}{2525} \times 3600 \times 32.2 = \boxed{0.683 \frac{\text{lb}}{\text{lb-hr}}}$$

Because $V_0 = 0$, $\eta_P = \eta_o = 0$

$$C = V_7 + \frac{(P_7 - P_0) A_7}{\dot{m}_7} = 1785 + \frac{(4720-2116)(2.111)}{7.816}$$

$$= 2{,}488 \text{ fps}$$

$$\eta_{th} = \frac{(1+f)C^2 - V_0^2}{2 f H_f} = \frac{1.0149(2488)^2}{2(0.0149)(4.636 \times 10^8)} = \boxed{0.456}$$

2 Up to P_{T5}, T_{T5} everything is the same.

$$P_{T6} = P_{T5} \cdot \pi_{AB} = 8934\,(0.94) = 8{,}398 \text{ psf}$$

$$T_{T6} = 3{,}200\,°R$$

$$f_{AB} = (1+f)\frac{C_p T_{T6} - C_p T_{T5}}{\eta_{AB} H_f - C_p T_{T6}} = (1+f)\frac{T_{T6} - T_{T5}}{\frac{\eta_{AB} H_f}{C_p} - T_{T6}}$$

$$= \frac{1.0149\,(3200 - 1591)}{\frac{0.90(4.636\times10^8)}{6006} - 3200} = 0.0246$$

$$P_{T7} = P_{T6} = 8{,}398 \text{ psf}$$

$$T_{T7} = T_{T6} = 3{,}200\,°R$$

$$\frac{P_0}{P_{T7}} = \frac{2116}{8398} = 0.2520 < 0.5283 \quad \text{CHOKED!}$$

$$P_7 = 0.5283\,(8398) = 4{,}437 \text{ psf}$$

$$T_7 = 0.8333\,(3200) = 2667\,°R$$

$$\rho_7 = \frac{4437}{1716(2667)} = 9.695\times10^{-4} \text{ slug/ft}^3$$

$$\dot{m}_7 = (1+f+f_{AB})\dot{m} = (1+0.0149+0.0246)(7.702)$$

$$= 8.006 \text{ slug/sec}$$

$$V_7 = \sqrt{2(6006)(1)(3200)(1-0.8333)} = 2{,}531 \text{ fps}$$

$$A_7 = \frac{\dot{m}_7}{\rho_7 V_7} = \frac{8.006}{(9.695\times10^{-4})(2531)} = 3.263 \text{ ft}^2$$

$$T_{net} = (8.006)(2531) + (4437-2116)(3.263)$$

$$= 20{,}264 + 7{,}570 = \boxed{27{,}835 \text{ lbs}}$$

$$T/\dot{m} = \frac{27{,}835}{7.702} = \boxed{3{,}614 \text{ fps}}$$

$$TSFC = \frac{f + f_{AB}}{T/\dot{m}} = \frac{0.0149 + 0.0246}{3614} \times 3600 \times 32.2$$

$$= \boxed{1.268 \ \frac{\text{lb}}{\text{lb-hr}}}$$

$$C = 2531 + \frac{(4437-2116)(3.263)}{8.006} = 3{,}477 \text{ fps}$$

$$\eta_{th} = \frac{(1+0.0149+0.0246)(3477)^2 - 0}{2(0.0149+0.0246)(4.636\times10^8)} = \boxed{0.343}$$

Thus...

$\dot{m}_f$ increases 165%

T increases 43%

TSFC increases 86%

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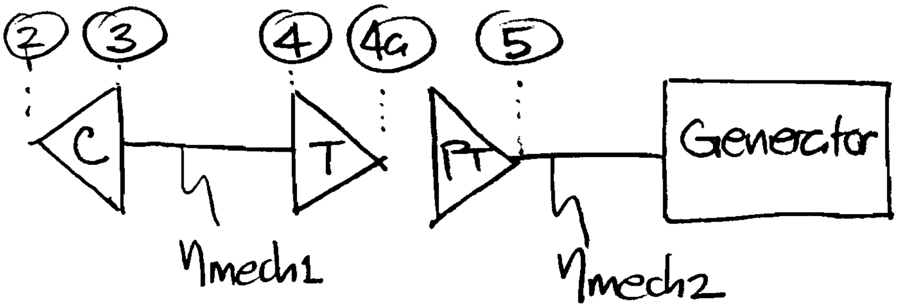

P_{T4a} = former P_{T5} = 8,934 psf

T_{T4a} = former T_{T5} = 1,591 °R

$V_7 \approx 0$ so $P_{T7} = P_7 = 2116 + 5(144)$

$= 2,836$ psf

If $P_{T5} = P_{T6} = P_{T7} = 2,836$ psf,

then $\pi_{PT} = \dfrac{P_{T5}}{P_{T4a}} = \dfrac{2,836}{8,934} = 0.3174$

$$T_{T5} = T_{T4a}\left[1 - \eta_{PT}\left(1 - \pi_{PT}^{\frac{\gamma-1}{\gamma}}\right)\right]$$

$$= 1591\left[1 - 0.89\left(1 - 0.3174^{\frac{\gamma-1}{\gamma}}\right)\right]$$

$= 1,182$ °R

$$W_{shaft} = \eta_{mech2} C_p (T_{T4a} - T_{T5})$$

$$= 0.98(6006)(1591 - 1182) = 2.407 \times 10^6 \ \frac{ft\ lb}{slug}$$

$$w_{elect} = \eta_{elect} \, w_{shaft}$$

$$= 0.75\,(2.407\times10^{6}) = 1.806\times10^{6}\ \frac{ft\,lb}{slug}$$

$$\dot{W}_{elect} = w_{elect}\,\dot{m} = (1.806\times10^{6})(7.702)$$

$$= 1.391\times10^{7}\ \frac{ft\,lb}{sec} = \boxed{18.86\ MW}$$

$$= 25{,}291\ HP$$

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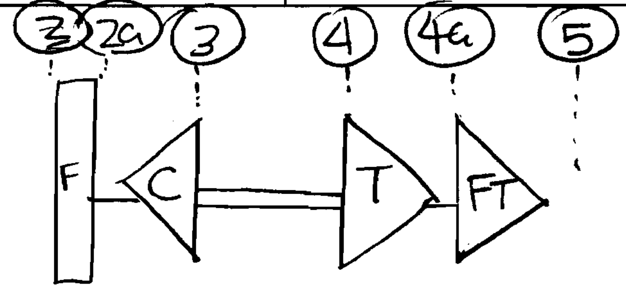

$$\pi_F = 3.8$$

$$\pi_c = \frac{23}{\pi_F} = \frac{23}{3.8} = 6.053$$

$$P_{T2a} = P_{T2} \cdot \pi_F = 2116(3.8) = 8{,}041 \text{ psf}$$

$$P_{T3} = 48{,}673 \text{ psf}$$

$$P_{T4} = 48{,}673 \text{ psf}$$

$$T_{T2a} = T_{T2}\left(1 + \frac{\pi_F^{\frac{\gamma-1}{\gamma}} - 1}{\eta_F}\right)$$

$$= 519\left(1 + \frac{3.8^{\frac{\gamma-1}{\gamma}} - 1}{0.85}\right) = 802.5\ °R$$

$$T_{T3} = T_{T2a}\left(1 + \frac{\pi_c^{\frac{\gamma-1}{\gamma}} - 1}{\eta_c}\right)$$

$$= 802.5\left(1 + \frac{6.053^{\frac{\gamma-1}{\gamma}} - 1}{0.87}\right) = 1{,}423\ °R$$

$$T_{T4} = 2{,}460\ °R$$

$$f = \frac{2{,}460 - 1{,}423}{\frac{(0.97)(4.636\times10^8)}{6006} - 2460} = 0.0143$$

$$\Pi_T = \left[1 - \frac{C_p T_{T2a}\left(\Pi_c^{\frac{\gamma-1}{\gamma}} - 1\right)}{(1+f) C_p \eta_c \eta_T \eta_{mech1} T_{T4}}\right]^{\frac{\gamma}{\gamma-1}}$$

$$= (1 - 0.2758)^{3.5} = 0.7242^{3.5} = 0.3232$$

$$P_{T4a} = P_{T4} \cdot \Pi_T = (48{,}673)(0.3232) = 15{,}730 \text{ psf}$$

$$T_{T4a} = T_{T4}\left[1 - \eta_T\left(1 - \Pi_T^{\frac{\gamma-1}{\gamma}}\right)\right]$$

$$= 2460\left[1 - 0.92(1 - 0.7242)\right]$$

$$= 1{,}836\ °R$$

[NOTE: Assumes core flow also goes through the fan!]

$$\Pi_{FT} = \left[1 - \frac{(1+\beta) C_p T_{T2}\left(\Pi_F^{\frac{\gamma-1}{\gamma}} - 1\right)}{(1+f) C_p \eta_F \eta_{FT} \eta_{mech2} T_{T4a}}\right]^{\frac{\gamma}{\gamma-1}}$$

$$= (1 - 0.2444)^{3.5} = 0.7556^{3.5} = 0.3750$$

$$T_{T5} = T_{T4a}\left[1 - \eta_{FT}\left(1 - \Pi_F^{\frac{\gamma-1}{\gamma}}\right)\right]$$

$$= 1{,}436\ °R$$

$$P_{T5} = P_{T4a} \cdot \Pi_{FT} = 15{,}730\,(0.3750) = 5{,}899 \text{ psf}$$

$$P_{T7} = P_{T6} = P_{T5} = 5{,}899 \text{ psf}$$

$$T_{T7} = T_{T6} = T_{T5} = 1{,}436\ °R$$

Core Exhaust

$$\frac{P_0}{P_{T7}} = \frac{2116}{5899} = 0.3587 < 0.5283$$
(CHOKED!)

$$P_7 = 0.5283(5899) = 3116 \text{ psf}$$

$$T_7 = 0.8333(1436) = 1197\ °R$$

$$\rho_7 = \frac{3116}{1716(1197)} = 0.001517 \text{ slug/ft}^3$$

$$\dot{m}_7 = 1.0143(7.702) = 7.812 \text{ slug/sec}$$

$$V_7 = \sqrt{2(6006)(1)(1436)(1-0.8333)} = 1{,}696 \text{ fps}$$

$$A_7 = \frac{7.812}{(0.001517)(1696)} = 3.036 \text{ ft}^2$$

$$T_{core} = (7.812)(1696) - 0 + (3116-2116)(3.036)$$
$$= 13{,}249 + 3{,}036 = \boxed{16{,}285 \text{ lbs}}$$

Less core thrust because of power drawn to drive the fan

Fan Exhaust

$$\frac{P_0}{P_{T2a}} = \frac{2116}{8041} = 0.2632 < 0.5283$$
(CHOKED)

$$P_8 = 0.5283(8041) = 4{,}248 \text{ psf}$$

$$T_8 = 0.8333(802.5) = 668.7\ °R$$

$$\rho_8 = \frac{4248}{(1716)(669)} = 0.003702 \text{ slug/ft}^3$$

$\dot{m}_8 = \beta \cdot \dot{m} = 0.4(7.702) = 3.081$ slug/sec

$V_8 = \sqrt{2(6006)(1)(802.5)(1-0.8333)} = 1,268$ fps

$$A_7 = \frac{3.081}{(0.003702)(1268)} = 0.6564 \text{ ft}^2$$

$$T_{fan} = (3.081)(1268) - 0 + (4248-2116)(0.6564)$$

$$= 3,907 + 1,399 = \boxed{5,306 \text{ lbs}}$$

$$T_{total} = T_{core} + T_{Fan} = 16,285 + 5,306$$

$$= \boxed{21,591 \text{ lbs}}$$

↳ More thrust

$$\frac{T}{\dot{m}} = \frac{21,591}{7.702} = \boxed{2,803 \text{ fps}}$$

$$TSFC = \frac{f}{T/\dot{m}} = \frac{0.0143}{2803} \times 3600 \times 32.2 = \boxed{0.591 \frac{\text{lb}}{\text{lb-hr}}}$$

$$T = \dot{m}_7 C - \dot{m} V_0$$

$$C = \frac{T + \dot{m} V_0}{\dot{m}_7} = \frac{21,591 + 0}{7.812} = 2,764 \text{ fps}$$

$$\eta_{th} = \frac{(1.0143)(2764)^2 - 0}{2(0.0143)(4.636 \times 10^8)} = \boxed{0.584}$$

Thus...

$\dot{m}_f$ decreases 4%

T increases 11%

TSFC decreases 13%